The simulation of wave filters having polynomial transfer functions on an analogue computer

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The simulation of filter characteristics is often required to form part of a signal processing operation carried out on an analogue computer. By adopting a simplified approach to the problem of direct mechanization of the polynomial transfer function involved, many of the required filter characteristics can be realized by the use of standard analogue elements. Groups of potentiometer settings can be calculated and made available in tabular form to cover a wide range of requirements. Matching of desired filter characteristics with analogue circuit configuration and gain/potentiometer setting may be simplified by reference to these tables.

1. Introduction

The problems of simulation for wave-filters on general purpose analogue computers has received scant attention in the literature. Hansen (1966) gives what is perhaps the most complete treatment at present available, but the main body of his paper is concerned with the design of filter networks to be associated with a single operational amplifier. Whilst this represents a considerable economy in circuit components it is not a very convenient approach to the user of the big machine, who generally does not have the engineering effort required for the construction of the wide variety of networks that would be needed in a range of signal processing problems.

The following treatment is concerned with the simulation of filters from analogue computing elements only and represents a completely flexible approach with few operational limitations.

The simulation of the characteristics of wave filters in an analogue computer involves the mechanization of a transfer function of a polynomial:

$$G_{(p)} = \frac{b_0 + b_1(p/\omega_c) + (p/\omega_c)^2 + \ldots + b_n(p/\omega_c)^n}{a_0 + a_1(p/\omega_c) + a_2(p/\omega_c)^2 + \ldots + a_n(p/\omega_c)^n} \quad (1)$$

where a_n and b_n are constants determining the filter characteristics

$\omega_c = 2\pi \times$ filter natural frequency

$p = d/dt.$

Stability requirements dictate that the order of the numerator must not exceed that of the denominator, and a simplification which permits most theoretical filters to be simulated is given below.

$$G_{(p)} = \frac{V_0}{V_i}(p) = \frac{1}{1 + a_1(p/\omega_c) + a_2(p/\omega_c)^2 + \ldots + a_n(p/\omega_c)^n} \quad (2)$$

This expression is still completely general and will enable the characteristics of low-pass, high-pass and band-pass filters to be obtained.

With one exception the filters described below exhibit the transfer characteristics given in (2).

The exception is a time averaging filter and is described by a form of (1) where the order of the numerator is two less than that of the denominator and n is always even viz.

$$G_{(p)} = \frac{[1 + (p/\omega_c)^2][1 + \tfrac{1}{4}(p/\omega_c)^2] \ldots \left[1 + \left(\frac{2}{n-2}\right)^2 (p/\omega_c)^2\right]}{1 + a_1(p/\omega_c) + a_2(p/\omega_c)^2 \ldots a_{n-1}(p/\omega_c)^{n-1} + (p/\omega_c)^n} \quad (3)$$

2. Simulation on the computer

Two forms of (2) have been considered. They are:

$$V_0 a_n(p/\omega_c)^n = V_i - V_0 - a_1 V_0(p/\omega_c) - a_2(p/\omega_c)^2 \ldots a_{(n-1)}(p/\omega_c)^{(n-1)} \quad (4)$$

and a "nested form":

$$V_0 = V_i - (p/\omega_c)\{a_1 V_0 + (p/\omega_c)[a_2 V_0 + (p/\omega_c)(a_3 V_0 + \ldots (p/\omega_c)[a_n V_0])]\} \quad (5)$$

To illustrate the practical difference a fourth-order filter is shown in **Fig. 1(*a*)** using (4) and **Fig. 1(*b*)** using (5). The latter is of value in element economy where n is large.

Referring to Fig. 1(*a*) a further dichotomy in circuit arrangement is possible. It will be apparent from the mechanization of (4) that potentiometers Q_{01} to Q_{03} set the frequency term directly (ω_c) whereas Q_{00} is set to the value ω_c/a_4.

Also $p_{00} = a_3$, $p_{01} = a_2$, $p_{02} = a_1$ and $p_{03} = 1 \cdot 0$. From this we see that p_{03} is not required for direct mechanization of (4). However, as will be seen later, for some filter conditions involving many stages and high Q values then this could lead to very large loop gains.

With the phase-shift margin of the computer amplifiers instability would be precipitated. To avoid this when the coefficient/gain value becomes large all terms on the

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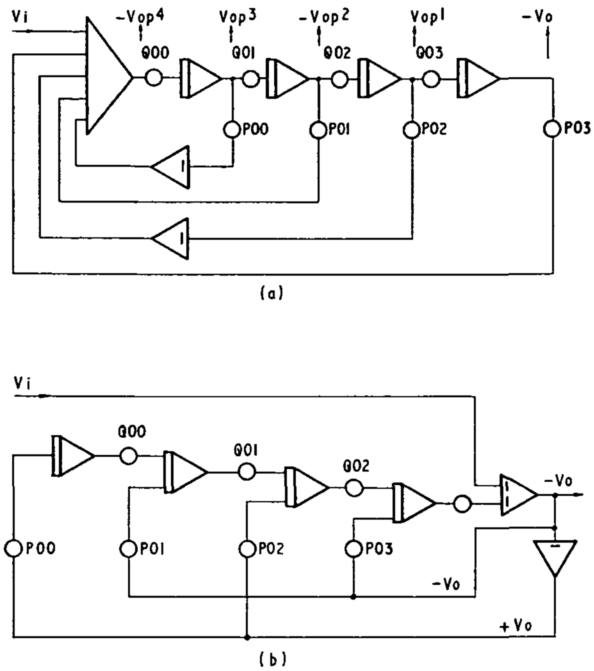

