The optimum arrangement of towers in an electric power transmission line

By J. C. Ranyard* and A. Wren†

A dynamic programming procedure is described which selects the sites and heights of towers in an overhead electric power transmission line in such a way that the construction costs are minimized, while the physical constraints are satisfied. The procedure has been programmed for a computer, and applied to an actual stretch of line. The results compare favourably with those obtained by current practice and implemented on the line considered. Several possible extensions of the method are suggested.

The object of this paper is to discuss a method of selecting the heights and locations of towers in an electric power transmission line in such a way that the construction costs of the line are minimized. The method was suggested by Shulman (1962), who does not report any implementation; it has been modified here to meet the specifications of the C.E.G.B. (Central Electricity Generating Board).

Once a route has been agreed for a transmission line, the C.E.G.B. divide it into straight sections. Within a section the conducting cables are supported by suspension towers which support the weight of the conductor, but are not designed to bear any horizontal force; the horizontal component of the tension in any section must therefore be constant. At the ends of each section there are tension towers, which are capable of withstanding a horizontal force, and so are stronger than the suspension towers, and therefore more expensive. There are several types of tension towers, dependent on the angle through which the line turns; for each type of tension tower, and for suspension towers, there are several different possible heights. When a straight line section is unusually long, tension towers may be sited at strategic points in order to take up any excess tension. For the present exercise it is assumed that the tension towers are fixed and that the problem is to site the suspension towers in the most economical way, as is normal C.E.G.B. practice. The cost of the conductors is assumed to be independent of the positions of the towers.

The following restrictions apply to a transmission line:

(i) A minimum clearance must be observed between the conductor and any point on the ground, and also between the conductor and buildings and other obstacles (clearance restriction).

(ii) The distance between adjacent towers must be less than a stipulated maximum (single span restriction).

(iii) The sum of the lengths of adjacent spans must be less than a stipulated maximum (double span restriction).

(iv) Every suspension tower must support at least 35% of the combined weight of the two spans it bears (weight restriction). This is to prevent excessive uplift in windy conditions. The weight supported by any tower is the weight of those parts of the conductor between the tower and the lowest points of the two adjacent spans. If it is assumed that the weight of the conductor is proportional to the horizontal component of its length, this restriction implies that the distance between the lowest points of two adjacent spans must be at least 35% of their combined length.

The above restrictions and the reasons for them are discussed qualitatively by Jackson (1961). The actual values of the clearances and maximum span lengths depend on the voltage of the line. The present study has been carried out on a 275 KV line. Data for an actual stretch of 10,975 feet was provided by the C.E.G.B. and used to test the final program.

Formulation of the problem

Let $\{H_p | p = 1, 2, \ldots, M\}$ be the set of M possible heights for suspension towers.

Let L be the length of the route.

Let E, F be the maximum single span and double span lengths respectively.

Let $k(x)$ be the minimum ground clearance required at point x.

Let $u(x)$, $v(x)$ be the elevations of the ground and conductor, respectively, above mean sea level.

Let $C(h, x)$ be the cost of erecting a suspension tower of height h at point x.

The problem is to choose a positive integer n, sites $x = s_1, s_2, \ldots, s_n$ and corresponding tower heights $h_1, h_2, \ldots, h_n$ so as to minimize the cost function

$$\sum_{i=1}^{n} C(h_i, s_i) \tag{1}$$

subject to $$h_i \epsilon H \qquad (i = 1, 2, \ldots, n) \tag{2}$$

$$0 = s_0 < s_1 < \ldots < s_n < s_{n+1} = L \tag{3}$$

* *Formerly at University of Leeds, now with National Coal Board.*
† *Electronic Computing Laboratory, The University, Leeds 2.*

a restriction on the length of a single span:

$$s_{i+1} - s_i \leqslant E \qquad (i = 0, 1, \ldots, n) \qquad (4)$$

a restriction on the combined length of two adjacent spans:

$$s_{i+1} - s_{i-1} \leqslant F \qquad (i = 1, 2, \ldots, n) \qquad (5)$$

a restriction on the clearance of the conductor:

$$v(x) \geqslant u(x) + k(x) \qquad (6)$$

a restriction on the proportion of two adjacent spans supported by the common tower:

$$y_i - y_{i-1} \geqslant 0 \cdot 35\,(s_{i+1} - s_{i-1}) \quad (i = 1, 2, \ldots, n) \quad (7)$$

where y_i is the x co-ordinate of the lowest point of the span (s_i, s_{i+1}).

