it is clear that there will be a long-term commitment, both inducements and illusion are needed—inducement through 'free' time and ability to use the system on useful work; illusion through good user documents and manuals which help dispel the chancy nature of any such development. The development group must change gears at some point in order to devote at least 50% of its time to users in order to understand their problems. Ways to use new tools are often obvious to the tool builder but not the mechanic (and later on vice versa).

2. Conflict between the objectives of maintaining production reliability and avoidance of new equipment hindered development. The treatment of the timesharing service as just another job could not be sustained without compromising performance. Measurement of interactive system performance was far more complex than expected and some performance measures need to be designed into the operating system.
3. Observation of service use points to more investigation of editing and file development as being more important than full interaction.
4. The application restructuring strategy worked, and thus we successfully built upon the massive investment in batch program development.

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Correspondence

To the Editor
The Computer Journal

Sir,

The recent article by McConalogue (1970) describes a method for curve drawing which appears to combine accuracy and simplicity, especially when the tangents are already known. When they are unknown, McConalogue employs a 'complementary algorithm', which expresses the x and y co-ordinates at P_{k-1}, P_k and P_{k+1} as separate Lagrange quadratics in a parameter equal to the chord length $D_k = \{(\Delta x_k)^2 + (\Delta y_k)^2\}^{\frac{1}{2}}$. This produces tangents which are invariant under axis rotation.

An alternative procedure, with the same invariance, utilises the circular arc defined by the same three consecutive points. The co-ordinates $(\bar{x}\bar{y})$ of the centre of this circle are found from

$$D\bar{x} = (x_{k-1} + x_k)\,\Delta x_{k-1}\,\Delta y_k - (x_k + x_{k+1})\,\Delta x_k\,\Delta y_{k-1} - (\Delta y_{k-1} + \Delta y_k)\,\Delta y_{k-1}\,\Delta y_k$$

$$D\bar{y} = (y_k + y_{k+1})\,\Delta x_{k-1}\,\Delta y_k - (y_{k-1} + y_k)\,\Delta x_k\,\Delta y_{k-1} + (\Delta x_{k-1} + \Delta x_k)\,\Delta x_{k-1}\,\Delta x_k$$

where

$$D = 2(\Delta x_{k-1}\,\Delta y_k - \Delta x_k\,\Delta y_{k-1})$$

and

$$\Delta x_k = x_{k+1} - x_k,\ \Delta y_k = y_{k+1} - y_k\,.$$

Thus the angle between the x-axis and the tangent at an arbitrary point (ξ, η) on the circular arc is $\theta = \tan^{-1}\{-(\bar{x} - \xi)/(\bar{y} - \eta)\}$; hence

$$N \sin\theta = \Delta y_{k-1}(\Delta x_k^2 + \Delta y_k^2) + \Delta y_k(\Delta x_{k-1}^2 + \Delta y_{k-1}^2) + D(\xi - x_k)$$

$$N \cos\theta = \Delta x_{k-1}(\Delta x_k^2 + \Delta y_k^2) + \Delta x_k(\Delta x_{k-1}^2 + \Delta y_{k-1}^2) - D(\eta - y_k)$$

where N is a normalising factor.

Setting $\xi = x_k$, $\eta = y_k$, the tangent at P_k is found to be precisely the same as that derived by McConalogue. This somewhat surprising result shows that McConalogue's algorithm is exact for points which lie on circular arcs.

However, if P_{k-1} is the first point on an open curve, the slope there must be estimated from the above formulae with $\xi = x_{k-1}$, $\eta = y_{k-1}$; and if P_{k+1} is the last point, $\xi = x_{k+1}$, $\eta = y_{k+1}$. In general, the slope at a point P is most simply calculated from

$$N \sin\theta = \alpha\Delta y_{k-1} + \beta\Delta y_k - \gamma\Delta x_{k-1} + \delta\Delta x_k$$
$$N \cos\theta = \alpha\Delta x_{k-1} + \beta\Delta x_k + \gamma\Delta y_{k-1} - \delta\Delta y_k$$

where $\alpha = (\Delta x_k)^2 + (\Delta y_k)^2$, $\beta = (\Delta x_{k-1})^2 + (\Delta y_{k-1})^2$ in all cases, and $\gamma = D$, $\delta = 0$ at P_{k-1} (if this is a starting point), $\gamma = 0$, $\delta = 0$ at P_k, and $\gamma = 0$, $\delta = D$ at P_{k+1} (an end point). Incidentally, the radius of curvature at P_k is N/D.

These results for the end points of open curves differ from those found by McConalogue, which may be written

$$N \sin\theta = \alpha'\Delta y_{k-1} + \beta'\Delta y_k$$
$$N \cos\theta = \alpha'\Delta x_{k-1} + \beta'\Delta x_k$$

where

$\alpha' = \alpha + 2\sqrt{\alpha\beta}$, $\beta' = -\beta$ at P_{k-1},
$\alpha' = \alpha$, $\beta' = \beta$ at P_k, and
$\alpha' = -\alpha$, $\beta' = \beta + 2\sqrt{\alpha\beta}$ at P_{k+1}.

It is suggested that the alternative formulation is preferable in that it is exact for circular arcs, does not require the use of square roots in finding the direction ratios, and avoids the somewhat arbitrary selection of cord length as a parameter.

Yours faithfully,
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Reference

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Erratum

In the paper 'The evaluation of eigenvalues and eigenvectors of real symmetric matrices by simultaneous iteration' by M. Clint and A. Jennings (this *Journal*, Vol. 13, No. 1, p. 76) there was an error in Table 1. The element of the fifth row and twelfth column should read 997·30 and not 977·30. We are grateful to Terry G. Seaks, Department of Economics, Duke University, Durham, North Carolina 27706, USA for pointing out the error.