The Solution of Railway Problems on a Digital Computer: 1

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Summary: This paper gives an account of railway problems solved on the English Electric DEUCE during the last three years and contains a review of current developments.

The paper is divided into two Parts, the first dealing with engineering and design applications and the second with operational applications. Part 1, below, covers problems in the design of traction motors and rectifiers, the calculation of locomotive performance to meet certain given conditions and the study of loads on a power supply system produced by an a.c. traction network.

In Part 2, to be published in July, the problems dealt with are the calculation of point-to-point running times, the use of linear programming methods for minimizing empty wagon movement, and a major current development—the preparation of timetables on digital computers.

1 INTRODUCTION

An earlier paper (Gilmour, 1956) outlined the possibilities of using digital computers in problems associated with electric traction, and, in particular, described a program for calculating the expected performance of a train of given characteristics hauling a given load over a given route. Since that paper was written there have been numerous developments in the application of digital computers to railway problems and the purpose of the present paper is to give a factual account of the use of the English Electric DEUCE in this field, with particular reference to programming methods used. This will include a description of some programs that are still in course of preparation. Considerable background information has been given to each problem so that the reader who is unfamiliar with railway engineering and operational practice may appreciate some of the possibilities of using computers for solving railway problems.

Railway problems may be divided into two types: first, those relating to design of equipment, which chiefly concern railway manufacturing companies and the design and research departments of operating companies; and, second, operational and commercial problems which are the concern of railway operating companies.

2 DESIGN PROBLEMS

2.1 *Calculation of Train Performance*

Railway manufacturing companies who are asked to submit tenders for the supply of locomotives or motor-coaches for a particular railway company are usually faced with the following type of problem:

> To calculate the performance of a locomotive hauling a given weight of train over a given track, the locomotive being so designed that in normal conditions the train will cover the route in a given time with a certain margin of power to spare.

The information required by the customer will vary, but usually the speed, time from the starting-point and fuel or energy consumption are required for various points along the route. These are needed for what is known as "all out" conditions (i.e. the locomotive exerting maximum power except where limited by track restrictions), but they are sometimes necessary for normal scheduled running, in which case a certain amount of coasting must be introduced into the run.

The designers will base their initial design of a locomotive on an estimate of the powers and accelerations required for the particular duties given in the specification. When this design has been prepared, sufficient information will be available to calculate the performance of the equipment in detail.

The designers will be able to provide information of the pull (T) that the locomotive can exert corresponding to a speed (v), the resistance to motion of the locomotive (R) as a function of speed, and the weight of locomotive (M). The customer will have provided already the particulars of gradients and speed restrictions, the weight of wagons or coaches to be hauled (m), and their resistance to motion (r). The speed of the train at any point of the track and the time taken to reach this point can then be calculated by solving the differential equations of motion,

$$(1 + \gamma)(M + m)\, v \frac{dv}{ds} = T - R - r - \frac{(M + m)g}{n} \qquad (1)$$

and

$$\frac{ds}{dt} = v \qquad (2)$$

where v is speed, s is distance, the slope of the gradient is 1 in n (and varies discretely along the track), and g is the acceleration due to gravity. γ is a factor to take account of the inertia of the rotating masses of the train (e.g. motors, axles, wheels).

To calculate the energy or fuel consumption of the locomotive the designer must provide a curve of rate of energy consumption (w) against speed, and the energy consumption at time t is then given by

$$W = \int_0^t w\,dt.$$

In the case of d.c. equipment the designer provides a curve of motor current (I_m) against speed, and line current (I_L) is a multiple of I_m, the multiplying factor depending upon whether motors are connected in parallel, series parallel or series. The speeds at which the motor groupings are changed are specified by the designer. The calculation of energy consumption then is

$$W = \int_0^t I_L V_L dt$$

where V_L is the mean line voltage.

