I do not understand, in relation to my article, Dr. Wilby's point on document transports, that there is no need to look hopefully across the Atlantic for a solution. I quite agree with his sentiment, but nowhere in my article did I suggest that we should look to America, and that British firms could not provide the equipment.

I wish success to Crosfield and look forward to reading a detailed article on the actual experience of the application of one of their machines, to a large integrated system such as Public Utility Billing.

To the Editor
The Computer Journal

The calculation of orthogonal vectors

Sir,
In his description of a new method for the calculation of orthogonal vectors (this *Journal*, Vol. 11, p. 302), M. J. D. Powell rightly claims a reduction in computing time compared to the method previously used by Rosenbrock in his minimisation procedure. Some tests which I have made have suggested that the new method has another important advantage in that it can actually accelerate the convergence of the minimisation process, measured in terms of the number of function evaluations performed.

A comparison has been made in a particular type of application of Rosenbrock's method which I employed in collaboration with J. Culhane while employed by Sigma (Science in General Management) Ltd. The problem is to find the best locations for a number of factories for the manufacture of a commodity, and a number of depots at which it is transferred from bulk transportation to local transportation. It is assumed in the simplest case that the cost of transportation is proportional to the amount carried and to the distance, and is at different rates for bulk and local carriage. Given a set of co-ordinates of consumers, and their rates of consumption, as well as a set of co-ordinates of factories and depots, the cost of distribution can be evaluated. It is assumed that each consumer is supplied along the most economical pathway, as is reasonable if there are no limitations on capacity of factories or depots. Rosenbrock's method has been applied to find new positions for a subset of the factories and depots so as to reduce the distribution cost. The number of variables in the optimisation is twice the number of movable entities. They consist of the x- and y-co-ordinates of these entities. There are no constraints on the solution.

In a specific example with eight customers, two factories (one movable) and three depots (two movable) the original distribution cost was 18,894 units. Rosenbrock's method in its original form reduced this to 12,753 in 19 stages, involving 835 evaluations, thereafter coming practically to a standstill.

When Powell's method of orthogonalisation was used, the improvement continued for about 43 stages, involving 1,831 evaluations, and reducing the cost to 12,479. When the method was operated without any reassignment of the directions the cost fell to 13,882 in 12 stages, involving 403 evaluations, then showed no further significant improvement.

Comparison of the two methods of orthogonalisation has only been carried out in the context of this particular type of problem and there is clearly a need for further investigation. It is interesting to look for some reason for the apparent superiority of Powell's method, and to consider whether there might be still further ways of assigning the orthogonal vectors which might be even more favourable to the progress of the optimisation. The angular separation between corresponding elements of successive vector sets tends to be less with Powell's method than with Rosenbrock's original one, but the difference is not great.

Following Powell's notation, let the set of orthogonal unit vectors used in the stage of optimisation prior to the reassignment be $d_1, d_2, \ldots, d_n$, and the advances made in their directions $\alpha_1, \alpha_2, \ldots, \alpha_n$. Both methods compute the first of the new vectors as

$$d_1^* = (\alpha_1 d_1 + \alpha_2 d_2 + \ldots + \alpha_n d_n)/denominator \qquad (1)$$

where the denominator is chosen to normalise the vector to unit magnitude.

Then in Rosenbrock's method a further $(n - 1)$ vectors are computed as follows:

$$\begin{aligned} d_2^1 &= \alpha_2 d_2 + \alpha_3 d_3 + \ldots + \alpha_n d_n \\ d_3^1 &= \qquad\quad\; \alpha_3 d_3 + \ldots + \alpha_n d_n \\ &\;\;\vdots \\ d_n^1 &= \qquad\qquad\qquad\qquad\; \alpha_n d_n \end{aligned} \qquad (2)$$

From each of the vectors in (2) a unit vector is derived, orthogonal to all that have come before it. This is done by subtracting from it its projection on each of the previous unit vectors and normalising the resulting vector to unit magnitude.

If instead of (2) the following set of $(n - 1)$ vectors is taken

$$\begin{aligned} d_2^1 &= d_1 \\ d_3^1 &= d_2 \\ &\;\;\vdots \\ d_n^1 &= d_{n-1} \end{aligned} \qquad (3)$$

the same process of subtracting projections and normalising leads to the set of vectors obtained using Powell's method.

It seems reasonable to suggest that the superiority of Powell's method is related to the fact that the second of his orthogonal vectors, d_2^*, is as near as it can be to the previous first vector d_1. By contrast, Rosenbrock's method forms the second vector in a way which excludes d_1 as far as possible.

At the end of the earlier stage of optimisation it is likely that the length of the trial-and-error steps in the direction d_1 was fairly large, since successful moves lead to an increase in step-length. Consequently, the position of the operating point at the end of the stage is likely to be such that significant improvement can be obtained by fine adjustment in the d_1 direction. In that case it could obviously be advantageous to have d_2^* close to d_1, and this might account for the improvement in performance when Powell's method of orthogonalisation is used. This is, however, largely conjecture, and the matter requires further investigation.

