Appendix 4

An examination of the case for rental as against purchase

Assumptions

1. Sum c available at commencement of a period to be employed on either rental or purchase.
2. An annual rental charge of a fraction v of the purchase price.
3. An annual decrease in the cost of machines of the same power to a fraction r.
4. An effective machine life of L years, after which it is worthless.
5. A constant annual interest on money deposits of i expressed as a compound interest ratio, *i.e.* $i > 1{\cdot}0$.
6. That the rented machine is changed each year, to take advantage of technological advance (or competition), for a machine of the same power.

For the first year, the rental cost for a machine of capital cost c is vc.

At the end of the first year the sum remaining is $ic - icv$.

For the second year the rental cost of a machine of equivalent power is vcr.

At the end of the second year the sum remaining is

$$i^2c - i^2cv - icvr\,.$$

For the third year the rental cost of a machine of equivalent power is vcr^2.

At the end of the third year the sum remaining is

$$i^3c - i^3cv - i^2cvr - icvr^2\,.$$

At the end of the $L - 1$th year the sum remaining is

$$i^{L-1}c - i^{L-1}\,cv - i^{L-2}\,cvr \ldots icvr^{L-2}$$

and if $$r/i = x$$
the sum remaining is

$$i^{L-1}c[1 - v(1 - x^{L-1})/(1 - x)]$$

The rental for the Lth year would be vcr^{L-1} and rental is cheaper than purchase if the sum remaining is greater than this, i.e.

if
$$i^{L-1}c[1 - v(1 - x^{L-1})/(1 - x)] > vcr^{L-1}$$
or
$$1 > vx^{L-1} + v(1 - x^{L-1})/(1 - x)$$
or
$$1 > v(1 - x^L)/(1 - x)$$
or
$$v < (1 - x)/(1 - x^L)\,.$$

The implications can be shown by quoting some general figures for L, v, r, and i.

Sharpe (1969, pp. 344, 353, footnotes) quotes Knight as suggesting that there is an annual decrease in cost of 23·6% for a given performance (scientific computation) and Patrick as suggesting a 23% decrease per year.

i.e. $r \sim 0{\cdot}77$
if i is taken to be 1·10 (10% interest rate, the test rate for government financing)
then x may be approximately 0·7
if L is taken to be 7 and x to be 0·7
then $(1 - x)/(1 - x^L) \sim 0{\cdot}33$.

Turning to Sharpe (1969, pp. 220, 254, 273) again for ratios of rental (excluding maintenance) cost to price for unlimited utilisation in 1966, we have

for CDC equipment
$$v = 12/46{\cdot}36 \times 1{\cdot}2 = 0{\cdot}3106$$
for IBM equipment
$$v = 12/50{\cdot}33 \times 1{\cdot}1 = 0{\cdot}2575$$

Thus by these figures, it would be cheaper to rent than buy both IBM and CDC machines.

Clearly, no account has been taken in these figures of discounts or duty, nor of the inconvenience and cost of frequent changes of machinery.

Reference

SHARPE, W. E. (1969). *The Economies of Computers*, New York and London: Columbia University Press.

Book review

Computers in Number Theory. Proceedings of the Science Research Council Atlas Symposium No. 2 held at Oxford from 18-23 August 1969. A. O. L. Atkin and B. J. Birch (editors), 1971; xvii + 433 pages. (*Academic Press, London and New York*, £8·00)

Many results in the theory of numbers, both before and after Euler's discovery of the law of quadratic reciprocity, have been discovered and confirmed by extensive numerical evidence long before they have been proved mathematically. So it is not surprising that number theorists have turned to modern computers to aid them in their investigations and that they have achieved notable success.

Computers in Number Theory gives detailed accounts of many recent researches in number theory and combinatorial theory. While a few of the papers seem to be on number theory with little or no reference to computers, most are concerned with results in the theory of numbers that have been suggested or discovered or proved by use of computers. Some of the results discovered by computers are given mathematical proofs, others remain uproved, and some of these (for example, those concerning linear relations connecting the imaginary parts of the zeros of the Riemann zeta function) look as though they may remain unproved for a long time. In quite a surprising number of contributions, computers have been used to provide rigorous proofs of difficult mathematical results, where it is by no means obvious, at first sight, that the computer can be of any use at all. A number of these are results in the Geometry of Numbers where conventional methods lead to the consideration of many different cases and subcases all of essentially the same form. It is less surprising that computers are useful in the study of Diophantine equations, but the papers on this topic display great ingenuity in the way the original problems are transformed to forms in which the computer is of use.

While most of the papers are about number theory, there are a few notable exceptions. D. H. Lehmer gives an introductory talk about the economics of number theoretic computation. I. J. Good and R. A. Gaskins develop results in the theory of numbers that are relevant to the art of pseudorandom number generation.

This volume is perhaps best regarded as an essential part of the periodical literature of mathematics. It is of especial interest to number theorists and to those computer scientists who are looking for interesting and unusual ways of using a computer. It has left me with a slight feeling of disappointment, in that, it could, I believe, have been made more interesting and useful if it had included a survey of some of the very striking results in this field that have been obtained over the last 10 years. The present work would have stood up quite well to the comparison with past achievements.

C. A. ROGERS (London)

Errata

Due to delays caused by a strike at the Université de Montréal, the author's proof corrections of the paper 'A note on the generalised Euler transformation' by P. Wynn (this *Journal*, Vol. 14, No. 4, pp. 437-441) were received after the printer's deadline. The following corrections should be made:

On page 438, column two, line 27 '$-\infty < a \leqslant \zeta \pm b < \infty$' should read '$-\infty < a \leqslant \delta \leqslant b < \infty$'.

On page 439, column one, line 11 the equation should be numbered (17).

On page 440, column one, line 12 '*if*' should read '**if**'.

On page 441, line−8 'NORLAND' should read 'NÖRLUND'.

The author's present address is:
Centre de recherches mathématiques, Université de Montréal, Montréal, Québec, Canada.