It is often necessary to obtain an estimate of motor heating to ensure that motors are not overheated during running, and occasionally customers specify certain calculations that must be performed to check on motor temperature. A close guide to temperatures expected in service is given by the root mean square of the motor or generator current, which over any time interval t_1 to t_2 is given by

$$I_{RMS} = \sqrt{\left(\int_{t_1}^{t_2} I_m^2 dt \Big/ \int_{t_1}^{t_2} dt\right)}.$$

Before 1939 these calculations were performed mainly by hand, by numerical or graphical step-by-step methods and, if energy consumption and R.M.S. current values were required, calculations for between two and ten miles of track could be performed in one hour. The speed of calculation depends, to a great extent, on the number of changes of gradient and speed restrictions, as at all these points some form of interpolation is required. Since inquiries from railway companies may require the performance of these computations for hundreds of miles of track for different loads, many weeks or months of work may be entailed. Even before 1939 one or two attempts had been made to mechanize these calculations, but during the last ten years, with the rapid development of analogue and digital computing machines, more manufacturers have been using mechanical and electronic aids for this work. A review of the use of special-purpose analogue machines and of a digital computer, DEUCE, for train performance calculations is contained in a recent paper (Graham and Van Dorp, 1957).

Two types of train performance programs have been developed on DEUCE. The first of these, which was described at the I.E.E. Convention on Digital Computers in April 1956 (ref. 4), used an interval of distance as the basic step for solving the differential equations (1) and (2), and solved these equations by the Runge Kutta method. Although the use of distance as independent variable had the advantage that no interpolation was necessary at gradient changes, it had a number of serious disadvantages. The main one was that at each step of the calculation the effect of braking had to be computed so that the train neither entered a speed restriction at too high a speed nor failed to stop at a station when required to do so. Also, since this program, which was one of the earliest engineering applications of DEUCE, was made, experience has shown that it could be very much improved, particularly in relation to the input of data. Nevertheless, in spite of these disadvantages, it was used for two years for the calculation of the performance of d.c. electric, a.c. electric and diesel electric trains relating to railway companies all over the

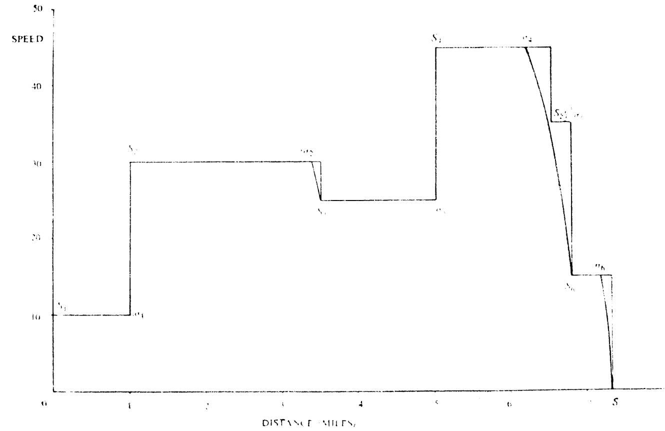

FIG. 1.—Speed restrictions and braking curves in speed/timing calculations.

world. In contrast with the speeds for hand computation already quoted, the speeds for this DEUCE program varied between 200 and 1500 miles per hour.

This program is now obsolete and has been replaced by a much faster one which uses speed as the independent variable. The problem of braking is solved by calculating the latest points on each speed restriction curve at which braking must commence so that the next restriction will be obeyed. This is done as the cards containing speed restriction data are being read into the machine. As Fig. 1 illustrates, it is not sufficient to find the point on one speed restriction at which braking must begin so that the next restriction is obeyed, because this might mean that the next restriction but one is not obeyed. The result of this stage of the program is that an envelope within which the speed distance curve must lie is stored in the machine, for example, in Fig. 1 the envelope would be formed by the straight lines and braking curves joining the points 0, S_1, σ_1, S_2, σ_2, S_3, σ_3, S_4, σ_4, S_6, σ_6, S. During the solution of the train motion equation, if a step of the computation crosses this envelope an interpolation is performed to find the crossing-point.

