```
begin
  real e, h1, h2, h3, h4, r1, r2, r3, r4, d1, d2, d3, c, s, int;
  integer i, j, k;
  int := e := s := c: = r4 := 0·0;
  j := if ia = n − 1 and n > 3 then n − 2 else if ia > 2 then ia else 2;
  k := if ib = 1 and n > 3 then 3 else if n > ib + 2 then ib + 1 else n − 1;
  for i := j step 1 until k do
  begin
    if i = j then
    begin
      h2 := x[j − 1] − x[j − 2];
      d3 := (f[j − 1] − f[j − 2])/h2;
      h3 := x[j] − x[j − 1];
      d1 := (f[j] − f[j − 1])/h3;
      h1 := h2 + h3;
      d2 := (d1 − d3)/h1;
      h4 := x[j + 1] − x[j];
      r1 := (f[j + 1] − f[j])/h4;
      r2 := (r1 − d1)/(h4 + h3);
      h1 := h1 + h4;
      r3 := (r2 − d2)/h1;
      if ia = 0 then
      begin
        int := h2 × (f[0] + h2 × (d3/2·0 − h2 × (d2/6·0 −
              (h2 + 2·0 × h3) × r3/12·0)));
        s := −h2↑3 × (h2 × (3·0 × h2 + 5·0 × h4) + 10·0 ×
              h3 × h1)/60·0
      end
    end
    else
    begin
      h4 := x[i + 1] − x[i];
      r1 := (f[i + 1] − f[i])/h4;
      r4 := h4 + h3;
      r2 := (r1 − d1)/r4;
      r4 := r4 + h2;
      r3 := (r2 − d2)/r4;
      r4 := r4 + h1;
      r4 := (r3 − d3)/r4
    end;
    if i ≤ ib and i > ia then
    begin
      int := int + h3 × ((f[i] + f[i − 1])/2·0 − h3 × h3 ×
            (d2 + r2 + (h2 − h4) × r3)/12·0);
      c := h3↑3 × (2·0 × h3 × h3 + 5·0 × (h3 × (h4 + h2) +
            2·0 × h4 × h2))/120·0;
      e := e + (c + s) × r4;
      s := if i = j then 2 × c + s else c
    end
    else e := e + r4 × s;
    if i = k then
    begin
      if ib = n then
      begin
        int := int + h4 × (f[n] − h4 × (r1/2·0 + h4 × (r2/6·0 +
              (2·0 × h3 + h4) × r3/12·0)));
        e := e − h4↑3 × r4 × (h4 × (3·0 × h4 + 5·0 × h2) + 10·0
              × h3 × (h2 + h3 + h4))/60·0
      end;
      if ib ≥ n − 1 then e := e + s × r4
    end
    else
    begin
      h1 := h2;
      h2 := h3;
      h3 := h4;
      d1 := r1;
      d2 := r2;
      d3 := r3
    end
  end i;
  int 4pt := int;
  er := e
end int 4pt
```


Reference

MILNE-THOMSON, L. M. (1933). *The calculus of finite differences*, Macmillan.

Book review

Essays on Cellular Automata, by Arthur W. Burks (editor), 1970; 375 pages. (*University of Illinois Press*, £6·00)

This book is a collection of 11 previously published papers and four new contributions concerned primarily with cellular automata. The cellular automaton model was originally devised by von Neumann in order to investigate the essential features required in a machine which has the capability of reproducing itself, and most of the papers in this book are concerned with this self-reproduction theme. A cellular automaton consists of an infinite rectangular grid, in each cell of which is placed the same finite state machine, the whole set of machines works synchronously, the next state of any machine being determined by its present state and those of its neighbours. One particular state is termed 'quiescent' and at any time only a finite number of the machines are not in this quiescent state. If the set of states occurring in one part of the plane appears at a later time in another part of the plane then reproduction is said to occur—care is needed to exclude trivial cases and a certain degree of generality is insisted on for the finite state machine.

As might be expected the quality of the papers in the book is varied, some seem to set up formalisms just for the sake of formalising, others set up formalisms and use them to obtain interesting non-trivial mathematical results. There is inevitably some overlap in content, although this is not a bad thing.

The paper by E. F. Moore is a very clear and readable account of cellular and other models of self-reproduction and is recommended to readers who wish to undersand the ideas of the formalism and to decide for themselves if they are interested in further reading. The introduction by A. W. Burks attempts to relate the various papers three of which are also written by Burks. These tend to be wordy and it is sometimes a little difficult to get at the heart of his thoughts. Papers are also included concerning the growth of patterns in a plane and in three dimensions given very simple growth rules, this leads to the study of fixed points of non-linear transformations and there is a paper on this topic. Finally, there are four papers by J. Holland on his 'iterative circuit computers'—these are designed with the idea that they could be usefully built, thus giving a highly parallel computer system. They are more general than cellular automata in that connections can be built so that non-neighbouring automata can influence each other directly.

It is stated in the introduction that, whilst the theory of cellular automata is the main theme of the book, the heuristic use of computers is a minor theme. Heuristics are suggested as a way of defining automata to do particular tasks or to find growth laws to achieve a certain purpose. Such automata or growth rules are partially specified, then simulated in a computer and on the basis of the results modification or extensions made to the system. The computer is only used as a simulator, the human supplies the heuristics and no attempt is made to formalise these. Thus, by the heuristic use of computers the editor means the use of a computer in a heuristic situation, not the use of heuristics in computers.

For research workers or others interested in this kind of ideas this is a useful collection of papers. However, this reviewer feels that the work is more likely to find application in the design and use of parallel machines than in the design of self-reproducing machines.

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