The wire is assumed to hang between the towers in a parabolic arc, so that

$$v(x) = v(y_i) + A(x - y_i)^2$$

for $\quad s_i \leqslant x \leqslant s_{i+1} \qquad (i = 0, 1, \ldots, n) \qquad (8)$

where A is a constant depending on the tension in the conductor.

Taking the extreme points of the span, we get from (8)

$$v(s_i) = v(y_i) + A(s_i^2 - 2s_iy_i + y_i^2) \qquad (9)$$

$$v(s_{i+1}) = v(y_i) + A(s_{i+1}^2 - 2s_{i+1}y_i + y_i^2) \qquad (10)$$

and hence (subtracting)

$$y_i = \frac{s_i + s_{i+1}}{2} - \frac{v(s_i) - v(s_{i+1})}{2A(s_i - s_{i+1})}. \qquad (11)$$

Now,

$$v(s_i) = u(s_i) + h_i \qquad (12)$$

$$v(s_{i+1}) = u(s_{i+1}) + h_{i+1} \qquad (13)$$

and if the elevation of the ground at points s_i and s_{i+1} is known, we may use (11), (12) and (13) to obtain $v(y_i)$ from (9) for a given span and pair of tower heights, so that (6) will become

$$A(x - y_i)^2 \geqslant u(x) + k(x) - v(y_i)$$

for $\quad s_i \leqslant x \leqslant s_{i+1} \qquad (i = 0, 1, \ldots, n) \qquad (14)$

with y_i and $v(y_i)$ known for the span in question.

Similarly constraint (7) may be fully specified if the sites and heights of the supports of the two relevant spans are known.

The above formulation of the problem introduces two sources of error; one because a wire freely suspended between two supports hangs in a catenary and not in a parabolic arc, and the other because the point on the conductor nearest to any point on the ground is not vertically above it, as assumed in the set of inequalities (14).

Approximate problem and solution

In order to simplify the problem we restrict the possible tower sites to a set of discrete points,

$$\{R_j | j = 1, 2, \ldots, N\}$$

with $\quad 0 = R < R_1 < \ldots < R_N < R_{N+1} = L.$

We also assume that the height of the ground is known only at a set of Q discrete "elevation points", $\{E_q | \, q = 1, 2, \ldots, Q\}$, which will include the set $\{R_j\}$. The clearance of the conductor will be checked only at these points. This will introduce a further error if the conductor is nearer to the ground at any position between adjacent elevation points than at these points themselves. In practice the points on the ground which are most likely to violate clearance restrictions will be chosen as elevation points, thus minimizing this source of error. The total effect of all errors was calculated to be about eight inches, if elevation points were chosen at intervals of 100 feet; an appropriate amount was added to the necessary clearance to allow for this. The clearance required is taken as constant, k, along the entire line. The elevation of buildings and other obstacles was adjusted to take account of this, their positions or bounds being chosen as extra elevation points.

The cost of erecting a tower is assumed to be independent of its site; this assumption is made in practice, and may be justified to some extent by remarking that if the terrain is difficult, then costs will be high irrespective of the exact sites used.

The problem is now to choose a positive integer n $(< N)$, sites $s_1, s_2, \ldots, s_n$ from $\{R_j\}$, and corresponding tower heights $h_1, h_2, \ldots, h_n$ from $\{H_p\}$, so as to minimize a cost function

$$\sum_{i=1}^{n} C(h_i)$$

subject to conditions (2), (3), (4), (5), (7) and, from (14),

$$A(E_q - y_i)^2 \geqslant u(E_q) - v(y_i) + k$$

for all $\quad q \ni s_i \leqslant E_q \leqslant s_{i+1} \qquad (i = 0, 1, \ldots, n) \qquad (15)$

The details of the method will now be described.

The minimum costs of building lines from R_0, where a tension tower of known height is assumed to be sited, to all feasible heights at R_1 are determined. These costs are simply the costs of the different tower heights permitted at R_1.

Next, the costs of building lines from R_0 to all possible heights at R_2 are determined. Feasible solutions may involve lines with towers at R_1 or lines direct to R_2 from R_0; where alternative solutions exist for any tower height at R_2, the cheapest is chosen. Minimum cost solutions are built up for lines to all heights at each of the points $R_1, R_2, \ldots, R_N$, and finally to the fixed height tension tower at R_{N+1}.