As a result of the methods of calculation adopted and various improvements in programming technique it has been possible to speed up the train performance program by a factor of four whilst increasing the scope of the calculation. In the earlier program for d.c. electric trains when the train was running at a constant speed due to the presence of a speed restriction, the energy consumption and motor current were calculated by assuming that constant speed was maintained by alternate steps of full power and coasting. In actual fact the driver of a d.c. electric locomotive maintains constant speed by notching between adjacent notches on his controller, and this action is now simulated in the new program.

As an example of the speed of this program, a recent calculation on the route Euston–Stafford took 90 seconds on the DEUCE, a speed of calculation of about 5000 m.p.h.

2.1.1 *Coasting*

When a customer requests details of speeds, times and energy consumptions for a normal schedule run with a make-up time of $x\%$, an all-out run, in which the locomotive exerts maximum power throughout except where restricted by track speed limits, is first calculated. For an efficient design meeting the customer's specification, the time for this run must be $x\%$ less than the normal schedule time. The usual way of computing the normal run is to introduce a certain amount of coasting into the run so that the time for each step-to-step section is increased by $x\%$. On a suburban run, this means finding points between stations at which to coast so that the interstation time is equal to the schedule time. This point is found by trial and error, and whether calculations are performed by hand or on a computer it can be quite a laborious process. On the earlier DEUCE program the position of the coasting-point was set up on hand switches, and at each step of the calculation the machine compared the distance travelled with the distance represented on the switches. If this point had been reached or passed the train would coast, otherwise it would run under full power. When the train stopped at a station a card was punched showing the time taken and energy consumption, and the computer stopped until the coasting-point for the next section was set up on the switches. At the end of the run the results were examined and, if the times were not the required ones, another set of coasting distances would be guessed and the whole calculation repeated.

With the new DEUCE program this is now an automatic process. A series of cards is read into the machine showing the all-out time, the make-up time required and a guess at the coasting-point for each section. The machine then performs the coasting calculation for the first section, compares the time obtained with the time required and, if these differ by more than a certain tolerance, modifies the coasting-point accordingly and repeats the calculation. When the required coasting-point has been obtained for the first section, the machine punches out a card containing the results for this section, and then performs a similar calculation for the next section.

2.2 *Traction Machine Design Calculations*

A digital computer can be used to produce the basic design data for train performance calculations and thus save the designer many hours of routine slide-rule work. A program to compute the relationships between tractive effort, speed, efficiency and motor current for traction motors is just being developed together with a similar program for diesel electric locomotives.

A program has been made for calculating the braking effort of an electric train which has a regenerative braking system. This braking system is often used on locomotive hauled trains on hilly routes where there are long down gradients. The motors act as generators to overcome the line voltage and pass current down the line to

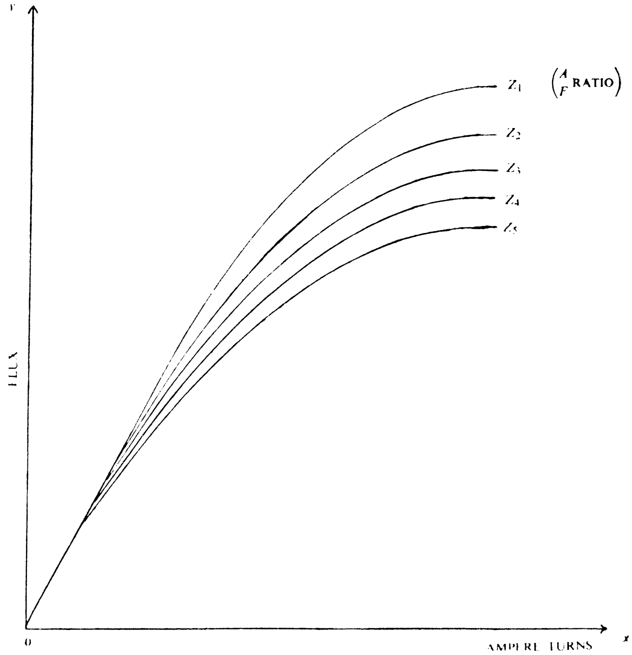

FIG. 2.—Saturation curves used in regenerative braking problem.

