Step 6: *Output Rule*, **Fig. 11.** A check is made to see if the state associated with its remaining integration sequence has an output. This is done by taking the remaining integration sequence, and finding the associated state in the INTEGRAL LIST and checking this state for an output in the state transition state which was read in as input.

(6a) If so, λ is added to the contents of INTEGRAL.

(6b) If not, continue.

Step 7: *Substitution Rule*, **Fig. 12.** A check is made to see if INTEGRAL contains any state for which integrals have already been determined. This is done by taking each state, one at a time, on the INTEGRAL LIST beginning with the first integral which was determined and checking to see if it is in INTEGRAL. This is continued until all the states on the INTEGRAL LIST have been tried.

(7a) If so,

(7a,1) All the sequences containing the state being checked are collected.

(7a,2) The corresponding integral is substituted in for this state.

(7b) If not, continue.

Step 8: *Star Rule*, **Fig. 13.** A check is made to see if INTEGRAL contains the state to which it is to be assigned. This is done by taking the state associated with the remaining integration sequence and checking to see if it occurs in the INTEGRAL.

(8a) If so,

(8a,1) All the sequences containing this state in INTEGRAL are collected and placed as the first sequence in the integral.

(8a,2) The state is removed, and the sequence starred and producted with the remaining sequences.

(8b) If not, continue.

Step 9: Taking the remaining integration sequence in *A*, find the state associated with it in the INTEGRAND LIST. This state and the contents of INTEGRAL are then placed in the INTEGRAL LIST.

Step 10: Is the remaining integration sequence in *A* and *B* equivalent.

(10a) If so, a check is made to see if this is the last integration. This is done by checking to see if both remaining sequences are equal to the null set.

(10a,a) If so, transfer to the Output Section.

(10a,b) If not, transfer to Step 1.

(10b) If not, move the contents of *B* to *A* and go to Step 4.

As stated before the output section consists of printing the derived regular expression. A check is also made to see if any more input is waiting. If so, the process begins again. If not, the program terminates.

References

Arden, D. N. (1961). Delayed logic and finite state machines, *Proc. 2nd Ann. Symp. on Switching Circuit Theory and Logical Design*, Detroit, Michigan, p. 133.

Brzozowski, J. A., and McCluskey, E. J. (1963). Signal flow graph techniques for sequential circuit state diagrams, *IEEE Trans. Elect. Comp.*, Vol. EC-12, p. 67.

Brzozowski, J. A. (1964). Derivatives of regular expressions, *J. Assoc. Comp. Mach.*, Vol. 11, p. 481.

Copi, I. M., Elgot, C. L., and Wright, J. B. (1958). Realization of events by logical nets, *J. Assoc. Comp. Mach.*, Vol. 5, p. 181.

Kleene, S. C. (1954). Representation of events in nerve nets and finite automata, *Automata Studies*, C. E. Shannon and E. J. McCarthy, eds. (Princeton, N.J.: Princeton University Press), study 34, p. 3.

McNaughton, R., and Yamada, H. (1960). Regular expressions and state graphs for automata, *IRE Trans. on Elect. Comp.*, Vol. EC-9, p. 39.

Book review

An Analysis of Complexity, by H. van Emden, 1971; 86 pages. (*Amsterdam: Mathematical Centre*, *Tract* 35, \$3.00)

In many branches of science—numerical taxonomy, pattern recognition, artificial intelligence are some examples—classification is a necessary precursor of theoretical study. Often the 'why' comes before the 'how' of classification. A fundamental difficulty is the choice of criteria that distinguish good classifications from bad ones. What might be good for one purpose may indeed not be good for another.

Dr van Emden proceeds to define complexity. A classification is a set of entities into mutually disjoint classes. A subset of the entities, one from each class, is a set of paradigms. If each of the remaining entities is assigned to the same class as its paradigm according to some measure, the classification is perfect if the same result is obtained for every possible set of paradigms. Thus classification depends on similarities or interactions between pairs of entities and between sets of entities. Measures of dissimilarity are defined to be matrics. Complexity is defined as the way in which 'a whole is different from the composition of its parts'. A mathematical definition of interaction in terms of the theory of information. An amount of variety, H, exists in a set so defined that H has the same properties as those which Shannon required the uncertainty of a random variable to have in information theory. Then the amount of complexity C(S) which a system S has is the difference between the sum of the varieties of the individual components of the system itself. This can be related to the interactions between sub-systems of S.

Pairwise interactions between entities to be classified when data is *qualitative* may be used to define a distance function without requiring the qualitative data themselves to constitute a matric space thus allowing a model of classification to be formulated in terms of information. When objects can be described by points in n-dimensional inner-product spaces, the covariance matrix of the set of points can be studied. The author gives a maximum entropy characterisation of the multivariate normal distribution with the aid of which he proposes a measure of the complexity of a covariance matrix. He finds that the condition number of the covariance matrix relates to the complexity.

In a final section he discusses interaction and computational complexity using Jacobi's iteration method for solving linear equations as an example. Here, I think, the author has most success. His work affords an insight into numerical procedures which promises to be valuable. Certainly one can get fresh understanding of processes such as Kron's method of tearing for dealing with large systems, and the various decomposition algorithm of linear programming by applying the author's ideas.

All in all, this little book is a well-written immensely readable introduction to a new and challenging topic.

A. Young (Coleraine)