

and the asymptotic error constant has the form

$$C = 2 \left( \frac{\alpha b_2}{d_1} + 1 \right) A_2^3 - 3 \left[ 1 + \frac{1}{2} \left( \alpha + \frac{2}{d_1} \right) \right] A_2 A_3 \\ + \left[ 4 \left( \alpha^3 a_2 + \frac{a_3}{d_1^3} \right) + 1 \right] A_4,$$

where  $d_1 = b_1 + b_2$ .

Now equations (4.3) represent a set of four equations in the six parameters  $a_1, a_2, a_3, b_1, b_2$  and  $\alpha$ , and hence we can solve for any four in terms of the remaining two.

However, it is convenient to write  $\mu = \frac{1}{b_1 + b_2}$  and solve the set for  $a_1, a_2, a_3$  and  $b_2$  in terms of  $\alpha$  and  $\mu$ . The general solution for  $\alpha \neq \mu, \alpha \neq -\frac{2}{3}, \alpha \neq 0, \mu \neq 0$  is

$$a_1 = \frac{3\mu(1 + 2\alpha) + 3\alpha + 2}{6\alpha\mu}, a_2 = \frac{3\mu + 2}{6\alpha(\alpha - \mu)}, \\ a_3 = \frac{3\alpha + 2}{6\mu(\mu - \alpha)}, b_2 = \frac{3(\mu - \alpha)}{2\alpha\mu(3\alpha + 2)},$$

and the asymptotic error constant can also be expressed in terms of  $\alpha$  and  $\mu$  in the form

$$C = \frac{3(\alpha + \mu) + 4}{(3\alpha + 2)} A_2^3 - 3[1 + \frac{1}{2}(\alpha + 2\mu)] A_2 A_3 \\ + \frac{1}{3}[6\alpha\mu + 4(\alpha + \mu) + 3] A_4.$$

## References

- JARRATT, P. (1966). Multipoint iterative methods for solving certain equations, *Computer Journal*, Vol. 8, p. 398.  
 TRAUB, J. F. (1964). *Iterative Methods for the Solution of Equations*, Englewood Cliffs, N.J.: Prentice-Hall.

## Book Review

*Introduction to Automata*, by R. J. NELSON, 1968; 400 pages. (New York: John Wiley and Sons, 112s.)

A recent review of another book on automata, in this *Journal*, pointed out that we are still lacking a systematic survey of the broad field. The present book fills that gap. The author points out that very little of the material is new. On the other hand very few important topics are omitted, nor is this wide cover achieved by treating topics sketchily, quite the reverse.

Following a review of basic mathematical requirements, Chapter 2 surveys the theory of recursive functions; in chapter 3 formal systems are defined and their basic properties developed. Propositional logic and program systems are discussed as examples, and finally automata and the three subspecies, acceptors, detectors and generators are defined in terms of semi-Thue systems.

Chapter 4 is devoted mainly to Turing machines. The relation with partial recursive functions and the ability to accept and generate the recursively enumerable sets is established. Decision procedures and unsolvability problems are discussed, a Universal Turing machine is constructed and a proof of the equivalence between Post and Turing machines is given.

Table 2

$n$	$a_1$	$a_2$	$a_3$	$b_1$	$b_2$	$\alpha$	ASYMPTOTIC ERROR CONSTANT
1	$\frac{1}{4}$	0	$\frac{3}{4}$	0	$-\frac{3}{2}$	$-\frac{1}{2}$	$A_2^3 - \frac{1}{4}A_2A_3 + \frac{1}{9}A_4$
2	0	$\frac{1}{4}$	$\frac{3}{4}$	0	-3	-1	$-\frac{1}{2}A_2A_3 - \frac{1}{9}A_4$
3	$\frac{1}{4}$	0	$\frac{3}{4}$	$-\frac{3}{4}$	$-\frac{3}{4}$	-1	$A_2^3 + \frac{1}{2}A_2A_3 + \frac{1}{9}A_4$
4	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{4}{6}$	$-\frac{1}{2}$	$-\frac{3}{2}$	-1	$\frac{1}{2}A_2^3$
5	$-\frac{1}{5}$	$\frac{3}{4}$	$\frac{9}{20}$	$\frac{1}{5}$	$-\frac{2}{5}$	$-\frac{1}{6}$	$\frac{4}{3}A_2^3$

Thus, provided equations (4.3) are satisfied, we can generate particular members of a family of fourth order methods by assigning specific values to the two free parameters  $\alpha$  and  $\mu$ . In Table 2 we have collected together solutions which simplify the form of the iterative formula or the asymptotic error constant.

One other reasonable choice of parameters is that for which  $b_1 = b_2$ , which minimises the possibility of cancellation in the denominator of  $\omega_3$ . The solution in this case is  $a_1 = \frac{1}{4}, a_2 = 0, a_3 = \frac{3}{4}, b_1 = b_2 = -\frac{3}{2}, \alpha = -1, \mu = -\frac{2}{3}$  and the iterative formula is

$$x_{i+1} = x_i - \frac{4f(x_i)}{f'(x_i) + 3f' \left[ x_i - \frac{4f(x_i)}{3\{f'(x_i) + f'[x_i - u(x_i)]\}} \right]}.$$

Chapter 5, accounting for a quarter of the book, deals with minimisation and decomposition of finite state machines, typically transition systems. Chapter 6 on networks treats the state assignment problem and state minimisation. The final two chapters on acceptors and generators, respectively, develop an extension of Kleene's regular sets, and introduce mathematical linguistics along the lines of Chomsky.

As formal textbooks go, this one is very readable; well chosen examples appear throughout to illustrate results, and problems (without solutions) are provided at the end of each major section within a chapter. Misprints are fairly numerous, but in the main obvious, and errors appear to be very few and trivial. The book is well produced although, on the reviewer's copy, a number of pages were ink deficient.

Based on the author's lectures at Case Institute of Technology, the book is addressed to advanced undergraduate and graduate students and the hope is expressed that the book will be of use to mathematicians and philosophers as well as computer scientists and engineers. (Some previous exposure to mathematical logic, algebra and switching theory is an asset though not a prerequisite.) To all of these and anyone teaching the subject I warmly recommend this book.

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