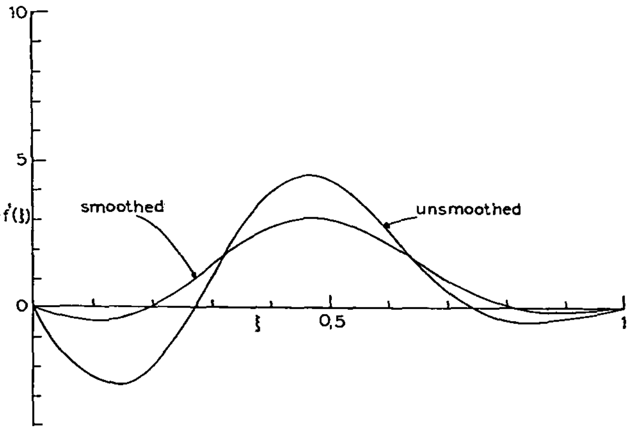

Fig. 5. Step function derivative. M = 5

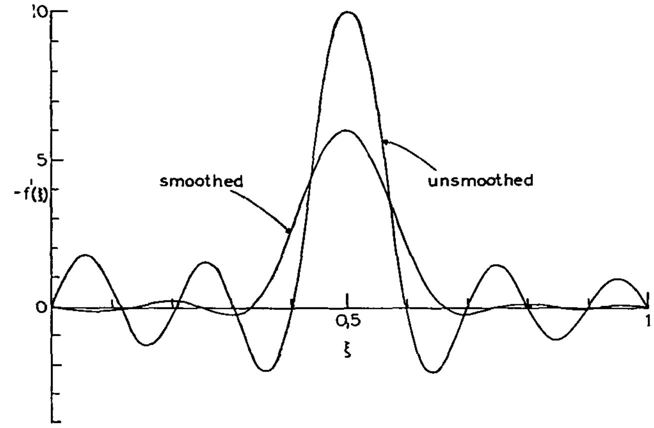

Fig. 6. Step function derivative. M = 10

examples verify that for both types of expansions the effects of the technique are very much the same, even for small values of ξ, where Bessel functions differ most from their trigonometric analogues. This suggests that it should be possible to adapt the smoothing technique to other forms of orthogonal expansion.

In cases where the original series converges rapidly, it has been found that the smoothing technique makes the accuracy *worse*. This is of some interest, since presumably the same thing must be true in the Fourier case, yet Lanczos has overlooked any such difficulties.

It has not been the intention of this paper to attempt to examine in any detail the convergence properties of this smoothing process, but simply to show (i) how it can be adapted to Fourier–Bessel expansions and (ii) by means of numerical examples, what sort of results may be expected from it.

More precise investigation leads almost at once to difficulties. Consider, for example, the particular form of expansion used in the above examples. The logical measure of error for expansions of this type, e_M, would presumably be defined by

$$e_M^2 = \int_0^1 \xi \left[f(\xi) - \sum_{m=0}^{M-1} A_m J_0(\lambda_m \xi)\right]^2 d\xi \qquad (27)$$

with e_{MS} as the equivalent quantity for the smoothed expansion. Using (27) and the properties of Bessel's functions, it is not difficult to show that $e_{MS}^2 \geqslant e_M^2$ for all A_m values. Indeed, one would expect this from the manner in which the A_m are normally defined. By this measure, then, the smoothed approximation is never "better". This agrees well with the results obtained in case (i), but not for those of cases (ii) and (iii), which, however, consider expansions for which the original assumption made in deriving the form of S_m become questionable anyway. If (27) is used then the sacrifice in accuracy at the discontinuities in cases (ii) and (iii) is always greater than the gain in accuracy away from the discontinuity, which is simply not realistic. Thus, before a criterion can be found (other than sheer experience) for deciding in any given case whether or not the smoothing technique should be applied, it will be necessary to find a measure of the sort used in (27) which gives more weight to the regions away from the discontinuities (and from which practical results can be obtained). Clearly much remains to be done.

In the meantime, the smoothing technique will be found an exceedingly simple and useful device for controlling the behaviour of the more awkward series, which so often arise in the solution of practical problems.

References

LANCZOS, C. (1956). *Applied Analysis*, Prentice Hall Inc., Englewood Cliffs, N.J.
JAHNKE, E., and EMDE, F. (1945). *Tables of Functions*, Dover: New York.

Errata

We regret that some errors occurred in Reeves, C. M.(1967). Description of a syntax-directed translator, *Computer Journal*, Vol. 10, p. 244. On p. 252, L.H. column, line 31, and p. 255, R.H. column, line 11 the symbols before the second K and before the L should be closing angled brackets and not "greater than" symbols.