Fig. 1.—Simulation of a fourth order filter

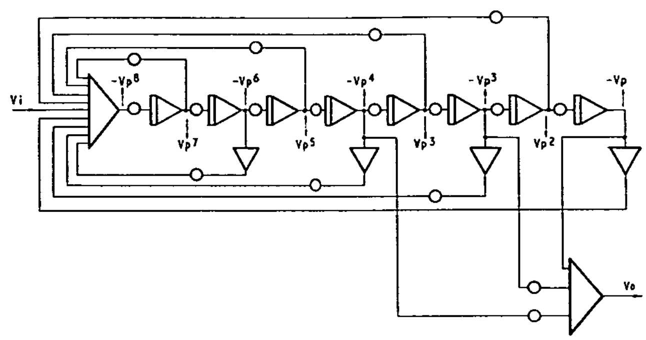

Fig. 2.—Simulation of an eighth order time-averaging filter

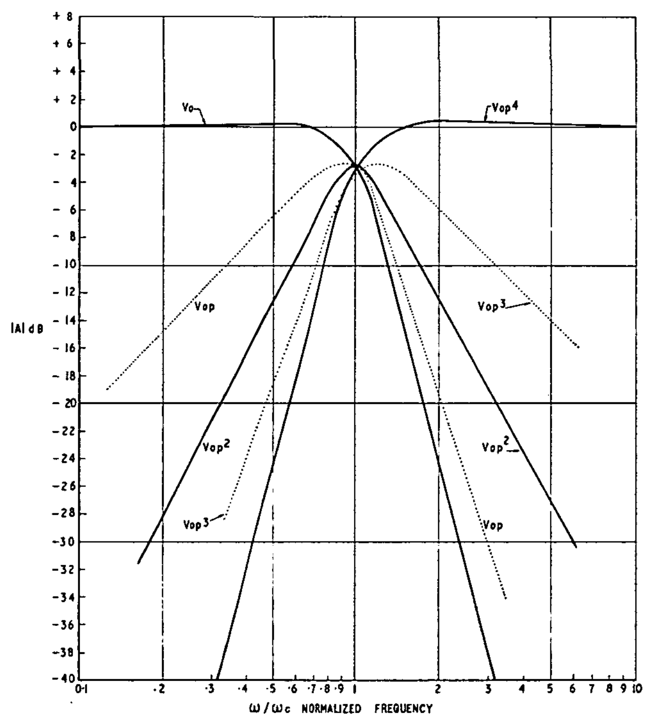

Fig. 3.—Fourth order Butterworth filter

right-hand side of (4) can be divided by a_n; i.e. the coefficient/gain values are normalized. This allows the values of potentiometers Q_n to be set to the same value (ω_c) for frequency determination only, whilst potentiometers p_n will be set to values

$$a_i/a_n \quad \text{where} \quad a_i = a_1, a_2, a_3, \ldots a_{(n-1)}$$

and determine the characteristics of the filter. This latter method is carried out in the design of narrow-band Chebyshev filters to be described later.

Equation (3) can be mechanized simply by its partitioned form (Noronha, 1964) where we write for (3)

$$V/V_i(p) = \frac{1}{1 + a_1(p/\omega_c) + a_2(p/\omega_c)^2 + \ldots a_{n-1}(p/\omega_c)^{n-1} + (p/\omega_c)^n} \tag{6}$$

and

$$V_0/V(p) = \frac{[1 + (p/\omega_c)^2][1 + \tfrac{1}{4}(p/\omega_c)^2] \ldots \left[1 + \left(\frac{2}{n-2}\right)^2 (p/\omega_c)^2\right]}{1} \tag{7}$$

Given V_i, (6) is mechanized. The derivatives are then used in the mechanization of (7) to derive the output V_0.

An example is given in **Fig. 2** for an eighth-order filter.

3. Filter characteristics

Since all the derivatives are available in Fig. 1 then the selection of the type of filter depends on the exit point chosen, e.g.

V_0	output gives Low-Pass type
V_{0p}^4	output gives High-Pass type
V_{0p}^2	output gives Band-Pass type
V_{0p} and V_{0p}^3	output gives Assymetrical Band-Pass type

Fig. 3 illustrates this for a fourth-order filter.

The slope of the L.P. and H.P. filter will be 6 db/octave/integrator used (i.e. 24 db/octave in this example).

