I do not understand, in relation to my article, Dr. Wilby's point on document transports, that there is no need to look hopefully across the Atlantic for a solution. I quite agree with his sentiment, but nowhere in my article did I suggest that we should look to America, and that British firms could not provide the equipment.

I wish success to Crosfield and look forward to reading a detailed article on the actual experience of the application of one of their machines, to a large integrated system such as Public Utility Billing.

To the Editor
The Computer Journal

The calculation of orthogonal vectors

Sir,

In his description of a new method for the calculation of orthogonal vectors (this *Journal*, Vol. 11, p. 302), M. J. D. Powell rightly claims a reduction in computing time compared to the method previously used by Rosenbrock in his minimisation procedure. Some tests which I have made have suggested that the new method has another important advantage in that it can actually accelerate the convergence of the minimisation process, measured in terms of the number of function evaluations performed.

A comparison has been made in a particular type of application of Rosenbrock's method which I employed in collaboration with J. Culhane while employed by Sigma (Science in General Management) Ltd. The problem is to find the best locations for a number of factories for the manufacture of a commodity, and a number of depots at which it is transferred from bulk transportation to local transportation. It is assumed in the simplest case that the cost of transportation is proportional to the amount carried and to the distance, and is at different rates for bulk and local carriage. Given a set of co-ordinates of consumers, and their rates of consumption, as well as a set of co-ordinates of factories and depots, the cost of distribution can be evaluated. It is assumed that each consumer is supplied along the most economical pathway, as is reasonable if there are no limitations on capacity of factories or depots. Rosenbrock's method has been applied to find new positions for a subset of the factories and depots so as to reduce the distribution cost. The number of variables in the optimisation is twice the number of movable entities. They consist of the x- and y-co-ordinates of these entities. There are no constraints on the solution.

In a specific example with eight customers, two factories (one movable) and three depots (two movable) the original distribution cost was 18,894 units. Rosenbrock's method in its original form reduced this to 12,753 in 19 stages, involving 835 evaluations, thereafter coming practically to a standstill.

When Powell's method of orthogonalisation was used, the improvement continued for about 43 stages, involving 1,831 evaluations, and reducing the cost to 12,479. When the method was operated without any reassignment of the directions the cost fell to 13,882 in 12 stages, involving 403 evaluations, then showed no further significant improvement.

Comparison of the two methods of orthogonalisation has only been carried out in the context of this particular type of problem and there is clearly a need for further investigation. It is interesting to look for some reason for the apparent superiority of Powell's method, and to consider whether there might be still further ways of assigning the orthogonal vectors which might be even more favourable to the progress of the optimisation. The angular separation between corresponding elements of successive vector sets tends to be less with Powell's method than with Rosenbrock's original one, but the difference is not great.

Following Powell's notation, let the set of orthogonal unit vectors used in the stage of optimisation prior to the reassignment be $d_1, d_2, \ldots, d_n$, and the advances made in their directions $\alpha_1, \alpha_2, \ldots, \alpha_n$. Both methods compute the first of the new vectors as

$$d_1^* = (\alpha_1 d_1 + \alpha_2 d_2 + \ldots + \alpha_n d_n)/\textit{denominator} \qquad (1)$$

where the denominator is chosen to normalise the vector to unit magnitude.

Then in Rosenbrock's method a further $(n - 1)$ vectors are computed as follows:

$$\begin{aligned} d_2^1 &= \alpha_2 d_2 + \alpha_3 d_3 + \ldots + \alpha_n d_n \\ d_3^1 &= \alpha_3 d_3 + \ldots + \alpha_n d_n \\ &\vdots \\ d_n^1 &= \alpha_n d_n \end{aligned} \qquad (2)$$

From each of the vectors in (2) a unit vector is derived, orthogonal to all that have come before it. This is done by subtracting from it its projection on each of the previous unit vectors and normalising the resulting vector to unit magnitude.

If instead of (2) the following set of $(n - 1)$ vectors is taken

$$\begin{aligned} d_2^1 &= d_1 \\ d_3^1 &= d_2 \\ &\vdots \\ d_n^1 &= d_{n-1} \end{aligned} \qquad (3)$$

the same process of subtracting projections and normalising leads to the set of vectors obtained using Powell's method.

It seems reasonable to suggest that the superiority of Powell's method is related to the fact that the second of his orthogonal vectors, d_2^*, is as near as it can be to the previous first vector d_1. By contrast, Rosenbrock's method forms the second vector in a way which excludes d_1 as far as possible.

At the end of the earlier stage of optimisation it is likely that the length of the trial-and-error steps in the direction d_1 was fairly large, since successful moves lead to an increase in step-length. Consequently, the position of the operating point at the end of the stage is likely to be such that significant improvement can be obtained by fine adjustment in the d_1 direction. In that case it could obviously be advantageous to have d_2^* close to d_1, and this might account for the improvement in performance when Powell's method of orthogonalisation is used. This is, however, largely conjecture, and the matter requires further investigation.

Yours faithfully,

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3 June 1969

To the Editor
The Computer Journal

Sir,

The letter from Messrs Larmouth and Whitby-Strevens (this *Journal*, Vol. 12, p. 200) on the use of the term 'processor' with a software connotation enunciates what seems to me to be a most dangerous principle. They say that to restrict the term to hardware connotations 'is an extravagant waste of a useful piece of terminology'.