Estimating magnetic disc seeks

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This paper discusses various conflicting methods of estimating magnetic disc seeks when their files are processed randomly, pseudo-randomly and serially. Several common methods are proven to be significantly inaccurate and alternative methods are developed analytically.

(Received July 1973)

This paper results from the CAM research project at the London School of Economics and Political Science which is investigating computer-aided methods of developing computer-based, information processing systems. The project was initially financed by the Science Research Council and outlined by Waters (1972a).

When computer systems of files and programs are being designed, alternative solutions should be developed, evaluated and compared; Waters (1973) indicates the possibly vast numbers of alternative solutions that can result from a systematic approach to computer systems design. Each design should be evaluated in terms of computer run time, amongst many other criteria, to ensure that response/turnaround time constraints are met, to ensure that the computer configuration has sufficient capacity to operate the design, to estimate the computing component of recurring cost and thence to compare alternative solutions.

A significant, and sometimes dominant, contribution to computer run time is incurred when transferring information between direct access devices and primary storage; further, this transfer time is often dominated by the seek (i.e. arm-movement) times of widely-used magnetic disc devices. These relatively slow seek times can be significant over the entire range of computer systems, from simple 'batch-processing' to complex 'quick response'; thus, it can be vital that sufficiently accurate methods be used to estimate these seek times.

In practice, several inaccurate methods are used to 'guess-timate' magnetic disc seek times at the computer systems design stage. In some cases, this may not matter because computer run time is heavily dominated by other factors. However, if computer run time is consequently grossly underestimated, then the computer system may collapse because response/turnaround times are not met or because insufficient computer time is available or because the computer system becomes uneconomic. Alternatively, if computer run time is consequently grossly overestimated, then perfectly valid designs can be rejected; as a result, an inferior design may be accepted, or, in the extreme case, the computer system may be totally rejected on the possibly false grounds that it is not feasible. Unfortunately, inaccurate estimating methods often remain undetected because computer systems are not sufficiently controlled; the operational computer system is often not evaluated against its original, planned design.

Magnetic disc devices

The wide range of current direct access devices is based on the concept of circular storage of magnetic information; circles, or tracks, of information are constantly rotating so that each block of information is periodically accessible to the fixed or movable read/write heads that service the track. These magnetic devices include fixed and exchangeable discs and disc packs, drums and card files; typically, the time to access a block of information varies from a few milli-seconds to a few seconds. This access time includes the seek time to position movable read/write heads, the latency or rotational delay time to spin the block of information to a read/write head, the transfer time to read or write the block of information and the optional cyclic check time to reread a block of information that has just been written.

This paper is concerned with the seek times of movable read/write head, fixed and exchangeable magnetic disc devices; it is assumed that there is only one read/write head to service all the tracks of a single disc surface and that seek time is dependent on the number of cylinders (of tracks) traversed by the read/write heads but independent of the source and destination cylinder numbers. Probably the most common such device is the exchangeable disc pack handler supplied as the IBM 2311 and ICL 2802; this device has 200 cylinders (of ten tracks each) where seek time is defined by the graph of **Fig. 1.**

Random seeks

Random seeks occur when the accesses to a magnetic disc file are unconstrained, for example:

1. Input and output messages that access the file are unsorted (with respect to the file) and each cylinder of the file has equal probability of being accessed by any message; it is assumed that a magnetic disc device is dedicated to the file. This situation often arises in 'one-shot', batch processing systems where inter-active messages are processed against several files in the same program run; it also arises in single-thread, quick-response systems and in multi-thread, quick response systems where queues are processed in FIFO (i.e. queue) or LIFO (i.e. stack) mode.
2. A magnetic disc device contains several files, possibly including systems software, that are continually being accessed with equal probability but in no sequence (with respect to the device). This situation can arise in a uni-programming environment but more commonly occurs in a multi-