When seeking the minimum cost route to any given height of tower at R_j, the minimum costs of building lines to all feasible tower heights at $R_0, R_1, \ldots, R_{j-1}$ are known. The method is then to determine which of these lines may be extended to the given tower height at R_j without violating any constraints. Of those which may be extended, that with the lowest associated cost is chosen; this cost, together with the cost of the tower under consideration at R_j is then the minimum cost of a line to this tower at R_j. When the minimum cost of a line to R_{N+1} has been found, the links forming this line are traced backwards to determine the sites and heights of the towers which are to be used.

This is a practical implementation of Bellman's principle of optimality (Bellman, 1957), which asserts that an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. In this case the "first decision" is the decision on the site and height of the penultimate tower (that ultimately linked to R_{N+1}); the initial state comprises the site and height of the tower at R_{N+1}. In order that the remaining decisions, those on the earlier towers in the line, should give an optimal policy no matter where the penultimate tower is chosen, the details of minimum cost lines from R_0 to all possible sites and heights are computed.

In exceptional cases the above formulation may not lead to the optimum solution of the approximate problem because of the nature of the double span restriction (5) and the weight restriction (7). In order to ensure that the optimum solution is obtained, suboptimal linkages should be retained at each stage in case future linkages are rejected solely because of (5) or (7). However, it was decided that the amount of computer time and storage necessary to retain and examine suboptimal links was unlikely to be justified by any saving in cost that was likely to be achieved. Subsequent experiments elsewhere (Mitra and Wolfenden, 1966) have confirmed this assumption.

The computer program

The method was programmed in Extended Pegasus Autocode (Barrett and Mitchell, 1963), the program being run on a Pegasus II computer. Lists of permissible tower heights, test sites (R_j) with elevations, and elevation points (E_q) are read into the machine together with details of the tension towers at the ends of the section. The minimum clearance, k, is given, as is the tension in the conducting cable at the external temperature at which the clearance is to be checked (specified by the C.E.G.B.); this tension is used to compute A, the constant for the parabola which most closely approximates to the curve of the conductor, used in constraint (15) and indirectly in constraint (7).

The minimum costs of lines to all possible towers at $R_1, R_2, \ldots, R_{N+1}$ are calculated and stored together with details of the site and height of the previous tower on each of these (minimum cost) lines. During computation it is only necessary to retain in the main store details of previous sites that might be linked to the current site, i.e. those within a distance equal to the maximum span length from the current site. Therefore in order to save internal storage, the details of minimum cost lines to towers are placed on magnetic tape as soon as they have been computed, and those which are no longer within range may be overwritten in the internal store by details of later towers.

In practice, spans of less than 600 feet are never used by the C.E.G.B., so to save time, the only sites, R_i, which are tentatively linked with R_j are those for which

$$600 \leqslant R_j - R_i \leqslant E \tag{16}$$

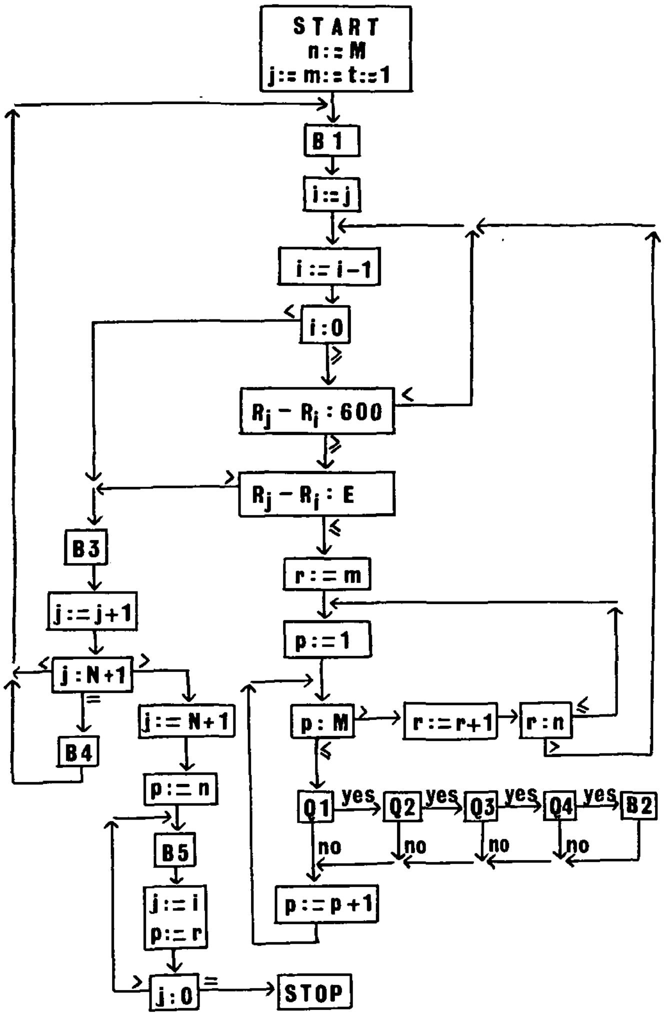
where all measurements are in feet. As a consequence it is unnecessary to have any test sites within 600 feet of R_0 or R_{N+1}.

