

This shows why  $\alpha$  'small' is a necessary condition for an accurate solution. (See also du Fort and Frankel [2], pp. 142–144.)

The reported results seem to indicate that out of the range of SLL and DUFF schemes (9) and (13) are to be preferred, and that a 'small value' of  $(h/k)$  would be less than  $1/40$ , say. For very small values of  $h$  ( $1/80$  or less) the schemes (9) and (13) can still therefore be competitive.

From the list of operations necessary per point, it can be seen that the Implicit and A.D.I. methods are not much more expensive than the Explicit schemes, and one

can use a large value of  $R$  in them with more safety than in SLL schemes. Although they do require more working space, this is only likely to be serious in three dimensional problems on a small computer.

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## Book Review

*Numerical Integration*, by P. J. DAVIS and P. RABINOWITZ, 1967; 230 pp. (Waltham, Massachusetts: Blaisdell Publishing Company, \$7.50.)

This monograph is intended to give a concise but comprehensive account of the available formulas and methods for the numerical evaluation of integrals. The subject is presented from the application point of view and the treatment is modern and computer oriented.

Chapter 1 contains an introduction to the concept of numerical integration and also gives some useful analytical background material. Chapter 2, by far the longest, deals with approximate integration over a finite interval and includes special sections on the treatment of periodic functions, rapidly oscillatory functions, contour integrals, improper integrals and indefinite integration. In Chapter 3 is a brief description of special methods and formulas for integration over an infinite range, while Chapter 4 covers the subject of error analysis and includes a treatment of the effect of round-off error following the general analysis given by J. H. Wilkinson. The effect of truncation error is analysed through Peano's theorem, differences, the theory of analytic functions,

and functional analysis. The topic of multi-dimensional integrals follows and includes a large section devoted to Monte Carlo methods. The last chapter deals with automatic integration schemes, which are classified as adaptive or non-adaptive and iterative or non-iterative, and has special sections devoted to Romberg's scheme and methods using Chebyshev polynomials. The book is concluded by five appendices containing a reprint of the article 'On the Practical Evaluation of Integrals' by M. Abramowitz, some FORTRAN programs, and bibliographies of ALGOL procedures, tables, books and articles.

The text is exceptionally well written, and very well organised, and the link between theory and application is, throughout, strongly forged with the help of numerous illustrative examples. The authors have carefully dug into the literature and concluded each section with a useful set of references. The book is a 'must' for any one interested in the practical evaluation of integrals, and will undoubtedly be of great value to all who wish to make a serious study of numerical integration.

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