The slope of the symmetrical B.P. filter will be 6 db/octave/pair of integrators used (assuming n is an even number).

The characteristics of the filter depend on the setting of potentiometers Pn. Groups of settings can be calculated to give the following filter characteristics:

Table 1

Potentiometer/gain coefficients for a Butterworth filter

(a)

n	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	
1	1·000								
2	1·414	1·000							
3	2·000	2·000	1·000						
4	2·613	3·414	2·613	1·000					
5	3·236	5·236	5·236	3·236	1·000				
6	3·864	7·464	9·141	7·464	3·864	1·000			
7	4·494	10·103	14·606	14·606	10·103	4·494	1·000		
8	5·126	13·138	21·848	25·691	21·848	13·138	5·126	1·000	

(b)

n	DENOMINATOR POLYNOMIAL
1	$(1 + p)$
2	$(1 + 1{\cdot}414\,p + p^2)$
3	$(1 + p)(1 + p + p^2)$
4	$(1 + 0{\cdot}7653\,p + p^2)(1 + 1{\cdot}8477\,p + p^2)$
5	$(1 + p)(1 + 0{\cdot}6180\,p + p^2)(1 + 1{\cdot}6180\,p + p^2)$
6	$(1 + 0{\cdot}5176\,p + p^2)(1 + 1{\cdot}4142\,p + p^2)(1 + 1{\cdot}9318\,p + p^2)$
7	$(1 + p)(1 + 0{\cdot}4449\,p + p^2)(1 + 1{\cdot}2465\,p + p^2)(1 + 1{\cdot}8022\,p + p^2)$
8	$(1 + 0{\cdot}3896\,p + p^2)(1 + 1{\cdot}1110\,p + p^2)(1 + 1{\cdot}6630\,p + p^2)(1 + 1{\cdot}9677\,p + p^2)$

Butterworth

The Butterworth function of order n is:

$$|Z_{12}(j\omega)|^2 = \frac{1}{1 - \omega^{2n}} \tag{8}$$

and can be approximated by expanding the function and taking n stages (ideally $n = \infty$ for perfect square L.P. response).

Over the pass band ω^{2n} should approximate to zero in range $0 < \omega < 1$ and infinity beyond this. The Butterworth filter attempts this by arranging that its first $(n - 1)$ derivates are at zero at zero frequency. It concentrates its approximating ability near $\omega = 0$. The result is a filter of maximally flat low-frequency response with good gain-v-frequency characteristics, approaching the ideal for large values of n.

As n increases, however, the transient response becomes poor.

Chebyshev (see Guillemin, 1957)

Here a function $F^2(\omega)$ is put in place of ω^{2n} in (8) to satisfy more closely the criterion given above.

i.e. $$|Z_{12}(j\omega)|^2 = \frac{1}{1 + \epsilon^2 V_n^2(\omega)}. \tag{9}$$

This implies two parameters, ϵ and n which can be adjusted to approximate to the ideal response. The gain over the pass band approaches unity not at zero frequency but at discrete frequencies distributed over the band.

The resulting response gives a sharper roll-off near the cut-off frequency but the transient response is more oscillatory than for the Butterworth. It is valuable as a narrow band-pass filter where this oscillatory response is unimportant.

Bessel (*or Thomson*)

This approximates the ideal phase-v-frequency characteristic in a similar manner to that attempted by the Butterworth in its amplitude-v-frequency response. In the Bessel filter the first $(2n - 1)$ derivatives are, with the exception of the first, zero at zero frequency. Where accurate phase response is required, or alternatively no amplitude over-shoot, the Bessel filter would be chosen.

Paynter

The Paynter filter (Paynter, 1963) approximates to the ideal phase-v-frequency characteristic in a similar manner to the Chebyshev by matching the phase angle at specific frequencies spaced throughout the pass band. Its transient response is superior to the Bessel filter.

Table 2

Potentiometer/gain coefficients for a second order Chebyshev filter

N = 2

E	K 1	K 2	R(DB)	N 1	N 2
.100	.599	.199	.043	3.008	5.025
.200	.794	.392	.170	2.024	2.550
.300	.905	.575	.374	1.575	1.740
.400	.966	.743	.645	1.301	1.346
.500	.994	.894	.969	1.112	1.118
.600	.999	1.029	1.335	.971	.972
.700	.989	1.147	1.732	.862	.872
.800	.968	1.249	2.148	.775	.801
.820	.963	1.268	2.233	.760	.789
.840	.958	1.286	2.319	.745	.778
.860	.952	1.304	2.404	.730	.767
.880	.947	1.321	2.491	.717	.757
.900	.941	1.338	2.577	.703	.748
.920	.935	1.354	2.663	.691	.739
.940	.929	1.370	2.750	.678	.730
.960	.923	1.385	2.837	.666	.722
.980	.916	1.400	2.923	.655	.715
.990	.913	1.407	2.967	.649	.711