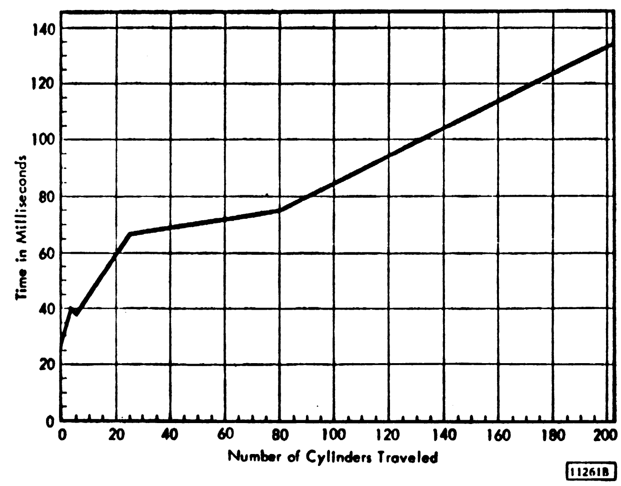

Fig. 1 Seek time graph of common disc pack device Courtesy IBM

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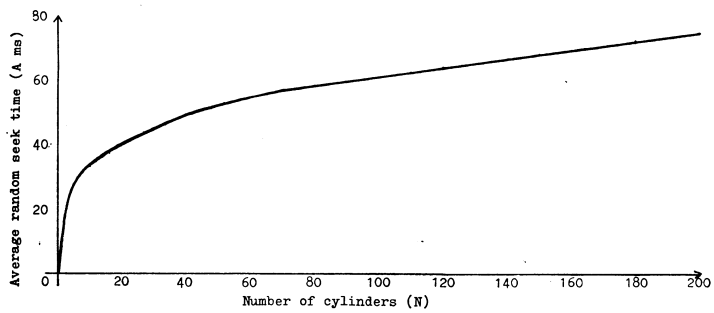

Fig. 2 Random seek time graph for common disc pack device

programming environment. The device may be treated as a single, randomly-processed file.

Thus, random seeks can occur in many situations and the factors that dictate the average seek time, say Ams, are the size of the file, say N cylinders, and the time taken to seek any number of cylinders, say $T(n)$ms.

Clifton (1969) suggests that $A = T(1)$ ms so that the average seek time is always that for a single-cylinder seek; clearly, this is false.

A widely-taught formula is $A = \frac{1}{2}T(N - 1)$ms on the basis that the average seek time is the average of the minimum and maximum seek times; this assumes that $T(n)$ is a linear function of n which is not generally true (e.g. Fig. 1).

A widely-used formula is $A = T\left(\frac{N-1}{2}\right)$ ms on the basis that the average seek distance is $\frac{N-1}{2}$ cylinders; although this method partially recognises that $T(n)$ is not a linear function of n, it overestimates the average seek distance and yields a pessimistic result (which is probably why the method is so popular).

Appendix 1 proves that the average seek distance is nearer $\frac{N}{3}$ cylinders therefore $A = T\left(\frac{N}{3}\right)$ ms is a more accurate formula.

Clearly, average seek time should be estimated by summing all possible seek times and dividing by the number of possible seeks. Appendix 2 develops a general formula on this basis which is not the integral of the seek time function, $T(n)$, as is commonly thought. **Fig. 2** plots this formula for the common disc pack device; this graph indicates significant variations over the previous formulae that estimate average seek distance and even the last formula is up to 25 per cent inaccurate; for example, the average seek time over a 71-cylinder file is 57 ms from the graph of Fig. 2 whereas the four formulae yield 25, 37, 68 and 67 ms, respectively.

Pseudo-random seeks

In practice, pseudo-random seeks are common; although the accesses to the file are not made in the file sequence these accesses are constrained by some pattern.

One common accessing pattern is that some records of the file are more likely to be accessed than the others; often, these highly-active records are contained in consecutive cylinders of the file to form a 'hit group'. In this case, the acessing pattern is usually defined as '(a high) percentage of all accesses to the file hit (a low) percentage of consecutive cylinders of the file'; for example, Pareto's extremely common case yields '80 per cent. of all accesses hit 20 per cent. of the file'.

Appendix 3 develops a formula for the average seek distance when the hit group is located at a general point of the file. Appendix 5 proves that this average seek distance is minimised by locating the hit group at the centre of the file and maximised by locating it at one of the file's extremities; unfortunately, in practice, the hit group is often inefficiently located at the start of the file. Appendix 5 further demonstrates that the average seek distance variation between locating the hit group at the start and centre of the file can be highly significant (which is not intuitively obvious and is therefore possibly not appreciated, in general). Frank (1969) indicates the technique of 'centrally locating dynamic records' to reduce seeking but underestimates its significance by suggesting that such reductions are only slight.