Wherever the minimum cost of a link to a tower exceeds that of a link to a higher tower at the same site, the higher tower will always be chosen in subsequent calculations, as it will allow a longer span at the next stage. When this situation is detected therefore, the lower height is marked as if it were unfeasible, so that unnecessary consideration of this height is eliminated at a later stage. In order to reduce computation the cost of a proposed link is tested before its feasibility; it is rejected immediately if a cheaper link has already been found for the same site and height.

The computational procedure is as shown in the flow chart in **Fig. 1**. All heights at R_0 are initially marked as unfeasible, except that of the actual tension tower, and all costs at other sites are set to infinity. An attempt is made for each j to link R_j to the nearest R_i which satisfies (16). First the lowest possible tower at R_j is considered and linked tentatively to the lowest tower at R_i which is not marked as unfeasible, if this would yield a cheaper solution than that already found for this height at R_j. If the link is not feasible the next height at R_i is investigated. As soon as a feasible link is found details of this are stored in place of details of any (dearer) link found earlier. It is not necessary to proceed to the next height at R_i, as it must be more expensive (the lower one would otherwise have been marked as unfeasible when R_i was originally dealt with). The next tower height at R_j is then treated in the same way.

When all heights at R_j have been examined, i is decreased by one and the process is repeated. The computation proceeds until a value of i is reached which violates (16). Any heights at R_j for which no link has been found, or for which the cost is not less than that of a higher tower, are marked unfeasible. The details of links to towers at R_j are recorded on magnetic tape, the index j is increased by 1, and the same procedure is adopted for the next test site, R_j.

When R_{N+1} has been reached, and details of the minimum cost link to the unique height there have been recorded, the magnetic tape is scanned in the reverse direction. Details of towers in the minimum cost line to R_{N+1} are recovered and printed.



B1 Set costs associated with all heights at R_j to infinity.

B2 Update information associated with H_r at R_j by setting the cost equal to the cost associated with H_p at R_i plus the cost of a tower of height H_r, and by recording p and i.

B3 Mark as unfeasible all heights at R_j which have an associated cost which is greater than the cost of a higher tower at R_j. Record all details for R_j on magnetic tape.

B4 Set m and n to the index of the fixed height of the tension tower at R_{N+1}.

B5 Look up i and r, the indices of the site and height linked to H_p at R_j. Print the co-ordinate of R_j, and the indices i and r. Search back along the magnetic tape and retrieve details of site R_i.

Q1 Is there a feasible link to a tower of height H_p at R_i?

Q2 Would the cost of a line to H_r at R_j be reduced if a link could be established through H_p at R_i?

Q3 Are the constraints (5) and (7) satisfied if the line to H_p at R_i is extended to H_r at R_j?

Q4 Is constraint (15) satisfied at all elevation points between R_i and R_j?

Fig. 1

Program testing and running

The program was tested on a short stretch of line using restricted sets of tower heights. Several timed computer runs were made using different numbers of heights and different densities of test sites and elevation points. The results showed that the spacing of test sites had the greatest effect on the running time, which varied nearly as the square of the density of sites, and that the number of elevation points made little difference.

The final run was carried out for an actual stretch of line 10,975 feet long. The twelve tower heights actually available to the C.E.G.B. were given as data; test sites were chosen at an average interval of 200 feet, and elevation points at 100 feet. The program ran for nine hours on Pegasus and produced results about 0·5% cheaper than those which had been obtained by conventional methods.

Comments

The small saving achieved was disappointing, but the authors felt that if test sites could be specified at smaller intervals, greater savings could be made. It was unfortunately not possible to obtain sufficient machine time on Pegasus to experiment in this way, but the authors estimated that if the method were reprogrammed for a more modern machine such as KDF9, 30 miles of line could be dealt with in one hour, with a spacing of 100 feet between test sites. The smallest possible saving by the reduction of a single tower is of the order of £100, so that only two small savings over 30 miles would recover the cost of the computer time. A refined version of the method has since been programmed elsewhere (Mitra and Wolfenden, 1966) for a modern machine and used on long stretches of line with considerable success.