Table 3

Potentiometer/gain coefficients for a fourth order Chebyshev filter

N = 4

E	K 1	K 2	K 3	K 4	R(DB)	N 1	N 2	N 3	N 4
.100	2.259	2.631	1.709	.796	.043	2.838	3.305	2.147	1.256
.200	2.553	3.567	2.504	1.569	.170	1.627	2.274	1.596	.637
.300	2.678	4.247	2.993	2.298	.374	1.155	1.843	1.302	.435
.400	2.715	4.790	3.288	2.971	.645	.914	1.613	1.107	.337
.500	2.697	5.238	3.447	3.577	.969	.754	1.464	.964	.280
.600	2.644	5.613	3.511	4.115	1.335	.642	1.364	.853	.243
.700	2.568	5.928	3.507	4.586	1.732	.560	1.292	.765	.218
.800	2.478	6.194	3.458	4.996	2.148	.496	1.240	.692	.200
.820	2.460	6.241	3.445	5.071	2.233	.485	1.231	.679	.197
.840	2.441	6.288	3.430	5.144	2.319	.474	1.222	.667	.194
.860	2.421	6.333	3.414	5.215	2.404	.464	1.214	.655	.192
.880	2.402	6.376	3.397	5.283	2.491	.455	1.207	.643	.189
.900	2.392	6.418	3.380	5.350	2.577	.445	1.200	.632	.187
.920	2.363	6.459	3.361	5.415	2.663	.436	1.193	.621	.185
.940	2.343	6.498	3.342	5.478	2.750	.428	1.186	.610	.183
.960	2.323	6.536	3.323	5.539	2.837	.419	1.180	.600	.181
.980	2.304	6.573	3.303	5.598	2.923	.412	1.174	.590	.179
.990	2.294	6.591	3.293	5.627	2.967	.408	1.171	.585	.178

Table 4

Potentiometer/gain coefficients for a sixth order Chebyshev filter

N = 6

B	K 1	K 2	K 3	K 4	K 5	K 6	R(DB)	N 1	N 2	N 3	N 4	N 5	N 6
.100	4.153	8.804	11.531	11.220	6.405	3.184	.043	1.305	2.765	3.622	3.524	2.012	.314
.200	4.460	10.640	14.629	16.719	9.576	6.274	.170	.711	1.696	2.332	2.665	1.526	.159
.300	4.557	11.871	16.277	21.049	11.553	9.192	.374	.496	1.291	1.771	2.290	1.257	.109
.400	4.545	12.812	17.116	24.674	12.760	11.880	.645	.383	1.078	1.441	2.077	1.074	.084
.500	4.464	13.568	17.430	27.759	13.426	14.305	.969	.312	.948	1.218	1.941	.939	.070
.600	4.341	14.188	17.389	30.395	13.708	16.456	1.335	.264	.862	1.057	1.847	.833	.061
.700	4.191	14.703	17.111	32.645	13.719	18.342	1.732	.228	.802	.933	1.780	.748	.055
.800	4.027	15.134	16.681	34.563	13.546	19.979	2.148	.202	.757	.835	1.730	.678	.050
.820	3.993	15.211	16.582	34.911	13.495	20.280	2.233	.197	.750	.818	1.721	.665	.049
.840	3.959	15.286	16.480	35.248	13.440	20.571	2.319	.192	.743	.801	1.714	.653	.049
.860	3.925	15.358	16.375	35.575	13.381	20.854	2.404	.188	.736	.785	1.706	.642	.048
.880	3.891	15.428	16.268	35.891	13.318	21.128	2.491	.184	.730	.770	1.699	.630	.047
.900	3.857	15.495	16.158	36.198	13.252	21.395	2.577	.180	.724	.755	1.692	.619	.047
.920	3.823	15.560	16.046	36.494	13.182	21.653	2.663	.177	.719	.741	1.685	.609	.046
.940	3.789	15.623	15.933	36.782	13.110	21.904	2.750	.173	.713	.727	1.679	.599	.046
.960	3.755	15.684	15.818	37.060	13.036	22.148	2.837	.170	.708	.714	1.673	.589	.045
.980	3.721	15.742	15.702	37.330	12.959	22.385	2.923	.166	.703	.701	1.668	.579	.045
.990	3.705	15.771	15.644	37.461	12.920	22.500	2.967	.165	.701	.695	1.665	.574	.044