Another common accessing pattern is that a magnetic disc device contains several files that are continually being independently accessed with different probabilities; for example, in a multi-programming environment, systems software might constitute several files on a device that also contains working files for several running programs (e.g. spooled data and results files, sort files, intermediate results files, etc.). The 'file hit group' approach above can be extended to this situation; the average seek over the multi-file device is reduced by locating the most frequently-accessed files centrally and the least frequently-accessed files at the extremities of the device; for example, given five files on the common disc pack device of 20, 30, 40, 50, 60 cylinders and accessing probabilities of 0·2, 0·1, 0·4, 0·2, 0·1, respectively, then the average device seek distance is minimised by locating the files on cylinders 61-80, 171-200, 81-120, 121-170, 1-60, respectively.

A restriction to this common accessing pattern yields the further common situation of a magnetic disc device containing several files that are continually being accessed with different probabilities but inter-dependently; thus, the probability of accessing a particular file depends on the previous file accessed and a 'file transition matrix' defines the probabilities of accessing any file from any other file. Verne and Bayes (1972) discuss several locating algorithms to reduce average device seek for this case; the optimum algorithm involves dynamic programming.

Serial seeks

Serial seeks occur when the accesses to a magnetic disc file are made in the same sequence as the file, for example:

1. Input and output messages that access the file are sorted to the same sequence as the file; often, this is deliberately arranged to minimise time-consuming seeks. Batch-processing systems often apply serial processing methods and multi-thread, quick-response systems often sort file access queues to apply serial scanning methods.
2. 'One-shot', batch-processing systems often appear to be processing some files randomly whereas these files are really being processed (repetitively) serially. For example, a process program that inputs customer orders and outputs customer invoices might update a customer file (held in customer number sequence) and a product file (held in product number sequence); if the customer/product order lines are merely pre-sorted to customer sequence, then the customer file is processed serially, say on magnetic tape, and the product file is processed randomly, say on magnetic disc; however, if these customer/product order lines are pre-sorted to product number sequence within customer number sequence then the product file is processed serially for each and every customer order. Another common example is a production explosion process program that inputs product requirements data in product sequence, refers to a product/resource usage file (held in that sequence) to calculate the resource requirements for each product and builds up a total resource requirements file for subsequent output; this latter file is processed serially for each product.

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Thus, serial seeks are extremely common both by design and by accident.

If a file is serially processed (and therefore scanned) just once during a batch-processing run, then the seek time is usually insignificant; for example, the time to scan all cylinders of the common disc pack device is approximately five seconds (i.e. 200 single-cylinder seeks). However, if the file is scanned repetitively, then seek time can be highly significant; for example, if customer orders to be processed or products to be exploded in the above examples number thousands, then total seek time can be measured in hours.

If all N cylinders of a file are scanned repetitively, as in serial processing, then total seek time for a single scan of the file is the time for a single return seek of $(N - 1)$ cylinders, from the end to the beginning of the file, plus the time for $(N - 1)$ minimal, single-cylinder seeks. If only $M(<N)$ cylinders of file are scanned repetitively as in skip-serial processing, then Appendix 6 develops a formula for estimating M given the record hit ratio and Appendix 7 indicates the total seek time for a single scan of the file. In both these cases, the return seeks can sometimes be eliminated by scanning the file in alternate directions so that one scan accesses the file in 'ascending' sequence and the next scan accesses the file in 'descending' sequence; for example, the customer/product order lines could be primarily sorted to customer number sequence and secondarily sorted to ascending or descending product number sequence depending on whether the customer order has an 'even' or 'odd' position in the primarily sorted, customer orders file.

Conclusion

This paper has estimated seeks for files stored on common magnetic disc devices and processed randomly, pseudo-randomly and serially. The following main results have been developed by simple, analytical methods:

1. The average random seek distance is one third the number of cylinders in the file; the average random seek time for the common disc pack device is plotted by Fig. 2.
2. The average pseudo-random seek distance varies significantly for a file containing a hit group (depending on where the hit group is situated); this seek is minimised by locating the hit group at the centre of the file.
3. Repetitive serial seeking can be incorrectly confused as random seeking; further, return seeks can be avoided by appropriate ascending/descending sorting.
4. The cylinders accessed by a skip-serial scan of a file disect the file into equal-length sections, on average.

Further, several established but invalid methods have been indicated.

