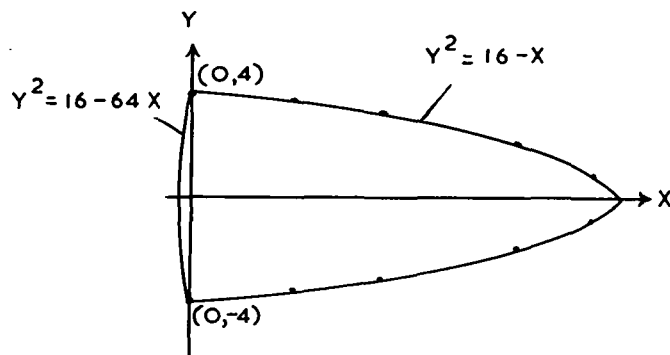


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61 DISTINCT, STABLE, LOCALLY MAXIMUM 65-GONS EXIST, WITH FROM 5 TO 65 VERTICES ON $Y = 16 - X$.

Fig. 3

$y^2 = 16 - x$, the larger the area of the 65-gon. Each of the 61 locally maximum 65-gons are stable, in the sense that if small enough perturbations are made of each of their points (even simultaneously), then the iterative process of adjusting the middle points of various triplets of vertices must converge back to the initial configuration. The sharp angles at the intersections $(0, \pm 4)$ prove to be impassable barriers to the migrations of vertices of N -gons for $N \leq 65$ (see Fig. 3).

Any point of a circle may be the vertex of a regular inscribed polygon. The circle may be projected onto any ellipse, so that the regular inscribed polygon is projected onto a locally maximum polygon of the ellipse (note that the projection preserves tangency and parallelism). It is probably characteristic of the ellipse (and circle) that any of its points may be used as a vertex of a locally maximum polygon of any order.

Yours faithfully,
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6 December 1971

To the Editor
The Computer Journal
Sir

Calculation of a double-length square root from double-length number using single precision techniques

I write to comment on the letter by D. W. Honey (this *Journal*, Vol. 14, Nov. 1971, p. 443) where he describes a method which he attributes to his colleague, Mr. J. Grabau. The method given is, however, quite well known, being Newton's method with rearrangement of terms to exhibit the correction to be made at any stage. The usual form of Newton's method for finding \sqrt{a} is

$$x_{i+1} = \frac{1}{2} \left(x_i + \frac{a}{x_i} \right)$$

which can be rewritten as

$$x_{i+1} = x_i + \frac{a - x_i^2}{2x_i}$$

to show the correction. Mr. Honey's (or Mr. Grabau's) technique is therefore seen to be equivalent to one more step of the Newton process after the single-length result has been obtained.

However, it is necessary to take care when this method is being used in fixed-point arithmetic, as overflow could result if the 'wrong' single-length square root is taken. It is not enough to take the unrounded (rounded down) value, because this leads to a value x satisfying

$$0 \leq a - x^2 \leq 2x,$$

and this can obviously give overflow. No such difficulty can arise if we take the rounded value, because this satisfies

$$-x \leq a - x^2 \leq x,$$

with a correction of at most $\frac{1}{2}$ unit, although it may be of either sign. In Mr. Honey's example, therefore, he *should* have used 14 as his initial guess at $\sqrt{192}$, which would have led to 13.86 as the better approximation, instead of 13.88. Since $(13.86)^2 = 192.0996$ this gives an error which is about $\frac{1}{2}$ of that quoted. Indeed, it is not difficult to show that the maximum relative error using the rounded single-length approximation will be about $\frac{1}{4}$ of the error that could arise from using the unrounded version. Choosing the rounded approximation is thus noticeably more accurate for the same amount of work.

Yours faithfully,
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14 December 1971

Mr. Honey replies:

I am obliged to Professor P. A. Samet for his letter commenting on 'Grabau's Method' for obtaining a double precision result using single precision techniques.

I think that I may have misled the readers by the lack of emphasis on the single precision. Professor Samet is quite correct in his observation that Newton's method is involved, although I had not appreciated this fact at the time. My main concern was that single precision techniques are used throughout the process and is something often overlooked by software designers with non-mathematical background.

I am also obliged for Professor Samet's further comment re 'overflow' (again sometimes overlooked), and his development of my worked example in decimal in which a rounded value is taken in preference to an unrounded value—a technique I shall remember in future.

I am sure that several readers will have gained benefit from our minor correspondence—which is its basic purpose. Thank you, Professor Samet, once again.