Table 5

Potentiometer/gain coefficients for an eighth order Chebyshev filter

N = 8

E	K 1	K 2	K 3	K 4	K 5	K 6	K 7	K 8	R(DB)	N 1	N 2	N 3	N 4	N 5	N 6	N 7	N 8
.100	6.120	19.003	37.556	55.780	57.517	50.251	25.126	12.778	.043	.479	1.487	2.939	4.365	4.501	3.933	1.966	.078
.200	6.426	21.802	44.518	73.989	76.866	78.908	37.899	25.226	.170	.255	.864	1.765	2.933	3.047	3.128	1.502	.040
.300	6.493	23.625	47.897	87.262	87.848	102.525	45.946	37.014	.374	.175	.638	1.294	2.358	2.373	2.770	1.241	.027
.400	6.433	24.998	49.358	97.929	93.949	122.831	50.912	47.899	.645	.134	.522	1.030	2.044	1.961	2.564	1.063	.021
.500	6.293	26.092	49.594	106.790	96.786	140.424	53.700	57.741	.969	.109	.452	.859	1.849	1.676	2.432	.930	.017
.600	6.101	26.986	49.019	114.241	97.375	155.638	54.929	66.492	1.335	.092	.406	.737	1.718	1.464	2.341	.826	.015
.700	5.878	27.727	47.912	120.536	96.429	168.744	55.057	74.174	1.732	.079	.374	.646	1.625	1.300	2.275	.742	.013
.800	5.640	28.344	46.474	125.863	94.465	179.995	54.429	80.858	2.148	.070	.351	.575	1.557	1.168	2.226	.673	.012
.820	5.592	28.455	46.160	126.827	93.985	182.044	54.236	82.083	2.233	.068	.347	.562	1.545	1.145	2.218	.661	.012
.840	5.543	28.562	45.840	127.759	93.482	184.031	54.025	83.274	2.319	.067	.343	.550	1.534	1.123	2.210	.649	.012
.860	5.494	28.666	45.513	128.661	92.958	185.957	53.797	84.429	2.404	.065	.340	.539	1.524	1.101	2.203	.637	.012
.880	5.445	28.766	45.182	129.534	92.415	187.823	53.554	85.552	2.491	.064	.336	.528	1.514	1.080	2.195	.626	.012
.900	5.397	28.862	44.847	130.379	91.856	189.633	53.298	86.641	2.577	.062	.333	.518	1.505	1.060	2.189	.615	.012
.920	5.348	28.955	44.508	131.197	91.282	191.387	53.028	87.699	2.663	.061	.330	.508	1.496	1.041	2.182	.605	.011
.940	5.300	29.045	44.166	131.989	90.695	193.088	52.748	88.726	2.750	.060	.327	.498	1.488	1.022	2.176	.595	.011
.960	5.251	29.133	43.822	132.755	90.097	194.736	52.457	89.723	2.837	.059	.325	.488	1.480	1.004	2.170	.585	.011
.980	5.203	29.217	43.476	133.497	89.488	196.334	52.157	90.690	2.923	.057	.322	.479	1.472	.987	2.165	.575	.011
.990	5.179	29.258	43.303	133.859	89.180	197.115	52.004	91.163	2.967	.057	.321	.475	1.468	.978	2.162	.570	.011

Time averaging filter

This is a filter derived from the Paynter which approximates a finite averaging process:

$$\bar{x} = 1/T \int_{t-T}^{t} f(t)dt. \qquad (10)$$

The result is a combined low-pass and notch filter which gives a high attenuation at the cut-off frequency with minimum overshoot.

4. Calculation of potentiometer constants

(i) *Butterworth*

These have been calculated previously by Baum (1948) and are reproduced in **Table 1(*a*)**. The coefficients correspond to those given in (4) and (5) and, since $a^n = 1$ in all cases, they would be used directly as fractional coefficient settings followed by the appropriate number of gain decades.

Table 1(*b*) gives the factorized form of the polynomial and is used in the derivation of Chebyshev constants as described below:

(ii) *Chebyshev*

The transfer function for a second-order or quadratic Chebyshev filter can be shown to be:

$$G_{(p)} = \frac{\sigma_K^2 + \omega_K^2}{(p/\omega_c)^2 + 2\sigma_K(p/\omega_c) + \sigma_K^2 + \omega_K^2} \qquad (11)$$

where σ_K and ω_K are the roots of the Chebyshev polynomial.

Hansen (1966) gives the relationships between the roots of the Butterworth and Chebyshev polynomials. If we take roots for the same angle θ we can write:

$$\theta = \cos^{-1}\left(\frac{Knm}{2}\right)$$

where K_{nm} is the Butterworth coefficient with n expressing the order of the polynomial and m the number of the factorized quadratic given in Table 1(*b*). Given the Chebyshev constant ϵ we can write:

$$\beta_{n\epsilon} = \frac{1}{n} - \sinh^{-1} 1/\epsilon.$$

The Chebyshev roots are then defined as:

$$\left.\begin{aligned} \sigma_K &= \sinh \beta . \cos \theta \\ \omega_K &= \cosh \beta . \sin \theta \end{aligned}\right\} \qquad (12)$$

If those values are substituted in (11) for each quadratic factor and the product of the quadratics taken, then after normalizing, an expression such as (2) can be obtained. This will give the value of the coefficients a_n for even-order filters and for different values of ϵ.

A digital computer program has been written to carry out these calculations, the results of which appear as **Tables 2, 3, 4** and **5**. These tables give first the potentiometer/gain coefficients K_1 to K_n, corresponding to the coefficients a_1 to a_n in (2). The normalized values are given as potentiometer/gain coefficients N_1 to N_n and correspond to the values a_i/a_n discussed earlier.

