K_{n+1} after one cycle of n iterations is then the generalized inverse A^+. For simplicity each difference Δx_i is taken as the ith unit vector with jth element δ_{ij}. Then Δf_i becomes the ith column of A. Also no storage need be assigned to the η sub-space because Δx_i is automatically orthogonal to $[\Delta x_{i-1}]$, so $\tilde{N}_i \Delta x_i = \Delta x_i$. Further simplicity accrues by taking K_1 as the zero matrix. The computation to force $K\tilde{\Delta f} = 0$ is then not required because this condition holds automatically. Furthermore after the ith iteration, K_{i+1} will have rows $i + 1$ to n with all zero elements, which can be taken into account when making the matrix multiplications. An interesting feature is that K_{i+1} is the generalized inverse for the first i columns of A and the updating formula is therefore a recurrence relation for the generalized inverses of matrices related in this way. Finally if $\text{rank}(A) = n$, then the computation requires approximately $1 \cdot 5mn(n + 1)$ multiplications. If the rank is less than n then this figure is an overestimate of the computation required.

References

Barnes, J. G. P. (1965). An algorithm for solving non-linear equations based on the secant method, *Computer Journal*, Vol. 8, p. 66.

Box, M. J. (1966). A comparison of several current optimization methods and the use of transformations in constrained problems, *Computer Journal*, Vol. 9, p. 67.

Broyden, C. G. (1965). A class of methods for solving non- linear simultaneous equations, *Mathematics of Computation*, Vol. 19, p. 577.

Fletcher, R., and Powell, M. J. D. (1963). A rapidly convergent descent method for minimization, *Computer Journal*, Vol. 6. p. 163.

Fletcher, R. (1965). Function minimization without evaluating derivatives—a review, *Computer Journal*, Vol. 8, p. 33.

Fletcher, R. (1966). Certification of Algorithm 251, Function Minimization, *Communications A.C.M.* Vol. 9, p. 686.

Freudenstein, F., and Roth, B. (1963). Numerical solutions of systems of non-linear equations, *Journal A.C.M.*, Vol. 10, p. 550.

Greville, T. N. E. (1959). The pseudoinverse of a rectangular or singular matrix and its application to the solution of sytems of linear equations, *S.I.A.M. Review*, Vol. 1, p. 38.

Householder, A. S. (1963). *Principles of Numerical Analysis*, McGraw-Hill, New York.

Penrose, R. (1955). A generalized inverse for matrices, *Proc. Camb. Phil. Soc.*, Vol. 51, p. 406.

Powell, M. J. D. (1965). A method for minimizing a sum of squares of non-linear functions without calculating derivatives, *Computer Journal*, Vol. 7, p. 303.

Spang, H. A. (1962). A review of minimization techniques for non-linear functions, *S.I.A.M., Review*, Vol. 4, p. 343.

Wells, M. (1965). Algorithm 251, Function minimization, *Communications A.C.M.*, Vol. 8, p. 169.

Wolfe, P. (1959). The secant method for simultaneous non-linear equations, *Communications A.C.M.*, Vol. 2, p. 12.

Book Review

Approximation of Functions: Theory and Numerical Methods, by G. Meinardus (Translated by L. L. Schumaker), 1967; 198 pages. (Berlin, Heidelberg, New York: *Springer*, DM.54.0, US $13.50.)

This is a slightly expanded translation of a book that was first published in German three years ago. The author's aim is "to collect essential results of approximation theory which on the one hand makes possible a fast introduction to the modern development of this area, and on the other hand provides a certain completeness to the problem area of Tchebycheff approximation". In this Professor Meinardus has succeeded brilliantly and Dr. Schumaker's excellent translation now makes this text available to the English-speaking world.

The whole concern of the book is the study of "best" approximations to functions by simpler functions (usually polynomials or rational functions). In recent years this topic has received considerable stimulus from the widespread use of computers: it has also undergone a revolution through the introduction of the methods of functional analysis. It is this elegant application of the most powerful tools of pure mathematics to problems of considerable practical concern that makes modern numerical analysis so attractive.

Part I treats Linear Approximation. The first five chapters deal with the theoretical problem, covering uniqueness, the minimax properties, error estimates, and a host of related problems. The last two chapter of this section go into more detail about approximation by polynomials and the numerical determination of best approximations. Points of interest for the computer user are the derivation of very good starting approximations for the exchange methods of Remez and Stiefel, and the derivation that the Remez algorithm has second order convergence. Non-linear approximation is the subject of Part II. This is mostly concerned with approximation by rational functions although there is also a brief section on exponential approximation.

All in all, this is a book that can be recommended without reservation to those interested in approximation theory.

P. A. Samet (London)