Yours faithfully,
A. M. Andrew

Department of Applied Physical Sciences
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3 June 1969

To the Editor
The Computer Journal

Sir,
The letter from Messrs Larmouth and Whitby-Strevens (this *Journal*, Vol. 12, p. 200) on the use of the term 'processor' with a software connotation enunciates what seems to me to be a most dangerous principle. They say that to restrict the term to hardware connotations 'is an extravagant waste of a useful piece of terminology'.

Whether the generalisation of the meaning of 'processor' is legitimate or not is not a question I would care to comment upon, but the argument that generalisation should be encouraged to avoid waste is surely untenable. Far too many terms have suffered generalisation in the past to such an extent that they are now incapable of conveying precise meaning; 'flip flop' is one example, 'routine' is another. Even if 'processor' is restricted to hardware it is still difficult to define precisely (and 'central processor' is worse).

I would not impose upon you, Sir, the burden of being a terminological watchdog, but please do not allow your authors to take the Larmouth–Whitby-Strevens dictum too seriously.

Yours sincerely,
I. H. GOULD

Sebright School
Cheverelles Green
Markyate
St. Albans
Hertfordshire
22 May 1969

To the Editor
The Computer Journal

Sir,
I support MM. Larmouth and Whitby-Strevens in their contention (this *Journal*, Vol. 12, p. 200) that the term 'processor' is valid for software. As further examples:

1. Firmware—which is an item of hardware that is obviously software.
2. My paper 'Checklist of Intelligence for Programming Systems', *CACM*, March 1959. This indicates that this usage was common enough a decade before the question has been raised anew.

Yours faithfully,
R. W. BEMER

General Electric Company
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Arizona
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10 June 1969

To the Editor
The Computer Journal

Comments on a line thinning scheme

Sir,
Among other items included in his review-paper on pattern recognition, D. Rutovitz (1966) also deals with methods of processing digital images by computer. He goes on to suggest a set of rules (rules I–V on page 512) the realisation of which yields an image-thinning algorithm. The purpose of this communication is to present briefly some comments on, and suggested corrections to these rules.

Attention is drawn to the fact that the algorithm in its present form does not completely reduce diagonal lines to a skeleton form. Such strokes remain two-elements thick. See **Fig. 1(a)**. It is suggested that the amendments given below be incorporated in order to deal with this deficiency.

If the crossing number at the point in question is 4, ($X = 4$; the definition of X remaining unaltered) and conditions I, IV, and V are satisfied then the said point is still deleted provided that the conditions given in either A or B below are satisfied.

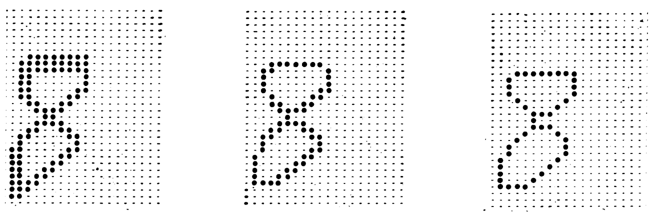
A.
$$\{\gamma(1) = 1, \gamma(7) = 1\}$$
$$AND$$
$$\{\gamma(2) = 1 \quad OR \quad \gamma(6) = 1\}$$
$$AND$$
$$\{\gamma(3), \gamma(4), \gamma(5), \gamma(8) = 0\},$$

B.
$$\{\gamma(3) = 1, \gamma(1) = 1\}$$
$$AND$$
$$\{\gamma(4) = 1 \quad OR \quad \gamma(6) = 1\}$$
$$AND$$
$$\{\gamma(5), \gamma(6), \gamma(7), \gamma(2) = 0\}.$$

Conditions A and B pertain respectively to diagonal strokes in the N.E.–S.W. and in the N.W.–S.E. directions. (Each argument in A is reduced by 2—on a period-of-8 basis—in B.)

Rule III becomes superfluous now that $X = 4$ forms one of the conditions for deletion. Hence its omission from the amendment. Furthermore, on using the above amendment one does not have to test for the conditions set out in the second part of rules IV and V.

It will not be inappropriate to mention that in his correspondence (privately) and elsewhere Dr. Rutovitz (1969) suggested yet a further modification to the original set of rules, whereby a point may not be deleted unless one of its axial neighbours is zero. This test is really superfluous. For having thus dispensed with the second part of rules IV and V, their first part—the conditions of which must be met before deletion can take place—express this very condition. The proposed axial-condition test is therefore unnecessary. **Fig. 1(b)** shows the image processed using all the above modifications.



INPUT (a) (b)
FIGURE 1

Yours faithfully,
E. S. DEUTSCH

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Dollis Hill
London
29 July 1969

References

RUTOVITZ, D. (1966). Pattern Recognition, *Journal of the Royal Statistical Society*, Series A, Vol. 129, pp. 504–530.
RUTOVITZ, D. (1969). Local Operations on Digital Images, Lecture given at the 6th meeting of the Pattern Recognition Group, April 15th, University College, London.