The two sets of tables are separated by a set of R(db) values which are described later in the paper. Where the normalized N values are used then the setting of the first frequency-determining potentiometer (Q_{00} in Fig. 1) may be calculated from ω_c/K_n or $\omega_c N_n$.

(iii) *Bessel*

The denominator polynomial function for (1), as applicable to a Bessel filter, can be obtained from the expression:

$$F_n(p/\omega_c) = (p/\omega_c)^2 F_{n-2} + (2n-1)F_{n-1} \qquad (13)$$

Table 6

Potentiometer/gain coefficients for a Bessel filter

(*a*) Unnormalized

n	a_0	a_1	a_2	a_3	a_4	a_5	a_6
1	1	1					
2	3	3	1				
3	15	15	6	1			
4	105	105	45	10	1		
5	945	945	420	105	15	1	
6	10395	10395	4725	1260	210	21	1

(*b*) Normalized

n	a_0	a_1	a_2	a_3	a_4	a_5	a_6
1	1	1					
2	1	1	0·3333				
3	1	1	0·4000	0·0667			
4	1	1	0·4285	0·0952	0·0952		
5	1	1	0·4434	0·1011	0·0159	0·00106	
6	1	1	0·4546	0·1212	0·0202	0·00202	0·000096

given that $F_c(p/\omega_c) = 1$ and $F_1(p/\omega_c) = (p/\omega_c) + 1$.

Table 6(*a*) gives the coefficients of this expansion up to the sixth order.

The large loop gains involved for orders $> n = 4$ render this method of doubtful value for stable simulation on the computer at large values of n. An improvement is possible if the coefficient values are normalized with respect to the a_0 coefficient as shown in **Table 6(*b*)**. The first frequency-determining potentiometer will now have to be set to a potentiometer gain coefficient of $\omega^0 . a_0$ (a_0 taken from Table 6(*a*)). This implies a large gain localized in one or a few amplifiers preceding the integrators, and the filter will remain stable for a higher order of n.

(iv) *Paynter*

These have been published elsewhere (Kohr, 1967) and are reproduced in **Table 7**. The form of the transfer function is as (2).

(v) *Time averaging*

The form of the transfer function is given by (1). The denominator coefficients are identical with those given for the Paynter filter and can be taken from Table 7. The numerator coefficients can be derived from the multiplication of the terms given in the numerator of (3). This has been carried out up to $n = 8$ in **Table 8**.

Table 7

Potentiometer/gain coefficients for a Paynter filter

n	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1·000							
2	1·571	1·000						
3	2·145	1·865	1·000					
4	2·721	3·333	2·041	1·000				
5	3·297	4·895	4·539	2·157	1·000			
6	3·874	7·001	7·363	5·755	2·239	1·000		
7	4·451	9·248	12·161	10·028	6·975	2·301	1·000	
8	5·028	12·005	17·533	18·800	12·833	8·198	2·348	1·000

Table 8

Potentiometer/gain coefficients for a time averaging filter

n	b_0	b_1	b_2	b_3	b_4	b_5	b_6
2	1						
4	1	0	1				
6	1	0	1·250	0	0·250		
8	1	0	1·361	0	0·389	0	0·277

Only the even order polynomials are applicable in this simulation.

5. Measured performance

A comparison between several four-stage low-pass filters is shown in **Fig. 4**. An equal lag filter is defined by the integer coefficients having equal spacing, e.g.:

$$G_{(p)} = \frac{1}{1 + 4p + 6p^2 + 4p^3 + p^4}.$$

The superiority of the Butterworth in respect of its

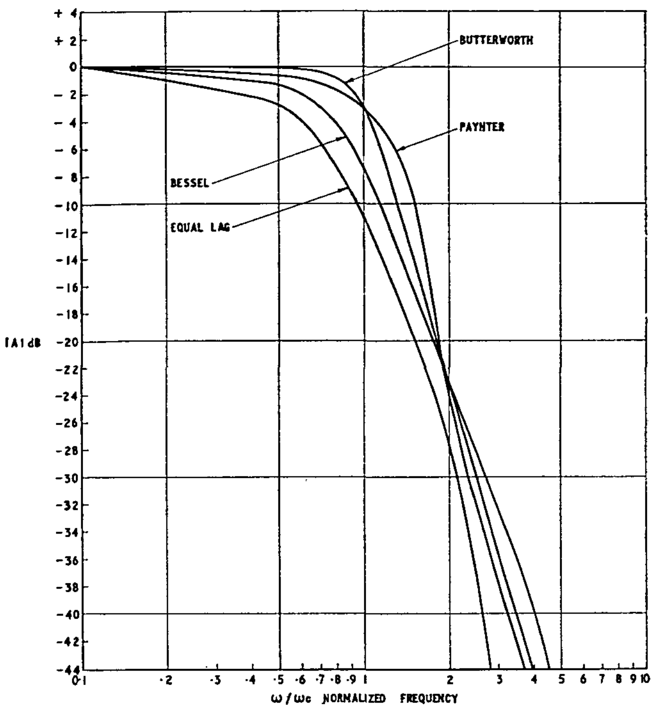

Fig. 4.—Fourth order L.P filters

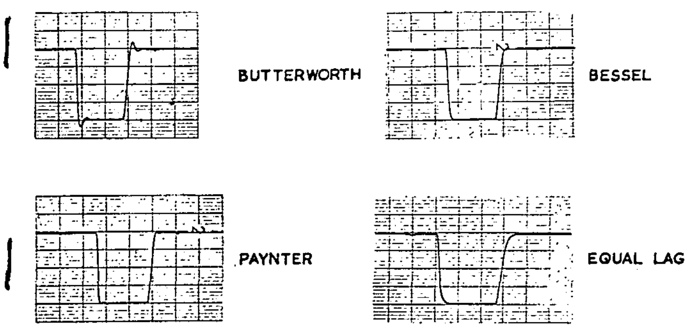

Fig. 5.—Fourth order L.P. filter response to +10 v, 70 sec pulse input

gain vs frequency characteristic may be seen in this diagram.

The transient response is shown in **Fig. 5**, also for a fourth-order filter. The Bessel filter is seen to give the closest approach to the ideal Gaussian impulse response, although very little overshoot is apparent with the Paynter filter.

The Chebyshev low pass filter approaches the ideal square response to frequency variation (**Fig. 6**) but is subject to a ripple in the pass band of maximum amplitude:

$$R = 20 \log_{10} \sqrt{(1 + \epsilon^2)} \text{ dbs} \qquad (14)$$

This had been calculated and included as R value in the computed Tables 2 to 5. Typical characteristics are shown in **Figs. 6** and **7**. The latter gives the response

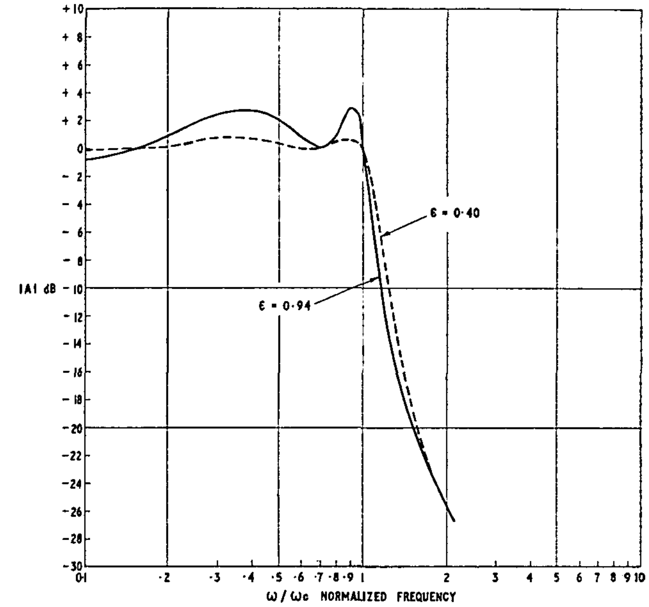

Fig. 6.—Fourth order Chebyshev L.P. filter

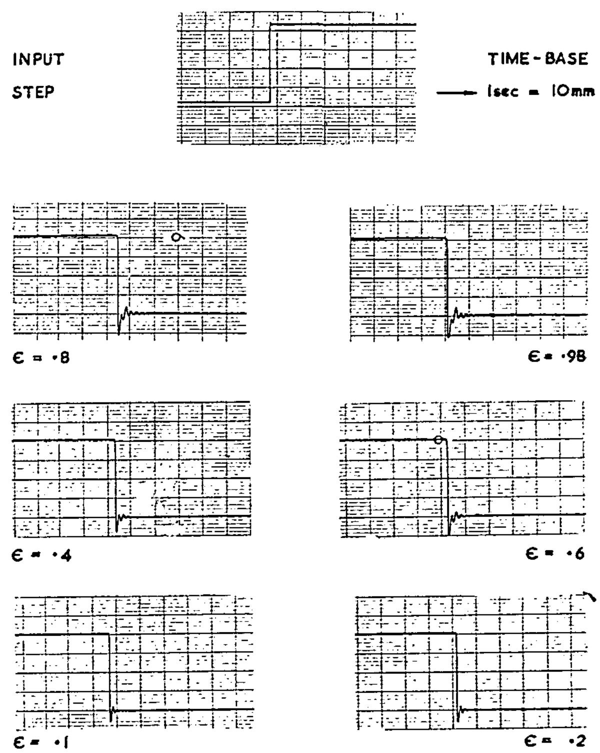

Fig. 7.—Fourth order Chebyshev L.P. filter (f_c = 10 c/s) response to step voltage input

to a step input waveform for different values of the Chebyshev constant ϵ.

