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Appendix

Relative errors

We wish to evaluate the integral to a relative accuracy ϵ ; that is, if E is the error in the quadrature and I is the integral then we require $|E/I|$ to be less than ϵ . If the integrand always has the same sign the problem is straightforward; we adopt the same principle in §2 and require that in each subinterval (p, q)

$$\left| \frac{E_{pq}}{I_{pq}} \right| < \epsilon + \sum_{r=a}^{s=p} \left[\epsilon - \left| \frac{E_{rs}}{I_{rs}} \right| \right] \left| \frac{I_{rs}}{I_{pq}} \right| \tag{A.1}$$

This allows $|E_{pq}/I_{pq}|$ to be as large as possible while still keeping $|\Sigma E_{pq}/I| < \epsilon$. When the integrand changes sign the problem is more difficult because I may be small and because I_{pq} may be close to zero. This last problem can usually be overcome by jumping to the next subinterval if I_{pq} is found to be small. On the other hand if any I_{pq} is negative then $R = \Sigma |E_{pq}| / |\Sigma I_{pq}|$ may be larger than ϵ . In this case the calculation can be repeated after replacing ϵ in (A.1) by ϵ^2/R .

The use of (A.1) has been found to be satisfactory in practice.

References

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Book Review

Information Theory for Systems Engineers, by L. P. Hyvärinen, 1968; 205 pages. (No. 5 in *Lecture Notes in Operations Research and Mathematical Economics*.) (Berlin: Springer-Verlag, \$3.80.)

Being based on lectures given at the IBM European Systems Research Institute, this book is composed in terms of the interests of computer users and computer designers. Its level is very well within the scope of a modern undergraduate course in computer science or communication engineering, since it assumes only a basic knowledge of calculus and of the theory of probability and statistics. The author points out that information may be *syntactic* (commonly known to communication engineers as *selective*), *semantic* or *pragmatic*. Any process which does not destroy syntactic information is reversible: this is a formulation which, incidentally, could be shown to be parallel with reversible operations in thermodynamics which do not increase entropy. But data-processing

Table 2
Number of function evaluations required to evaluate $\int_0^1 f(x)dx$ to a specified accuracy

$f(x)$	ACCURACY ϵ	CCR-METHOD	ATLAS ROUTINE	4-PT GAUSS
$\frac{1}{1-0.998x^4}$	0.5 (-3)	125	122	216
	0.5 (-4)	137	181	238
	0.5 (-5)	133 ^b	311	260
	0.5 (-6)	241	548	414
	0.5 (-7)	277	a	480
$\frac{1}{1+100x^2}$	0.5 (-8)	397	a	678
	0.5 (-3)	41	23	62
	0.5 (-4)	53	32	84
	0.5 (-5)	61	52	84
	0.5 (-6)	61	92	106
	0.5 (-7)	97	157	194
	0.5 (-8)	145	248	216
	0.5 (-9)	193	432	260
	0.5 (-10)	253	742	348

a No convergence
b This number is correct although smaller than the number above it. In both cases the interval was finally subdivided in exactly the same way; in the upper case the failure of an early error test was detected using 4 additional function evaluations.

is often concerned with reducing the quantity of data in order to make it easier to grasp what remains, a process of reducing the syntactic information in order to increase the pragmatic information.

The treatment of coding is rather sketchy: it is largely confined to the types of parity check which are used within the structure of computer systems and barely mentions data transmission. The types of burst-correcting codes most suitable for transmission are not mentioned, but k -out-of- m codes (e.g. 2 out of 5 and 4 out of 8) are recommended for protection against bursts of errors. There are also more sophisticated methods of coding decimal inventory numbers than the simple complement digit described in this book.

In summary, the book is useful to a computer user who wishes to understand the customary parity check procedures but is not sufficient for a computer designer or programmer who wishes to devise more sophisticated error-protection.

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