Acknowledgements

The author wishes to acknowledge the assistance of his colleagues in the LSE Systems Research Group, particularly Miss L. Julian, Mr. F. F. Land and Mr. J. Rosenhead of the Statistics, Mathematics, Computing, Operational Research and Demography Department. Finally, the author would be pleased to discuss the details of this paper with any interested organisation.

Appendix 1

To prove that the average random seek distance for a file of N cylinders is $N/3$ cylinders, approximately.

Let the source cylinder of the read/write head be $i(i = 1, 2, \ldots, N)$ and let its destination cylinder be $j(j = 1, 2, \ldots, N)$.

The seek distance from cylinder i to cylinder j is $|i - j|$ cylinders, therefore the sum of all possible seek distances, numbering N^2, is given by:

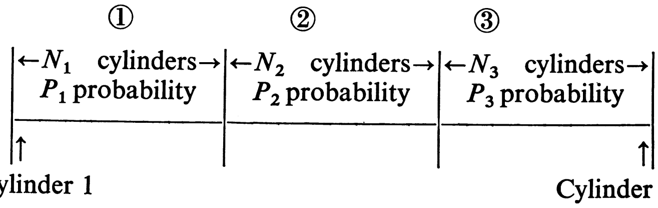

Fig. 3 File with generally-located hit group

Seek	*Probability*	*Average Seek Distance (cylinders)*
① → ①	P_1^2	$\frac{N_1}{3}$
② → ②	P_2^2	$\frac{N_2}{3}$
③ → ③	P_3^2	$\frac{N_3}{3}$
① → ②, ② → ①	P_1P_2	$\frac{N_1 + N_2}{2}$
② → ③, ③ → ②	P_2P_3	$\frac{N_2 + N_3}{2}$
① → ③, ③ → ①	P_1P_3	$\frac{N + N_2}{2}$

Fig. 4 Analysis of seeks for file with generally-located hit group

$$\sum_{i=1}^{i=N} \sum_{j=1}^{j=N} |i - j|$$
$$= \sum_{i=1}^{i=N} 2 \sum_{j=1}^{j=i} (i - j)$$
$$= 2 \sum_{i=1}^{i=N} \left[i^2 - \frac{i(i + 1)}{2} \right]$$
$$= \sum_{i=1}^{i=N} (i^2 - i)$$
$$= \frac{N(N + 1)(2N + 1)}{6} - \frac{N(N + 1)}{2}$$
$$= \frac{N(N + 1)(N - 1)}{3} \text{ cylinders .}$$

Thus, the average random seek distance is given by:

$$\frac{N(N + 1)(N - 1)}{3} \div N^2$$
$$= \frac{N^2 - 1}{3N}$$
$$= \frac{N}{3} \text{ cylinders approximately, since } N > 1$$

QED

Frank (1969), Martin (1967) and Sharpe (1969) developed this formula using different approaches.

Appendix 2

To estimate the average random seek time for a file of N cylinders, where $T(n)$ ms is the time to seek n cylinders.

Let the source cylinder of the read/write head be $i(i = 1, 2, \ldots, N)$ and let its destination cylinder be $j(j = 1, 2, \ldots, N)$.

The seek from cylinder i to cylinder j is $|i - j|$ cylinders, therefore the sum of all possible seek times, numbering N^2, is given by:

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$$\sum_{i=1}^{i=N}\sum_{j=1}^{j=N} T(|i-j|)$$

$$= \sum_{i=1}^{i=N} 2 \sum_{j=1}^{j=i} T(i-j)$$

$$= 2 \sum_{i=1}^{i=N} [T(1) + T(2) + \ldots + T(i-1)]$$

$$= 2[(N-1)\,T(1) + (N-2)\,T(2) + \ldots + 2T(N-2) + T(N-1)] \text{ ms} .$$

Thus, the average random seek time is given by:

$$A = \frac{2}{N^2}[(N-1)\,T(1) + (N-2)\,T(2) + \ldots + 2T(N-2) + T(N-1)] \text{ ms} .$$

Appendix 3

To estimate the average seek distance for a file of N cylinders which contains a generally-located hit group.

Fig. 3 illustrates the file as being split into three sections of N_1, N_2 and N_3 cylinders such that the middle section is a high-activity hit group of general length and location; the probabilities of accessing each section of the file are P_1, P_2 and P_3.