A fourth-order time-averaging filter response is shown in **Fig. 8**. This shows very good notch characteristics at the cut-off frequency ω_c. When the filter forms the feedback network of an amplifying circuit then a Q-

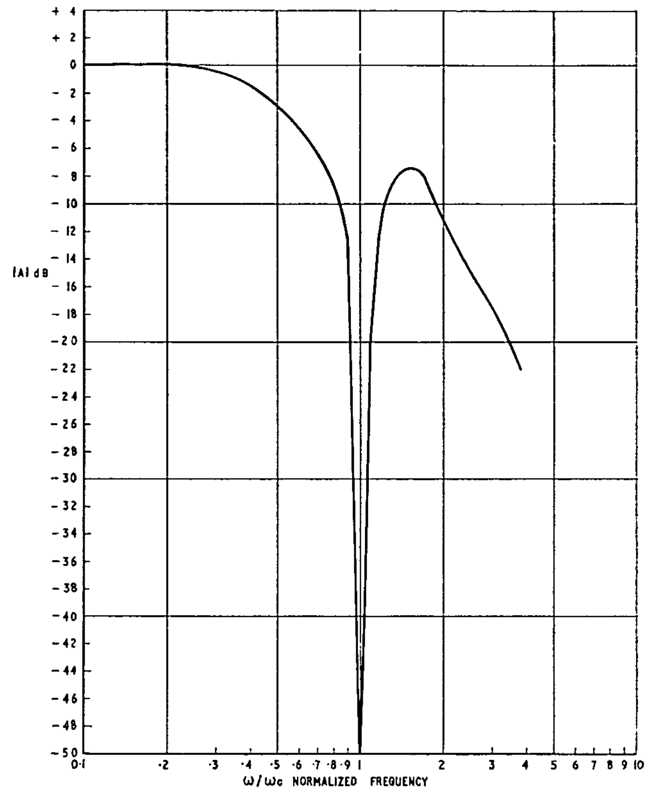

Fig. 8.—Fourth order time-averaging filter

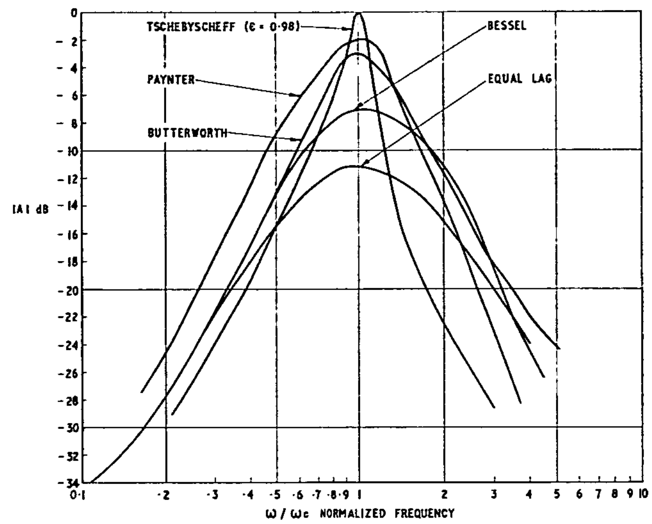

Fig. 9.—Fourth order band-pass filters

factor at ω_c superior to that obtained by a Chebyshev filter of similar order may be obtained.

The performance of bandpass filters of the fourth order is compared in **Fig. 9**. The Q-factor of the filter (ω_c/bandwidth at -3 db points) is determined by the order of the filter n and coefficient ϵ. This is shown in the relationship given in **Fig. 10**. An extrapolated curve is given for $n = 8$ since instability is likely to prevent the realization of this filter simulation at the higher Q values.

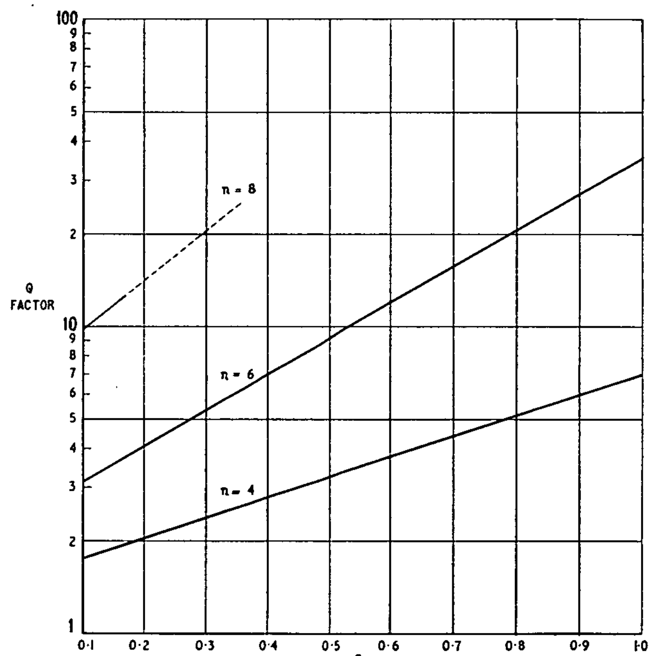

Fig. 10.—Chebyshev band-pass filter variation of Q-factor with ϵ and η

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