

not be pursued here.

If the system disturbance $u(k-1)$, $k = 1, \dots, N$, is not present in (1), the term involving $\|u(i-1)\|_{Q^{-1}(i-1)}^2$ in V_N^* does not appear and the set of constraints in (5) is replaced by the set

$$x(i) - f(x(i-1), i-1) = 0$$

where $i = 1, \dots, N$. The procedure in Section 3 still applies in this case with certain obvious simplifications.

If the statistical description which was given in the problem formulation is not available, the computational method of Section 3 can still be applied by adopting a modified least-squares approach as was done by Detchmندی and Sridhar (1966) for continuous-time nonlinear filtering. In this case, the estimate is constrained to be of the form

$$\hat{x}(k) = f(\hat{x}(k-1), k-1) + u(k-1)$$

for $k = 1, \dots, N$, where $\hat{x}(0)$, $\hat{x}(N)$ and $u(k-1)$ are determined so that

$$\begin{aligned} V_N = & \frac{1}{2} \|x(0) - \bar{x}(0)\|_{A(0)}^2 \\ & + \frac{1}{2} \sum_{i=1}^N \{ \|z(i) - h(\hat{x}(i), i)\|_{B(i)}^2 \\ & + \|u(i-1)\|_{C(i-1)}^2 \} \end{aligned}$$

is minimized. Here $\bar{x}(0)$ is a preliminary estimate of $x(0)$, and $A(0)$, $B(i)$ and $C(i-1)$ are arbitrary weighting matrices. All of these parameters are chosen on the basis of engineering considerations associated with the particular physical system of concern.

References

- BELLMAN, R. E., and KALABA, R. E. (1965). *Quasilinearization and Nonlinear Boundary-Value Problems*, Elsevier: New York.
- BRYSON, A. E., Jr., and HO, Y. C. (1969). *Applied Optimal Control*, Blaisdell: Waltham, Massachusetts.
- COX, H. (1964). On the Estimation of State Variables and Parameters for Noisy Dynamic Systems, *Trans. IEEE Auto. Cont.*, AC-9, pp. 5-12.
- DETCHEMENDY, D. M., and SRIDHAR, R. (1966). Sequential Estimation of States and Parameters in Noisy Nonlinear Dynamic Systems, *Trans. ASME, Ser. D, J. Basic Eng.*, Vol. 88, pp. 362-368.
- GELFAND, I. M., and FOMIN, S. V. (1963). *Calculus of Variations*, Prentice-Hall: Englewood Cliffs, New Jersey.
- GURA, I. A., and HENRIKSON, L. J. (1969). A Unified Approach to Nonlinear Estimation, Report No. TDR-0200(4306-02)-2, Aerospace Corporation, Los Angeles, California.
- MEDITCH, J. S. (1968). A Successive Approximation Procedure for Continuous-Time Nonlinear Data Smoothing, Boeing document D1-82-0803, Boeing Scientific Research Laboratories, Seattle, Washington. Also, *Proc. Symp. on Information Processing*, Purdue University, Lafayette, Indiana, May 1969 (to appear).
- SAGE, A. P. (1968). *Optimum Systems Control*, Prentice-Hall: Englewood Cliffs, New Jersey.

Book review

A Collection of Matrices for Testing Computational Algorithms, by Robert T. Gregory, D. L. Karney, 1969; 154 pages. (John Wiley and Sons Ltd., £4.75)

In recent years considerable efforts have been made to develop algorithms of wide application in the field of linear algebra. It is important that such algorithms should be subjected to very stringent tests and for this purpose a well designed set of test matrices is required.

In this book the authors have assembled a wide selection of matrices which were distributed throughout the literature, together with collections provided by J. Elliott, S. Voigt, R. Greenwood and J. Westlake. The test matrices are divided into two main classes. The first are appropriate for algorithms for solving systems of linear equations and computing determinants and inverses, and the second for computing the eigensystems of matrices, both symmetric and asymmetric.

For anybody involved in developing algorithms in this field it is an invaluable collection of material. I have only two minor criticisms. First, the set of matrices for testing

algorithms for the unsymmetric eigenvalue problem includes results for the FRANK matrix of order twelve. It would have been far more useful if the authors had shown how to use algorithms for finding eigenvalues of symmetric tridiagonal matrices to find accurate eigenvalues of the unsymmetric FRANK matrices of all orders, thus providing the reader with a whole series of extremely valuable tests.

Secondly the book concludes with a specific DOLPH-LEWIS complex matrix of order twenty. This matrix has played an important role in the history of the development of eigenvalue algorithms. (Both the authors and the reviewer have been concerned with it.) However, the matrices are 'full' and the elements are given as 19 digit decimal numbers; this makes it a prohibitively tiresome matrix to use. As far as possible test matrices should be easy to generate and should usually contain parameters, so that a large number of tests is provided in a simple manner. This is true of most of the examples given.

I hope this book will be regarded as a first attempt in this important field and that its success will stimulate the authors and others to further efforts.

J. H. WILKINSON (Teddington)