individual points are each used once only, in calculating the values of b for the zero degree polynomial, and the values of d. The coordinates, and the values of the orthogonal polynomials at the data points, need not be stored. Thus considerable store space is saved, especially if the number of points is large.

Let n be the degree of the polynomial approximation required, and m be the number of data points. In the case of curve fitting, the modified method needs approximately $5n$ locations of store. A method which stored the values of the polynomials at each of the data points would need approximately $4m + n$ locations.

In the case of surface fitting, the number of store locations needed for the two methods are approximately $\frac{6}{5}(n + 1)^3$ and $m(2n + 3) + \frac{2}{3}(n + 1)^3$ respectively.

In the case of curve fitting, the two methods take approximately the same amount of computer time. For surface fitting, the modified method requires less time. The time saving may be illustrated by an example in which a surface of degree 8 was fitted to 150 points. On an ICL 1905E computer, a program written by the author which used the values of the orthogonal polynomials at each of the data points took 23 seconds. A program using the modified method took 8 seconds, of which 2 seconds were spent finding the values of d, and the values of b for the zero degree orthogonal polynomial.

Numerical problems

When curve fitting, one can normalise the x-coordinates of the points so that they lie in the range $(-1, 1)$. If the points are not correctly normalised, and if any points lie outside this range, this causes ill-conditioning. Ill-conditioning also occurs if any large portion of the range is unoccupied by points.

When surface fitting, one must normalise the points so that they lie within the square $|x| < 1$, $|y| < 1$, and so that there are no large regions within this square unoccupied by points. If the points occupy a region in the shape of a parallelogram, then such normalisation is possible. However, if, for example the points occupy the interior of a triangle, then they cannot be satisfactorily normalised. The best one can do is to choose a parallelogram whose boundary lies as close as possible to the boundary of a triangle, and normalise the points as if they occupied the interior of that parallelogram. In the former case numerical difficulties are not likely to arise. In the latter case it may be necessary to restrict the degree of polynomial approximation to prevent numerical difficulties.

The modified method of surface fitting makes use of products of Chebyshev polynomials in x and y. These polynomials have been chosen because they may be calculated using a simple recurrence relation. If the data points can be satisfactorily normalised, they do not cause ill-conditioning. It may be possible to prevent ill-conditioning in certain other cases by using some other polynomials in x and y. The author has not investigated this possibility.

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Book Review

Decision Analysis, by Howard Raiffa, 1968; 309 pages. (*Addison-Wesley*.)

The author, a self confessed 'Bayesian' and a very persuasive one I might add, has transposed a series of lectures on decision analysis into a very readable text book. These lectures were given at various times and not always in the continuous entity that comprises this work. This, however, does not mitigate against cohesiveness and the text flows in a calm, logical and very acceptable manner. Almost unique is the author's amiability in footnoting any section or chapter not vital to the theme of his argument so that the reader may bypass it if he so wishes. In fact, Mr Raiffa openly declares when he digresses and rides a personal hobby horse. One could wish for this in all authors.

One could also wish that the subject of 'Decision Analysis in Conditions of Uncertainty', as outlined so admirably, might find an audience and indeed be practised in the senior management levels of most UK companies. The ideas of PERT, network analysis and resource allocation are gaining adherents rapidly, and so they should. However, the decision point must have been reached before these disciplines can be of use. One suspects that in many cases the decision point may well have been arrived at by almost caveman-like processes of thought or hunch or just plain stubbornness.

A first casual flick through the pages might daunt the non-mathematician for there is an apparent profusion of algebraic formulae. This is not the case; in the main it is quite basic and easily comprehended. The work progresses from the first analysed elements of a basic decision, highlighting decision points or where chance prevails, through probability assessments, payoffs and the use of judgemental probability. The concluding two chapters deal with implementation of real rather than experimental situations and a final bibliographical 'walk through' that is far better than a mere list of references. This book can be recommended as reading for all practising decision makers for it is not merely an exercise in philosophical argument. For those who have a sincere desire for a disciplined approach to decisions, it is a must; for those who are inclined to jump in with both feet it will be a salutary exercise.

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