Clearly, N_2 is given, N_1 is the variable that defines the location of the hit group and $N_3 = N - N_1 - N_2$.

Further, P_2 is given and, since the hit group has high activity,

$$P_2 > \frac{N_2}{N} .$$

Also,

$$P_1 = \frac{N_1}{N_1 + N_3}(1 - P_2) = \frac{N_1}{N - N_2}(1 - P_2)$$

and

$$P_3 = \frac{N_3}{N_1 + N_3}(1 - P_2) = \frac{N - N_1 - N_2}{N - N_2}(1 - P_2)$$

Fig. 4 lists all possible seeks that can occur, the probability that each occurs and its average seek distance; the first three use the approximated result of Appendix 1 and the last three use a result derived in Appendix 4.

Thus, the average seek distance over the entire file is approximately given by

$$S = \tfrac{1}{3}(P_1^2 N_1 + P_2^2 N_2 + P_3^2 N_3) + P_1P_2(N_1 + N_2) + P_2P_3(N_2 + N_3) + P_1P_3(N + N_2) \text{ cylinders} .$$

This result is consistent with Appendix 1 in that $S = \frac{N}{3}$ in the limiting cases where the hit group disappears ($N_2 = P_2 = 0$) and when the hit group becomes the entire file ($N_2 = N$, $P_2 = 1$).

Appendix 4

To prove that the average seek distance between two files sharing a magnetic disc device is the number of cylinders between their mid-points.

Let the two files be F_1 and F_2 such that file Fi commences on cylinder Ci and spans Ni cylinders; assume that F_1 occupies lower cylinder numbers than F_2 so that $C_2 \geqslant C_1 + N_1$.

Let the source cylinder of the read/write head be i on F_1 ($i = C_1, C_1 + 1, \ldots, C_1 + N_1 - 1$) and let its destination cylinder be j on F_2 ($j = C_2, C_2 + 1, \ldots, C_2 + N_2 - 1$).

The seek distance from cylinder i to cylinder j is $(j - i)$ cylinders therefore the sum of all possible inter-file seek distances numbering N_1N_2, is given by

$$\sum_{i=C_1}^{i=C_1+N_1-1} \sum_{j=C_2}^{j=C_2+N_2-1} (j-i)$$

$$= \sum_{i=C_1}^{i=C_1+N_1-1} \left[\frac{(C_2 + N_2 - 1)(C_2 + N_2)}{2} - \frac{(C_2 - 1)C_2}{2} - i\,N_2\right]$$

$$= \frac{N_1}{2}[(C_2 + N_2 - 1)(C_2 + N_2) - (C_2 - 1)C_2] - \frac{N_2}{2}[(C_1 + N_1 - 1)(C_1 + N_1) - (C_1 - 1)C_1]$$

$$= \frac{N_1}{2}[N_2(2C_2 + N_2 - 1)] - \frac{N_2}{2}[N_1(2C_1 + N_1 - 1)]$$

$$= \frac{N_1N_2}{2}[(2C_2 + N_2) - (2C_1 + N_1)] \text{ cylinders} .$$

Thus, the average seek distance is given by

$$\frac{N_1N_2}{2}[(2C_2 + N_2) - (2C_1 + N_1)] \div N_1N_2$$

$$= \left(C_2 + \frac{N_2}{2}\right) - \left(C_1 + \frac{N_1}{2}\right) \text{ cylinders}$$

QED

Appendix 5

To prove that the average seek distance function, S, of Appendix 3 is (*a*) maximised by locating the hit group at the start (or end) of the file and, (*b*) minimised by locating the hit group at the centre of the file.

The average seek distance S of Appendix 3 can be written as a function of the variable N_1 as

$$S = \tfrac{1}{3}\left(\frac{1 - P_2}{N - N_2}\right)^2 N_1^3 + \tfrac{1}{3}P_2^2 N_2 + \tfrac{1}{3}\left(\frac{1 - P_2}{N - N_2}\right)(N - N_1 - N_2)^3 + \left(\frac{P_2(1 - P_2)}{N - N_2}\right)(N_1 + N_2)N_1 + \left(\frac{P_2(1 - P_2)}{N - N_2}\right)(N - N_1)(N - N_1 - N_2) + \left(\frac{1 - P_2}{N - N_2}\right)^2 (N + N_2)(N - N_1 - N_2)N_1$$

$$= 2\frac{1 - P_2}{(N - N_2)^2}(P_2N - N_2)N_1^2 - 2\frac{1 - P_2}{N - N_2}(P_2N - N_2)N_1 + \tfrac{1}{3}[N + (P_2N - N_2)(1 - 2P_2)] \text{ cylinders} .$$

Since $P_2 > \frac{N_2}{N}$ and $N_1 \leqslant N - N_2$ then the maximum value of this function, S_{max} is attained when N_1 (or similarly, N_3) is zero. QED(a).

