

Sir
The 'P.S.' to Mr. David Silber's letter to (this *Journal*, Vol. 15, No. 3) is indeed interesting. I too find it odd that there is as yet no specialist group on Time Sharing.

Could it be that 'Time Sharing' is a big myth and unsupportable in fact?

Yours faithfully,
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To the Editor
The Computer Journal

Sir
In general I agree with B. L. Meek (this *Journal*, 15, 267) that the ALGOL 68 looping structure is quite a good one. But the syntax he suggests for putting it into FORTRAN will not do. This is because FORTRAN has no reserved words, and is space independent (other than in a Hollerith datum), within columns 7-72.

Thus
DO 15 I = M TO N

could mean that the value of a variable called MTON was to be assigned to a variable called DO15I.

In ANS FORTRAN, the statement

DO 15 I = M TO N BY K

must be a DO-statement since MTONBYK has 7 letters and cannot be an identifier. However the righthand side could still mean M to NBYK (in steps of 1), in addition to its obvious meaning.

The only way out would seem to be to copy the .AND. .OR. .NOT. device and require .TO. .BY. .WHILE., but this would lead to a proliferation of 'dotted' system words, which would be much worse than the underlining of ALGOL, in that one would have to be constantly on one's guard to remember which words were dotted and which were not. The existing eleven dotted words are quite enough.

Yours faithfully,
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Brian Meek replies:

David Hill is quite right that the control words should be dotted, and I am grateful to him for pointing out the error. Unlike him,

however, I find the use of dots quite acceptable; the FORTRAN camel has so much in the way of rules to remember heaped upon it that I cannot regard this as the last straw. And anyone who dislikes dotted words could still use commas.

To the Editor
The Computer Journal

Sir
Lagrangian Interpolation at $\cos(\nu\pi/n)$, $\nu = 0(1)n$
A recent paper by Salzer (1972) discusses five unnoted advantages in Lagrangian interpolation at the Chebyshev points

$$x_{n,\nu} \equiv \cos(\nu\pi/n), \nu = 0(1)n.$$

The writer now wishes to add an important sixth advantage, also hitherto unnoted, namely, that of optimally stable extrapolation and extrapolatory differentiation for any order derivative.

When the function $f(x)$ is known, observed, measured, or found experimentally to limited accuracy for x within $[-1, 1]$, the computing error in $(n+1)$ -point Lagrangian extrapolation or extrapolatory differentiation, where $x > 1$, could often be more significant than the truncating error. Then for

$$f^{(k)}(x) \sim \sum_{\nu=0}^n L_{\nu}^{n+1(k)}(x) f(x_{\nu}), k \geq 0,$$

where

$$L_{\nu}^{n+1}(x) = \prod_{\mu=0, \mu \neq \nu}^n (x - x_{\mu}) / \prod_{\mu=0, \mu \neq \nu}^n (x_{\nu} - x_{\mu}),$$

a criterion for optimal computational stability is the minimisation of

$$L \equiv L(n, k, x_0, x_1, \dots, x_n; x) = \sum_{\nu=0}^n |L_{\nu}^{n+1(k)}(x)|$$
 by proper choice

of x_{ν} , $\nu = 0(1)n$, within $[-1, 1]$. It turns out that L is minimised by $x_{\nu} = x_{n,\nu}$ for every $x > 1$ and every $k \geq 0$.

The proof follows from

$$\sum_{\nu=0}^n |L_{\nu}^{n+1(k)}(x)| = \left| \sum_{\nu=0}^n (-1)^{\nu} L_{\nu}^{n+1(k)}(x) \right| = T_n^{(k)}(x),$$

the first equality holding whenever $x > 1$, $k \geq 0$ (even when $x_{\nu} \neq x_{n,\nu}$, as long as the x_{ν} are labelled to increase with ν), and the second equality holding when $x_{\nu} = x_{n,\nu}$. But $T_n^{(k)}(x)$ is a lower bound for L for all sets of x_{ν} , $\nu = 0(1)n$, which are in $[-1, 1]$, and since it is assumed here for $x_{\nu} = x_{n,\nu}$, those points minimise L .

Yours faithfully,
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Reference

SALZER, H. E. (1972). Lagrangian interpolation at the Chebyshev points $x_{n,\nu} \equiv \cos(\nu\pi/n)$, $\nu = 0(1)n$; some unnoted advantages, *The Computer Journal*, Vol. 15, pp. 156-159.