It should also be possible to use a timetabling program for studying traffic flow problems. From time to time very bad hold-ups have occurred on the railways due to fog and other mishaps, and it should be possible to use this program to study the effects of delays at various points. In other words, the computer could be used to simulate the operation of a railway system and, because of the high speed of the machine, a great deal could be learnt about the way in which the system behaves which could only be learnt otherwise by years of experience.

4 CONCLUSIONS

Digital electronic computers are now regarded as essential tools in the newer industries of aircraft and atomic power engineering. The problems described in this paper show that there is great scope for the use of computers in a much older industry, the railway industry, and that eventually there may be as much need here for these machines as there is in the new industries.

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A Modified Congruence Method of Generating Pseudo-random Numbers

A widely used method of obtaining pseudo-random numbers is from the recurrence relation

$$u_{j+1} = ku_j \pmod{m}. \quad (1)$$

For binary computers it is convenient to have

$$m = 2^n \quad (2)$$

where n is the number of binary places available in the computer. The numbers produced from equation (1) must repeat themselves sooner or later. When $m = 2^n$ the length of the cycle is at most 2^{n-2}. In this note we discuss a simple elaboration of (1) which can give a cycle of 2^n.

We first recapitulate some of the properties of the sequence u_j as defined by equations (1) and (2). Not all of these have been found explicitly in the literature, but they are not difficult to prove and are presumably "known." To obtain the maximum cycle of 2^{n-2} it is necessary and sufficient that

$$k \equiv \pm 3 \pmod{8} \quad (3)$$

and that u_0, the arbitrary first number, is odd. [A commonly used form for k is an odd power of 5; since $5^{2n-1} = (1+4)^{2n-1} \equiv -3 \pmod{8}$, this is included in (3).] The fact that the cycle is of length 2^{n-2} is associated with the property that, in the n-digit binary representation of u_j, only $n-2$ digits change. The least-significant digit is always unity and the least but one, for $k \equiv -3 \pmod{8}$, or the least but two, for $k \equiv +3 \pmod{8}$, does not change.

The proposed modification consists in carrying out the operation (1) on a hypothetical $(n+2)$-digit number,

Continued on page 86