between $p[m]$ and $r[m]$, with a weighting dependent on $p[m]$. If the variation in the weighting over the region of averaging is neglected, (6) may be replaced by

$$D = \frac{N}{2} \sum_m a[m]^2/r[m]. \tag{25}$$

If now the region of averaging is taken as being identical to the range of application of $p[m]$ where $p[m] = r[m]$, the average value of (25) is given by (16), substituting the optimum value of x from (22), as

$$(M - 1)/2 \tag{26}$$

Combining (24) and (26) gives the expected additional cost incurred with relative frequencies of states $r[m]$ as

$$\left(\frac{M-1}{2}\right)(\ln N/12 + 1) - \tfrac{1}{2} \sum_m \ln r[m] - \ln (M - 1)! \tag{27}$$

This expression represents the expected total cost additional to the message length which would be required to give the states of the N things using optimum label lengths of $-\ln r[m]$. The expected total message length is therefore

$$\left(\frac{M-1}{2}\right)(\ln N/12 + 1) - \tfrac{1}{2} \sum_m \ln r[m] - \ln(M-1)! - N \sum_m r[m] \ln r[m]$$
$$= \left(\frac{M-1}{2}\right)(\ln N/12 + 1) - \ln (M-1)! - \sum_m (n[m] + \tfrac{1}{2}) \ln r[m] \tag{28}$$

where $n[m]$ is the number of things in state M. The above form is essentially that used in the body of the paper for the information needed to record the class of each thing, and to record the values of multistate variables.

It is worth noting that in size and shape, the region over which an M-tuple $p[m]$ is applied is essentially similar to the region of expected error in the estimation of the probabilities of a multinomial distribution based on a sample of size N. Thus the message transmitted to nominate the M-tuple is sufficient to convey essentially all the available information about the probability of occurrence of each state.

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Correspondence

To the Editor
The Computer Journal

A new method for solving polynomial equations

Sir,

Reading the article about solving polynomial equations by Garside, Jarratt and Mack (this *Journal*, Vol. 11, p. 87), I wonder if the method can be further improved by successive long divisions of $F(Z)$ into a continued fraction with the repeated roots consequently removed. In general, for an nth order polynomial their

$$F(Z) = \cfrac{a_1}{Z + b_1 + \cfrac{a_2}{Z + b_2 \cfrac{a_n}{Z + b_n}}}$$

For a repeated root a_n should $=0$ and $-b_n$ be the repeated root. Should a_n be very small it could cause unnecessary trouble were it purely a round-off error. Consequently, if small values of the coefficients are not to be ignored, they should all be calculated to double length with only the single length used later for evaluation of the continued fraction. This would seem a small price to pay for separating out the repeated roots.

It may happen that part of the way through the repeated division process a leading coefficient (or coefficients) of the numerator becomes zero. This will result in the next division giving a second (or higher) order polynomial with correspondingly less undone divisions. This must be catered for in the program. Also if the leading coefficient is very small, it could cause very large coefficients in the next stage. Should this happen, all the coefficients should be sealed and this may result in the leading one becoming zero.

In calculating the continued fraction, it may happen that one denominator vanishes. This must make the next fraction zero.

Yours sincerely,

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(*Further correspondence appears on pp.* 172 *and* 240)