Further, taking S as a function of a continuous variable, N_1, then $\frac{dS}{dN_1} = 2\frac{1 - P_2}{N - N_2}(P_2N - N_2)\left[\frac{2}{N - N_2}N_1 - 1\right]$ which is zero if, and only if, $N_1 = \frac{N - N_2}{2}$; thus, the minimum value of this function, S_{min}, is attained when

$$N_1 = \frac{N - N_2}{2} = N_3 .$$

QED(b).

Clearly,

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$$S_{max} = \tfrac{1}{3}[N + (P_2N - N_2)(1 - 2P_2)]$$

and

$$S_{min} = \tfrac{1}{6}[2N - (P_2N - N_2)(1 + P_2)]$$

In Pareto's case, $P_2 = \frac{4}{5}$ and $N_2 = \frac{N}{5}$

$$\therefore \quad S_{max} = \frac{16}{75}N$$

and

$$S_{min} = \frac{23}{150}N$$

which yields a variation in average seek distance of almost 40 per cent.

Another common case, often met in client accounting systems, is '60 per cent. of all accesses hit 5 per cent. of the file' so that $P_2 = \frac{3}{5}$ and $N_2 = \frac{N}{20}$

$$\therefore \quad S_{max} = \frac{89}{300}N$$

and

$$S_{min} = \frac{14}{75}N$$

which yields a variation in average seek distance of almost 60 per cent.

Appendix 6

To develop a formula for estimating the number of cylinders accessed, given the record hit ratio and the number of records in each cylinder.

Consider a magnetic disc file organised in several levels of storage; commonly, records form blocks, blocks form tracks, tracks form cylinders and cylinders form devices. The record hit ratio, R, measures the proportion of records accessed during a scan of the file therefore the hit ratio, S, for a higher level of storage (e.g. block, track, cylinder or device) can be estimated as follows:

$R = \dfrac{\text{Number of records accessed}}{\text{Total number of records}}$

= Probability any particular record is accessed, assuming accessed records are randomly distributed over the file

$\therefore (1 - R)$ = Probability any particular record is not accessed

$\therefore (1 - R)^L$ = Probability any particular higher level of storage is not accessed, where L is the number of records in that higher level

$\therefore 1 - (1 - R)^L$ = Probability any particular higher level of storage is accessed

$\therefore \; S = 1 - (1 - R)^L$.

This result was misprinted in Waters (1972) and in the subsequent erratum.

Thus, the cylinder hit ratio is given by

Scylinder = 1 − (1 − Record hit ratio) ↑ Number of records in cylinder thus, the number of cylinders accessed during a scan of the file is given by

$M = N \times S$cylinder, where N is the total number of cylinders in the file

= $N - N$(1 − Record hit ratio) ↑ Number of records in cylinder rounded up to the next integer.

Fig. 5 tabulates the cylinder hit ratio for common values of the record hit ratio and number of records in each cylinder.

Appendix 7

To indicate a method for estimating the total seek time for a single scan of a skip-serially processed file; assume the file has N cylinders and $M(<N)$ cylinders are accessed and these accessed cylinders are randomly distributed over the file.

Consider the simplest case of $M = 2$; let the first cylinder accessed be $i(i = 1, 2, \ldots, N)$ and let the second cylinder accessed be $j(j = i, i + 1, \ldots, N)$.

