$$\int_{-1}^{1} \frac{T_r(x)T_s(x)dx}{(1-x^2)^{1/2}} = \begin{cases} \pi, & r = s = 0 \\ \pi/2, & r = s \neq 0 \\ 0, & r \neq s \end{cases} \qquad (20)$$

and, if $x'_k = \cos(k\pi/n)$, $k = 0(1)n$, then for $r, s \leqslant n$,

$$\sum_{k=0}^{n}{}'' T_r(x'_k)T_s(x'_k) = \begin{cases} n, & r = s = 0, n \\ n/2, & r = s \neq 0, n \\ 0, & r \neq s \end{cases} \qquad (21)$$

We observe here that, for r, $s \leqslant n$, these two orthogonality relations are entirely equivalent inasmuch as the discrete point orthogonality relations (21) are the orthogonality relations (20) discretised at these abscissas by the Gauss–Chebyshev quadrature formula of the closed type. For, replacing the integral in (20) by the quadrature formula (12), we have for r, $s \leqslant n$,

$$\frac{\pi}{n}\sum_{k=0}^{n}{}'' T_r(x'_k)T_s(x'_k) + E_{P_{n+1}}(T_rT_s) = \begin{cases} \pi, & r = s = 0 \\ \pi/2, & r = s \neq 0 \\ 0, & r \neq s \end{cases} \qquad (22)$$

and since $T_n^2 = \frac{1}{2}(T_0 + T_{2n})$, from (13) we obtain

$$E_{P_{n+1}}(T_rT_s) = \begin{cases} 0, & r + s \leqslant 2n - 1 \\ -\pi/2, & r = s = n \end{cases} \qquad (23)$$

the discrete point orthogonality relations (21) now follow from (22) and (23).

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Book review

Methods of Multivariate Analysis, by K. Hope, 1969. (*University of London Press.*) With *Handbook of Multivariate Methods Programmed in Atlas Autocode*, 288 pages, 63s. (Paperback, without handbook, 168 pages, 30s.)

As computer systems have become easier to use throughout the past two decades so it has become easier for an experimentalist to program statistical techniques straight from any textbook he chooses. The most commonly disastrous failing of this means of performing statistical analysis is that not all books are equally competent in their descriptions of the techniques. Furthermore, while inappropriate analyses are performed, time which might be spent obtaining the advice of a statistician is commonly spent overcoming some deficiency of a computer system. The more recent attempts to provide statistical analysis systems comprising suites of programs for standard statistical techniques or statistical languages should seek to eliminate not only the labour of re-programming similar techniques but, in addition, the hazard from 'do-it-yourself' analysis. The book under review in part describes such a system, but does not seem to remove many of the potential dangers.

The hard cover edition of this new book on multivariate methods has a novel presentation consisting of two parts. In the first part the author describes certain multivariate techniques and the second part is a handbook giving the specifications of eleven computer programs which are written in Atlas Autocode and are available at the Atlas Computer Laboratory at Chilton.

The range of techniques covered by this book is ambitious from the analysis of variance (including factorial designs) and analysis of covariance through principal components and geometric descriptions including spherical mapping to regression, discriminant and canonical analyses. There is some useful practical discussion of these topics together with a reasonably comprehensive list of references but little depth of theory or background is given. The techniques are described in matrix notation throughout and there is a short introductory first chapter to matrices and vectors. The user orientation of the whole presentation is helped by the small number of brief worked examples which are used as illustrations both of the techniques in the first part of the book and also of the specimen data and results formats from the computer programs in the handbook. Unfortunately, these examples show multivariate techniques only in psychological and educational applications and there are no examples for the student to work for himself.

There is no mention of residuals which are necessary for verifications of underlying assumption. All possible tests are performed and results provided by the programs and this practice is likely to lead the user astray unless he reads very carefully the lengthy texts which are output with his results. The book is well presented and provides a valuable practical introduction to certain multivariate techniques and an adequate description of a worthwhile set of programs.

R. W. Hiorns (Oxford)