substations or to other trains that are taking power. The calculation of braking effort as a function of speed and current involves a lengthy series of computations, including much looking up of points on families of curves, such as motor saturation curves and loss curves. This is a type of problem which may only occur in a design office twice or three times a year but which may take an engineer a number of weeks to perform. The program for regenerative braking calculations was made by using the DEUCE General Interpretive Scheme. Although the time of operation of the program was slightly greater than it would have been if a special purpose one had been made, the time spent on making it was very much less. It was also a problem for which matrix methods were quite applicable since it was desirable to carry out similar calculations in parallel. Empirical data given in curve form was stored as a series of points, and for a single curve they would be stored as two (1 by n) vectors of x values and a vector of y values. For a family of curves, such as the motor saturation curves given in Fig. 2, the y values of each curve would be read at the same x values, but these need not be spaced at equal intervals and the curves would be stored in the machine as a (1 by n) vector of x's, a (1 by m) vector of z's and a (m by n) matrix of y's. A special "brick" for the Interpretive Scheme was written for looking up a series of y values corresponding to a set of x's and z's, and if these did not coincide with the stored points the corresponding y values were found by linear interpolation. Although the brick was specially made for the regenerative braking program it is clearly of great use in many engineering and scientific calculations. In addition, another special brick was made for finding the intersections of a single curve with a family of curves, and this also has more general application than that for which it was made.

This program is of particular interest because it is often thought that linear algebra interpretive schemes are only used in strictly linear algebra applications, e.g. problems involving the solution of simultaneous equations, matrix multiplication, matrix inversion or evaluation of latent roots of a matrix. This application shows the value of matrix methods in typical routine design office calculations.

2.3 *A.C. Electrification Problems*

The use of industrial frequency alternating current for the distribution of power to trains has grown rapidly in importance during the last ten years because of the great success achieved by the French railways and the decision of British Railways in 1956 to proceed with major electrification schemes using the 50 c/s single-phase system. One of the most favoured types of locomotive for a.c. electrification schemes is the locomotive containing rectifiers so that power is converted to d.c. on the locomotive, and the most efficient type of motor, the d.c. traction motor, can be used. In a paper to the Institution of Electrical Engineers (Calverley and Taylor, 1957) it has been shown that the approximate methods which have been used in the past for calculating the performance of multi-phase rectifiers, as used in industrial installations, may result in serious errors when applied to bi-phase and single-phase bridge connected rectifier circuits as used on rectifier locomotives. The authors have shown that a more rigorous method is necessary and describe a method involving the step-by-step solution of the differential equations representing the performance of the circuit. Numerous calculations of this form, using special purpose programs, have been performed on DEUCE and the paper referred to contains some interesting comparisons between rectifier performance predicted by the approximate method, by the rigorous method used in the DEUCE program, and by the actual performance recorded during tests carried out on the Lancaster–Morecambe–Heysham line of British Railways. One aspect of rectifier performance which is important to both the designer and user of rectifiers is the harmonic content of the output waveform. The digital computer was used for the harmonic analysis of both the calculated waveforms and those obtained from tests.

In planning a major electrification scheme at the industrial frequency it is necessary to know the effect that the loads taken at the various traction substations will have on the main industrial supply system. A particular problem of this nature, solved on DEUCE, has been the investigation of the degree of unbalance in a 3-phase supply system produced as a result of supplying single-phase power to a.c. traction substations. In this study a generation schedule was supplied and the voltage and current unbalances found for various types of loading occurring at the traction substations due to the different positions and power demands of trains at different times of the day. This was a matrix problem involving the manipulation and inversion of complex matrices. This is another type of problem showing the matrix interpretive scheme to great advantage. Once the preliminary mathematical analysis had been performed to set up the matrix equations for the system, the program was constructed in less than a day. The calculations for 20 power stations and 14 sub-stations for eight different types of substation loading were carried out on the DEUCE in $2\frac{1}{2}$ hours.

(*To be continued*)

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