$$\frac{\log_2 S}{\log_2 \left(\frac{n}{2}\right)} < \frac{n-1}{2(n-2)} \log_2 S\,,$$

$$\log_2 \left(\frac{n}{2}\right) > 2\left(1 - \frac{1}{n-1}\right)$$

$$\frac{n}{2} > 2^{2\left(1 - \frac{1}{n-1}\right)}$$

$$n > 2^{3 - \frac{2}{n-1}}$$

i.e.

$$n > 6{\cdot}09$$

Thus for six tapes or less, the polyphase sort is faster than the balanced merge sort.

Acknowledgement

The author is indebted to Dr. M. Bloxham of the Mathematics Department, University of Essex, for suggesting the proof in the Appendix.

Appendix 1

To show that $p(z) = z^n - z^{n-1} - z^{n-2} - \ldots - z - 1$ has one real root in (1, 2) and $(n - 1)$ roots of modulus less than one.

$p(z)$ is continuous, and since $p(1) = -(n-1) < 0$ and $p(2) = 1 > 0$ there is a real root in (1, 2).

Since $p(z)$ may be expressed as

$$p(z) = \begin{cases} \frac{z^n(z-2)+1}{(z-1)} & \text{for } z \neq 1 \\ -(n-1) & \text{for } z = 1\,, \end{cases}$$

it follows that the zeroes of $p(z)$ coincide with the zeroes of $f(z) = z^n(z - 2) + 1$ with the exception of the root at $z = 1$ which is clearly not a root of $p(z)$. It is required, therefore, to show that $f(z)$ has precisely n zeroes in $|z| \leqslant 1$.

One proof of this employs a well-known theorem of complex analysis, a preamble to Rouché's theorem sometimes called the principle of the argument, which is used here in a form possibly most familiar as Nyquist analysis among electronic engineers.

Set $g(z) = z^n(z - 2)$. The image of the unit circle $|z| = 1$, described anticlockwise, is a curve that spirals out from $|g| = 1$ to $|g| = 3$ and back again, making n revolutions in all. This analysis is straightforward; a direct intuitive route is to see the image as the path of a moon that makes one orbit of its planet while the planet completes n around the sun. It crosses the negative real axis n times, at $g = -1$, $-(1 + \delta_1)$, $-(1 + \delta_2)$ etc., where δ_1 is a finite positive quantity, calculable for a given n.

Now consider $h(z) = g(z) + 1 + \varepsilon, 0 < \varepsilon < \delta_1$. The image of the unit circle C under the mapping $h(z)$ is the curve we have just described, displaced $1 + \varepsilon$ units to the right. It thus crosses the negative real axis $n - 1$ times (the intercept formerly at -1 now having moved across to $+\varepsilon$). Since the image of C now loops the origin $n - 1$ times there are, by the theorem, $n - 1$ zeroes of $h(z)$ inside C.

Finally, we note that with $\varepsilon \to 0$, $h(z)$ becomes $f(z)$ with a prescription that the zero of f at $z = 1$ is to be counted as lying outside C (it appears near $1 + \varepsilon/(n - 1)$ before the limit is taken).

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Correspondence

To the Editor
The Computer Journal

Sir,

Two impressions following the BCS conference on *Computer Performance* just completed are, first, that debugging and performance measurement are very much the same thing and require similar techniques; and, second, that hardware designers have so far done precious little to help. I would like to suggest a facility which promises to help a great deal.

A common bug which can be difficult to find is the sudden appearance of a nonsensical value somewhere in the data space. With current hardware one must normally resort to intellectual exercise or executing the entire program interpretively. What I would like to suggest is hardware address traps which can be set by program. For example one might provide a set of program accessible registers, each containing two addresses, which would cause the hardware to interrupt to one address each time a reference to the other address was encountered. Such a facility would, I suggest, greatly facilitate the provision of decent debugging and monitoring facilities without large run-time or software implementation overheads.

Yours faithfully,

R. J. DAKIN

UKAEA Culham Laboratory
Abingdon
Berkshire
18 September 1972

To the Editor
The Computer Journal

Sir,

Words like 'compactifying' and 'digressionally' ought, in our opinion, to be thoroughly editorised.

Yours faithfully,

D. WHEELER and R. NEEDHAM

University of Cambridge Computer Laboratory
Corn Exchange Street
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12 October 1972

Editor's comment:

Whilst agreeing with the sentiments expressed by the writers of this letter, I wonder if they really feel that the editor has the right to change words which already have been author-ised.