

deduce the same result for Algorithm B unless expressions (6) and (7) are identical. Instead we have the more favourable theorem:

THEOREM 2

The matrix  $D^*$ , defined by equation (9) and Algorithm B of Section 2, is such that, for any set of directions  $d_1, d_2, \dots, d_n$ , we have the inequality

$$\Delta(D^*) \leq \Delta(D). \quad (16)$$

PROOF

For Algorithm B the multipliers  $\alpha_1, \alpha_2, \dots, \alpha_n$  are defined by equation (2),  $d_1^*$  is defined by equation (7), and for  $t \geq 2$  the definition of  $d_t^*$  is given by equation (8). It follows that the vector  $d_1^*$  is normalised, and is orthogonal to the other calculated vectors,  $d_2^*, d_3^*, \dots, d_n^*$ . Therefore from the definition (11) we obtain the equation

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## Book Review

*Asymptotic Methods in the Theory of Linear Differential Equations*, by S. F. FESHCHENKO, N. I. SHKIL' and L. D. NIKOLENKO, 1967; 270 pp. (Barking, Essex: Elsevier Publishing Co. Ltd., 140s.)

This book was originally written in Russian and translated very efficiently by Scripta Technica, Inc. It contains an advanced treatment to a rather specialised topic and is only suitable for mathematical graduates. It describes some methods for obtaining approximate solutions to linear differential equations in which the coefficients vary slowly in time. If  $t$  denotes the normal independent variable and  $\epsilon$  is some small positive parameter, then a slow time variable  $\tau = \epsilon t$  is introduced, and equations are considered whose coefficients are functions of  $\tau$ . This occurs, for example, in a single equation in which the coefficient of the highest derivative is small, or in Sturm–Liouville equations in which large values of the eigenvalue are being considered. The basis of the methods is to find an approximate solution in the form of an asymptotic series in  $\epsilon$  combined with an oscillatory term.

$$\Delta(D^*) = \sum_{i=2}^n \sum_{j=2}^n [(d_i^*, d_j^*) - \delta_{ij}]^2. \quad (17)$$

Let  $\Omega$  be the orthogonal matrix that is consistent with equations (8) and (10), and let

$$\tilde{D} = D\Omega. \quad (18)$$

Because the last  $(n-1)$  columns of  $\tilde{D}$  are identical to the last  $(n-1)$  columns of  $D^*$ , equation (17) provides the inequality

$$\Delta(\tilde{D}) \geq \Delta(D^*). \quad (19)$$

Now by the argument of Theorem 1 we obtain the identity

$$\Delta(\tilde{D}) = \Delta(D), \quad (20)$$

so Theorem 2 is an immediate consequence of inequality (19).

Both the algorithms and the stability theorems have been checked by numerical examples, and they confirm that the method of this paper is satisfactory.

After a brief historical introduction, in Chapter 1 is given a method for solving a single second-order ordinary equation in cases of both resonance and non-resonance. This is generalised in Chapter 2 to simultaneous second-order equations. In Chapter 3 it is shown how to decompose a set of simultaneous first-order equations  $\dot{x} = Ax + b$  into a number of sets of mutually independent equations, and the complications of solving these when  $A$  has some multiple characteristic values with non-linear elementary divisors is given in Chapter 4. The method is generalised in Chapter 5 to the solution of equations in Banach space. Finally, in Chapter 6 the solution of a particular hyperbolic partial differential equation is described in which the method of separation of variables is used to produce a system of ordinary differential equations which are solved by the method given in Chapter 5.

The book is well printed and only a small number of misprints were noticed.

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