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Correspondence

To the Editor
The Computer Journal

Generation of time delays on analogue computers

Sir,

I have one or two comments on the paper by Riley and Walker (1968) dealing with rational approximations to $\exp(-sT_D)$ and their use in generation of time delays for analogue computers (this *Journal*, Vol. 11, p. 72).

First, a matter of terminology: the paper uses the term 'Padé approximation' to mean any suitable rational approximation. This term should be confined to a rational function for which the first N terms of its Maclaurin expansion agree with those of the function being approximated; N is the number of independent parameters. Thus in the paper only Set 1 is a Padé approximation; the expansion agrees with that of $\exp(-sT_D)$ up to the term in s^8.

Since the Padé approximation devotes all its parameters to securing desired behaviour at the origin, it is not surprising that it is not the best if one wants the delay of the corresponding system to be approximately constant over a range of frequencies. If 'approximately constant' is given some precise meaning, then a well-defined problem exists.

Much work has been done in this area by circuit theorists (Kiyasu, 1943; Thomson, 1949; Abele, 1960; Ulbrich and Piloty, 1960). The first two deal with the Padé approximation (although not in fact using this term) and include analytical formulae for the delay. The others give numerical solutions for systems with an equal-ripple approximation to constant delay. Two results, derived from Abele's tables, may be of interest; a_1 to a_4 and T_D have the same meaning as in Riley and Walker, and the group delay lies within the range $T_D(1 \pm c)$ for $0 \leqslant f \leqslant f_0$.

Another approach is that of Hausner and Furlani (1966), who give design tables for equal-ripple approximations both for phase and for phase delay.

c	T_D/a_1	T_D^2/a_2	T_D^3/a_3	T_D^4/a_4	f_0T_D.
0·01	2·020	9·052	79·69	1126	1·040
0·02	2·041	8·975	80·63	1054	1·133

The coefficients of Sets 2 and 3 of the paper are in the same neighbourhood as these.

Yours faithfully,
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14 May 1968

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(*Further correspondence appears on pp.* 194 *and* 240)