

and integral equations, and compares quite favourably with the other two methods under consideration in the computational examples we have considered. The general second-order differential equation  $y'' = P(x)y' + f(x)y + g(x)$  can also be treated by the above technique provided one uses either the well known transformation [5] to eliminate the  $y'$  term from the

above differential equation or treats the  $P(x)y'$  term by integration by parts, depending upon whether or not  $P(x)$  is explicitly integrable.

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#### Book Review

*Optimal Adaptive Control Systems*, by David Szwed, 1966; 187 pages. (New York: Academic Press, 68s.)

This book examines the mathematical formulation of the engineering problem of devising control algorithms in discrete time which are optimal in some sense. This takes into account the imprecision with which the characteristics of the controlled object are known initially. Two main alternative approaches are described: a Bayesian formulation in which *a priori* distributions are assigned to the values of the parameters and a "game theoretic formulation". In the latter the asymmetry of the man v. nature game is well pointed out and so, while the control problem is not a typical game, some existing results of game theory, particularly those related to the convexity of admissible sets, can be adapted to fit the problem.

The examples given are developed from conventional discrete time optimal control problems. An explicit solution is given to the problem of bringing a linear system to zero state with parameters specified initially by an *a priori* Gaussian distribution. The presentation draws upon the formalism of Fel'dbaum's "dual control theory" which is the subject of another monograph in the same series. Problems are treated later which involve variation of the parameters as Markov processes.

The main interest in the book is the demonstration of the tractability of certain quite sophisticated problems of control, but since these have been chosen through analytical considerations, it is unlikely that anyone will find direct help if he is seeking a solution to a practical adaptive problem.

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