rule for moving halfway toward the centroid be retained to deal with constraint violation.

(3) Obviously the method fails when the centroid falls into a non-feasible region as often occurs when searching non-convex spaces. We have found that it is desirable to continue moving toward at least a local optimum, and to effect this the method can be modified as follows. The centroid of the remaining points should be checked for feasibility before rejecting a point. If the centroid is found to be unfeasible, then the procedure is to discard all except the best point of the complex and to construct a new complex according to $x_i = x_0 + r_i(x_c - x_0)$. Here x_0 is the best point of the old complex, x_c is the unfeasible centroid, and r_i is a random number over the interval (0, 1). The construction of the new complex is restricted to a more favourable subspace of the original region and it will continue movement in the direction of an optimum. In constructing the new complex, rule (2) is invoked for constraint violation.

In actual practice the three modifications listed above were found to allow the complex method to reach the optimum in situations where it would have ordinarily terminated.

Correspondence

To the Editor,
The Computer Journal.

Gödel's theorem

Sir,

I would like to mention a correction to a statement of mine in a review of Arbib's book (this *Journal*, Vol. 8, p. 88).

It has been held by some people that Gödel's theorem shows that a man's reasoning transcends that of any Turing machine. I denied this in my review and suggested that the catch might be that both machine and man might not have enough internal states to complete Gödel's construction in all cases. This conjecture is false, as pointed out by Alan Tritter, but I still do not believe that a mathematical theorem can prove a metaphysical statement. I think I have given a proper discussion of the problem in my article "Human and Machine Logic", *British Journal of Philosophy and Science*, Vol. 18 (1967), pp. 144–147.

Yours sincerely,

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24 October 1967.

To the Editor,
The Computer Journal.

Solution of linear differential equations

Sir,

A recent paper (Davison, 1967) proposes a step-by-step method for solving the set of simultaneous first-order linear time-invariant equations

$$\dot{x} = Ax + Bu(t). \tag{1}$$

For an nth order formula, with step size h, the truncation error is $O(h^{n+1})$, n even, or $O(h^{n+2})$, n odd. An alternative approach gives equations of the same form, but with truncation error $O(h^{2n+1})$. These equations are (taking $t = 0$ to $t = h$ as a typical step)

$$\begin{aligned} x(h) = {} & \left[\sum_{j=0}^{n}(-)^j K(n,j)(Ah)^j\right]^{-1} \times \left[\left(\sum_{j=0}^{n} K(n,j)(Ah)^j\right)x(0)\right. \\ & + \sum_{k=0}^{n-1}\left(\sum_{j=0}^{n-1-k} K(n,j+k+1)(Ah)^j\right)Bu^{(k)}(0)h^{k+1} \\ & \left. + \sum_{k=0}^{n-1}\left(\sum_{j=0}^{n-1-k}(-)^{j+k}K(n,j+k+1)(Ah)^j\right)Bu^{(k)}(h)h^{k+1} + C_n\right] \end{aligned} \tag{2}$$

where
$$K(n,j) = \frac{n!(2n-j)!}{(2n)!(n-j)!j!}$$

and
$$\begin{aligned} C_n &= (-)^n \frac{(n!)^2}{(2n)!(2n+1)!}h^{2n+1}x^{(2n+1)}(\tfrac{1}{2}h) + \ldots \\ &= (-)^n \frac{(n!)^2}{(2n)!(2n+1)!}\delta^{2n+1}x(\tfrac{1}{2}h) - \ldots \end{aligned}$$

represents the truncation error.

The well-known trapezoidal formula, with a truncation error $O(h^3)$, is given by $n = 1$; $n = 2$ gives

$$\begin{aligned} x(h) = {} & (I - \tfrac{1}{2}Ah + \tfrac{1}{12}A^2h^2)^{-1} \times \\ & \left[\left(I + \tfrac{1}{2}Ah + \frac{1}{12}A^2h^2\right)x(0) + \left(\tfrac{1}{2}I + \frac{1}{12}Ah\right)Bu(0)h\right. \\ & + \left(\tfrac{1}{2}I - \frac{1}{12}Ah\right)Bu(h)h + \frac{1}{12}B\dot{u}(0)h^2 - \frac{1}{12}B\dot{u}(h)h^2 \\ & \left. + \frac{1}{720}\delta^5x(\tfrac{1}{2}h) - \ldots\right]. \end{aligned} \tag{3}$$

Davison's proposed formula involves polynomials in A up to A^3 and derivatives of u up to the third order; the leading term of the expression corresponding to C_n is $\frac{1}{120}\delta^5x(\frac{1}{2}h) - \frac{1}{48}A^4h^4\delta x(\frac{1}{2}h)$. Thus equation (3) is to be preferred, since it is easier to calculate and has a comparable truncation error.