A number of extensions of the method are possible.

(i) The method can easily be modified to deal with a route section by section. The appropriate tension tower will be specified for marked sites at the ends of each straight section, and the relevant list of possible tower heights and costs will be consulted when these sites are encountered in the general procedure. Links which bypass these marked sites will be rejected.

(ii) Consideration may be given to the use of tension towers in sharp dips in the route where constraint (7), which only applies to suspension towers, might easily be violated.

(iii) Where constraints are violated only slightly it might be possible to move test sites within specified limits for the particular link being considered.

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References

SHULMAN, H. (1962). "The pole spotting problem", *SIAM Review*, Vol. 4, p. 12.

JACKSON, G. B. (1961). *Network for the nation: a non-technical guide to overhead transmission lines*, London: Central Electricity Generating Board.

BELLMAN, R. (1957). *Dynamic programming*, Princeton: Princeton University Press.

MITRA, G., and WOLFENDEN, K. (1966). "A computer technique for optimizing the sites and heights of transmission line towers —a dynamic programming approach", University of London Institute of Computer Science, Technical Report TR10.

BARRETT, W., and MITCHELL, A. J. (1963). "An extended Autocode for Pegasus", *The Computer Journal*, Vol. 6, p. 237.

Book Review

Computer Control of Industrial Processes, by E. S. Savas, 1966; 400 pages. (Maidenhead: *McGraw-Hill Publishing Co. Ltd.*, 128s.)

The book is intended for the engineers and production managers who are becoming increasingly involved in computer control projects. It aims to provide an introduction and basic understanding of the subject for this diverse audience, and should go some way to meet the needs of those who are production orientated but would not claim to be computer specialists. Equally those who are concerned with the use of computers from other aspects should find the control engineering and applied material useful.

"Computer Control" is normally taken to mean process control but the author is not loath to expand his title to bring in information systems and control of whole industrial organizations. This being so, some discussion on economic modelling and industrial dynamics would not have been out of place. Otherwise the subject coverage is good with regard to process control. It includes the normal introduction and explanation of concepts, mathematical modelling, statistical techniques, steady state and dynamic optimization and control including optimization techniques. The economics and management of computer control projects, computer hardware and programming, and instrumentation, are also covered. There is a long chapter completely given to case studies and examples.

The book is arranged in two sections, Principles of Computer Control, and Computer Control in Practice. The second contains the chapters on economics and management of projects, computer and instrumentation hardware, and case studies. It also contains a chapter on direct digital control which somehow seems out of place following instrumentation and preceding computer programming which is in danger of becoming disassociated from computer hardware. Perhaps direct digital control should have been spread between computer control concepts in the first half of the book and instrumentation in the second rather than taking a complete chapter in an unexpected location.

The depth to which individual topics are covered varies as is to be expected in a book in which nearly half the chapters have been contributed. The explanation of concepts and terminology is excellent, being thorough but not boring. For example many campaigners against unexplained jargon will rejoice at the handling of "off-line", "on-line", "in-line", open loop, closed loop, feedback and feed forward. Equally the concept of mathematical modelling is very well handled. The model of a fluid catalytic cracking unit is given in detail as an example, while the management and control of a computer control project is related throughout to a network diagram given in the text.

In some places the apparently unavoidable generalizations invite criticism. For example, the vital condition given that for computer control one must have the prior development of the control strategy which is valid over the anticipated range of conditions, is later undermined by generalizations implying that any difficult process is a sitting duck for computer control. Similarly it is optimistically suggested that the problems of obtaining process data are solved by the use of an on-line computer for recording. Again in the brief section on business planning and control, the "bill of materials" in steelmaking is sweepingly and dubiously likened to the parts explosion in light engineering. The example of scheduling refers to developed algorithms that are yet to be used in this country. However, these are small criticisms taking the book as a whole.

Few chapters are likely to bemuse those not already versed in the topics in question. Amongst the chapters which might be criticized on this score are statistical methods, instrumentation and direct digital control. In particular that on statistics is rather difficult for the uninitiated and tends to use results which have not been previously introduced or explained. Overall Dr Savas and his contributors have produced a useful introductory textbook in the main for engineers and production personnel. Those concerned in the use of computers for the same and related topics but trained in other disciplines should also find some of the material useful.

G. C. CUDDEFORD