factor $1 - \frac{h^2}{12}c^2$ cancels throughout and equation (19) reduces to

$$\delta^2 y_j + \frac{h}{2}c(y_{j+1} - y_{j-1}) + h^2\left(1 + \frac{1}{6}\delta^2\right)g_j y_j = h^2\left(1 + \frac{1}{6}\delta^2\right)r_j \quad (21)$$

which again has a local truncation error of order $\frac{1}{12}\delta^4 y$ (as in case (a)) and is the best three-point approximation to

$$y'' + cy' + g(x)y = r(x). \quad (22)$$

On solving equations (19) in conjunction with the boundary conditions (6) for the y_j, the full spline solution is obtained by direct substitution into equation (15) or (17) and (1). Derivatives at the nodal points may be calculated from equations (2) or (3).

We observe that a boundary condition of the form $\alpha y' + \beta y = \gamma$ at say $x = x_0$ may be approximated, on using equations (15), (17) and (2) by a two-term relationship connecting y_0 and y_1, so that the tri-diagonal structure of the equations for determining y_0 to y_n is retained.

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Book Review

Stochastic Approximation and Nonlinear Regression, by A. E. Albert and L. A. Gardner; 204 pages. (*MIT Press*, 154s.)

Statistical inference may be defined as the art of drawing sensible conclusions from variable data, and the history of statistical theory is to some extent a development of ways of describing the variability of data and of assessing the relative merits of various possible conclusions. The problem, in anything like complete generality, is vast. Under certain strong restrictions substantial progress has been made, particularly in the 'stationary parametric' case, when the data may be regarded as realisations of mutually independent random variables with a common probability distribution of known functional form which involves one or more unknown parameters. Here the 'sensible conclusion' that is sought is a useful approximation for the unknown parameters, usually expressed as a fallible statement (or 'estimate') of what their actual values seem to be, together with some information about the degree of reliability of this statement.

The theory of this approach was brought to its full flowering by the late Sir Ronald Fisher. Crippling though the imposed restrictions appear to be, the theory nevertheless had (and has) a wide field of strictly practical applicability. Where the functional form of the underlying probability distributions is not known, or cannot reasonably be guessed, a closely related procedure is available in which the experimental data are expressed as random deviations from their average values, these average values being known (or postulated) functions—often linear—of given experimental conditions and unknown parameters. This so-called regression approach is widely used, particularly when the data are thought to be influenced by several experimental factors. The parameters to be estimated (usually by 'least squares') are then measures of the sensitivity of the system to changes in the experimental conditions. In this type of situation a body of experimental data may be likened to a parcel of gold-bearing ore: the quantity of golden information potentially available is finite, and the most efficient mathematical processes must be used to extract it, however protracted the computations. In the last couple of decades, however, increasing attention has been paid to the more complex situation where, instead of being concerned with the analysis of a completed experiment, the inferences we are interested in are related to an evolving system in which data becomes available sequentially, and analyses have to be made repeatedly—and rapidly—as fresh data comes in. Here an inference leads to the immediate action of modifying the system: indeed the whole purpose of making a fresh observation is to decide how next to modify the system—the 'system' being, for example, the progress of a chemical industrial reaction. In this type of situation the older static concept of efficiency of estimation may well have to be abandoned since the speed with which the estimate can be computed may be an overriding factor.

As a result of research and development in this area the subject is beginning to define itself and books are now beginning to appear with titles such as *Optimization and Control in Stochastic Systems*. It is to this class that the work under review belongs. Given data $\{Y_n\}$ from a time-series whose mean-value function $\{F_n(\theta)\}$ is of known form but involves an unknown parameter-vector θ, the problem of estimating θ in this regression-type problem by an efficient and rapid method is tackled by a 'differential correction' recursive approach in which the estimate t_{n+1} of θ at the $(n + 1)$-th stage is defined in terms of t_n by an equation of the form

$$t_{n+1} = t_n + a_n\{Y_n - F_n(t_n)\}$$

where $\{a_n\}$ is a suitably chosen sequence of 'smoothing' vectors. The main aim of the monograph is to consider the effect of various choices of smoothing vectors on the estimates obtained.

EMLYN LLOYD (Lancaster)