Equation (2) may be proved thus: consider

$$C_n = \sum_{j=0}^{n}(-)^j K(n,j)h^j[f^{(j)}(h) - (-)^j f^{(j)}(0)] \tag{4}$$

put

$$f^{(j)}(h) - (-)^j f^{(j)}(0) = D^j[\exp(\tfrac{1}{2}hD) - (-)^j\exp(-\tfrac{1}{2}hD)]f \tag{5}$$

where D represents the differential operator at time $t = \frac{1}{2}h$. The resultant expression can be put in the form

$$C_n = (-)^n\{n!/(2n)!\}\pi^{1/2}(hD)^{n+1/2}I_{n+1/2}(\tfrac{1}{2}hD)f \tag{6}$$

where $I_{n+1/2}$ is a modified spherical Bessel function of the first kind; on expansion

$$C_n = (-)^n\frac{n!}{(2n)!}(hD)^{2n+1}\sum_{j=0}^{\infty}\frac{(n+j)!}{j!(2n+2j+1)!}(\tfrac{1}{2}hD)^{2j}f. \tag{7}$$

Results equivalent to this have been derived previously, at least for $n = 1, 2, 3$; see Section 8.11 of Buckingham (1957).

If f is replaced by x, and derivatives of x replaced by

$$x^{(j)} = A^j x + \sum_{k=0}^{j-1} A^{j-1-k} B u^{(k)}, \tag{8}$$

which can be derived from equation (1), equation (2) follows.

It is interesting to compare equation (2) with the formally exact solution

$$x(h) = \exp(Ah)x(0) + \int_0^h \exp\{A(h-t)\}Budt. \tag{9}$$

It will be seen that $\exp(Ah)$ is approximated by a rational function; the expression is, in fact, the Padé approximation, which is best in the sense that, given the degrees of numerator and denominator, the coefficients are chosen so that the Taylor series for the approximation agrees with that of $\exp(Ah)$ for as many terms as possible, up to the term in $(Ah)^{2n}$ in the present case.

Yours faithfully,

W. E. Thomson

P.O. Research Station,
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August 1967

References

Davison, E. J. (1967). A high-order Crank–Nicholson technique for solving differential equations, *The Computer Journal*, Vol. 10, p. 195.

Buckingham, R. A. (1957). *Numerical Methods*, London: Pitman.

Management information systems

A series of one-day seminars entitled "Keys to Management Control" were presented in November at London, Bristol, and other cities, by the Electronic Data Processing Division of Honeywell Controls Ltd. The speakers included members of Honeywell, Boston, staff. A booklet bearing the same title containing a synopsis of the speeches may be obtained, while stocks last, from the divisional headquarters in the U.K. at Great West Road, Brentford, Middx., but it lacks some of the vital statistics presented in the seminar.

It was reported that in USA there were many companies whose computer costs absorbed between \$2 and \$12 per \$1000 of sales, with a median around \$5.4. In one class of business, hardware rentals, depreciation and maintenance accounted for 38%, systems planning 19% and operating 43%, of total computer costs. Among a group of 33 companies, only 8 had so far planned to centralize their management information system, 14 were definitely decentralized, and 11 were undecided, or had not yet defined the information needed.

The economics and productivity, of the data collected, processed and information produced, must be analysed so that improvements could be made towards greater productivity. Managers must define their information requirements clearly for each job, each decision, each long-term planning operation. Precision of data, timeliness in input, rapid processing and meaningful presentations of resultant information were problems for the system designer. A senior executive should be given top responsibility for co-ordinating the enquiry and development of a complete system covering all functions. The job must be "managed full time by the kind of guy who gets things done!"

This speaker also referred to the "Cornucopia of Confusion" found in industry (U.S.?) by accumulated attempts to introduce, in turn, integrated data processing, total systems, information and control systems, on-line real-time systems, etc. Management at all levels must develop sufficient knowledge of computer potential, to define their requirements of information so that those responsible for designing an efficient system could see them in proper perspective.

The next speaker displayed four large charts, each covering an area of data processing in a manufacturing company, for which it was claimed, application packages are already developed. Files previously held in several departments to cover product specifications, materials required, etc., with delayed updating and frequent problems of incompatible detail, could conveniently be centralized on magnetic tapes or discs, promptly amended and reissued. There are packages also to analyse and forecast demand and control stocks. But all these depend on adequate and prompt data collection, communication and input. Data collection presupposes adequate classification, counting and recording facilities at critical stages in manufacture: it emerged in discussion that certain highly automated factories found it difficult to record work-in-progress accurately: with continuous processes and infrequent physical inventories, the absence of accurate work-in-progress data could be misleading to three of the four functional areas charted. It remains the most difficult and challenging area for data collection in factories: the mechanical devices for centralized recording of what the operators think they see (and remember to enter) may, on occasion, merely accelerate the consumption of incorrect figures, unless well-designed editing routines are included in stock record and w.i.p. updating programs.

The direct access storage requirements of any complete system requiring such rapid access were considerable. The problems of designing an adequate system for a company with many manufacturing branches was considerable. It was recommended that no one should attempt to go too soon into a fully integrated on-line system. Progress would best be achieved by batch processing; dedicated data collection would follow, using on-line entry (with programs which edited the data during entry) to reduce cost of punching and verifying and intermediate media. The wheel seems to have come full circle from the days when we were all urged to get absolutely accurate data carded or taped before entering the (slow) computer area. The economies depended on the total cost of alternative systems, and urgency considerations. It is claimed, in the penultimate page of the booklet, that on-line data communication for most applications costs less, causes fewer delays and creates fewer errors than off-line data conveyance. Further discussion of experience in this area is invited for this *Journal*.

H. W. G.