

be of the same order of magnitude as the maximum eigenvalue of  $M$ .

The simple numerical example chosen has  $r = \frac{\Delta t}{h^2} = 5$  and this large value gives a condition number of approximately 10 for the third order method.

## References

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## Book Review

*Chebyshev Polynomials in Numerical Analysis*, by L. Fox and I. B. PARKER, 1968; 205 pages. (Oxford University Press, 42s.)

Although Chebyshev polynomials play an important role in modern numerical analysis, they still attract little attention in most of the mathematics degree courses in Britain. This book is intended to meet the need for a comprehensive treatment at a level suitable for undergraduates.

The field is covered in seven chapters in which the authors explain the relevance of Chebyshev polynomials to problems of polynomial approximation, develop their properties, and apply them to several fundamental problems of numerical analysis. If a criticism of the choice of material can be made, it is that a quarter of the whole book is devoted to the solution of ordinary linear differential equations with polynomial coefficients. (Though interesting, this topic is of limited application, and probably only the enthusiastic specialist will wish to follow the detailed comparisons between the various, and closely similar, refinements of Lanczos'  $\tau$ -method.) The material is generally presented in a pleasantly readable manner; it is appropriate that in such a book the main aim should be to interest and inform the reader, rather than to achieve complete rigour.

Unfortunately the book is marred by defects which, though few in number, are sometimes serious in content. Perhaps the most disconcerting appear in Chapter 2, where the convergence of Chebyshev and Fourier series are compared. We read that for smooth functions the order of magnitude of the  $k$ th coefficient in the Chebyshev series is '... considerably smaller for large  $k$  than the  $k^{-3}$  of the best Fourier series'. This is not an isolated slip; similar remarks occur elsewhere in this chapter. It is not easy to see the authors' intention here—coefficients in a Chebyshev series are, of course, coefficients in a corresponding Fourier series (as the authors note in Exercise 3, p. 46) and may converge arbitrarily rapidly.

The weighty Chapter 5 also has its flaws. On p.109 there is a system of linear algebraic equations which allegedly causes embarrassment to the method of backward recurrence. It is clear, however, that this method will readily deal with the system obtained by simply omitting the first two equations;

The size of the ratio  $r$  will always be restricted because of the need to match the errors in the space discretisation and the time integration and so although there is a possibility that the linear systems could be ill-conditioned this is unlikely to happen.

these can be solved subsequently to give  $a_0$  and  $a_1$ . On the following page is an account of 'a type of problem in which there is no need to satisfy an initial condition'. In this section the relaxation of rigour has gone too far; it is essential in such a problem to ensure that no unwanted solution intervenes. The fact that 'the relevant initial condition  $y(0)=1$  is inherent' in the differential equation would not have helped if the authors had considered the function  $e^{-x}\text{Ei}(x)$ , for example, instead of  $e^x\text{Ei}(-x)$ . In this case another boundary condition (at  $x=1$ ) would be required, and *apparent* convergence of the numerical process would provide no indication of success.

The account of Lanczos' canonical polynomials could perhaps be simplified. An appropriate starting point might be the fact (not mentioned in the book) that a polynomial  $Q_m(x)$ , canonical with respect to the linear operator  $L$ , satisfies  $LQ_m(x) = x^m$ .

In the example given on p.148 we should then find, using the inverse operator  $L^{-1}$  formally (which use can be justified subsequently), that since

$$\begin{aligned} Ly &= (1+x)y' + y, \\ Lx^m &= (m+1)x^m + mx^{m-1}, \\ \text{and hence } x^m &= (m+1)Q_m + mQ_{m-1}. \end{aligned}$$

Thus we obtain  $Q_0 = 1$ ,  $Q_1 = \frac{1}{2}(x-1)$ ,  $Q_2 = \frac{1}{3}(x^2-x+1)$ , . . . .

Whichever development is used, however, it must be stressed that the canonical polynomials do not necessarily exist for all  $m$ ; nor, if they do exist, are they necessarily unique. These points are not mentioned, though they are perhaps hinted at in Exercise 22 on p.154. Incidentally, Exercise 19 on this page sets a formidable task: 'Find a rational approximation to  $e^z$ , valid at every point in the complex plane.'

One should not dwell too long on these infelicities, however. That they have been unearthed is due in no small measure to the interest which the authors aroused in at least one reader. The book is certainly the best that has yet appeared in English on this topic and at this level. With a little careful editing the second edition could be a very good book indeed.

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