The seek distance from cylinder i to cylinder j is $(j - i)$ cylinders, therefore the sum of all possible seek distances is given by

$$\sum_{i=1}^{i=N}\sum_{j=i}^{j=N}(j - i)$$

$$= \sum_{i=1}^{i=N}\left[\frac{N(N + 1)}{2} - \frac{i(i - 1)}{2} - i(N - i + 1)\right]$$

$$= \tfrac{1}{2}\sum_{i=1}^{i=N}[N(N + 1) + i^2 - i(2N + 1)]$$

Record Hit Ratio (R)	Number of Records in Each Cylinder 10	20	30	40	50	60	70	80	90	100	150	200	250	300	400
0·01	0·10	0·19	0·27	0·34	0·40	0·46	0·51	0·56	0·60	0·64	0·78	0·87	0·92	0·96	0·99
0·02	0·19	0·34	0·46	0·56	0·64	0·71	0·76	0·81	0·84	0·87	0·96	0·99			
0·03	0·27	0·46	0·60	0·71	0·79	0·84	0·89	0·92	0·94	0·96	0·99				
0·04	0·34	0·56	0·71	0·81	0·88	0·92	0·95	0·97	0·98	0·99					
0·05	0·41	0·65	0·79	0·88	0·93	0·96	0·98	0·99							
0·06	0·47	0·71	0·85	0·92	0·96	0·98	0·99								
0·07	0·52	0·77	0·89	0·95	0·98	0·99									
0·08	0·57	0·82	0·92	0·97	0·99										
0·09	0·62	0·85	0·95	0·98											
0·10	0·66	0·88	0·96	0·99											
0·15	0·81	0·97													
0·20	0·90	0·99													
0·25	0·95														
0·30	0·98														

(otherwise 1·00)

Fig. 5 Cylinder hit ratios

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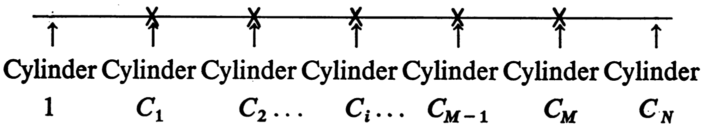

Fig. 6 File with some of its cylinders skip-serially accessed

$$= \tfrac{1}{2}\left[(N+1)N^2 + \frac{N(N+1)(2N+1)}{6} - (2N+1)\frac{N(N+1)}{2}\right]$$

$$= \frac{N(N+1)}{6}[3N - (2N+1)]$$

$$= \frac{N(N+1)(N-1)}{6} \text{ cylinders}.$$

The number of all possible seeks is given by

$$\sum_{i=1}^{i=N}\sum_{j=i}^{j=N} 1$$

$$\sum_{i=1}^{i=N}(N-i+1)$$

$$= N(N+1) - \frac{N(N+1)}{2}$$

$$= \frac{N(N+1)}{2}.$$

Thus, the average seek distance for the simplest case of $M = 2$ is $\frac{N-1}{3}$ cylinders; thus, in the average case, the accessed cylinders trisect the file.

In general, let the average case be illustrated by **Fig. 6** where cylinders Ci are accessed ($i = 1, 2, \ldots, M$). The average final seek distance (from C_{M-1} to C_M) can be expressed as the sum of all possible final seek distances divided by the number of all possible final seeks; thus,

Average

$(C_M - C_{M-1}) =$

$$\frac{\sum_{C_1=1}^{C_1=N}\sum_{C_2=C_1}^{C_2=N}\cdots\sum_{C_{M-1}=C_{M-2}}^{C_{M-1}=N}\sum_{C_M=C_{M-1}}^{C_M=N}(C_M - C_{M-1})}{\sum_{C_1=1}^{C_1=N}\sum_{C_2=C_1}^{C_2=N}\cdots\sum_{C_{M-1}=C_{M-2}}^{C_{M-1}=N}\sum_{C_M=C_{M-1}}^{C_M=N} 1}$$

Let

$$\Sigma' \equiv \sum_{C_1=1}^{C_1=N}\sum_{C_2=C_1}^{C_2=N}\cdots\sum_{C_{M-1}=C_{M-2}}^{C_{M-1}=N}$$

then the above dividend can be written as

$$\Sigma' \sum_{C_M=C_{M-1}}^{C_M=N}(C_M - C_{M-1})$$

$$= 2\Sigma' \sum_{C_M=C_{M-1}}^{C_M=N} C_M - \Sigma' \sum_{C_M=C_{M-1}}^{C_M=N} C_{M-1} - \Sigma' \sum_{C_M=C_{M-1}}^{C_M=N} C_M$$

$$= 2\sum{}' \left[\frac{N(N+1)}{2} - \frac{(C_{M-1}-1)C_{M-1}}{2}\right] - \Sigma' C_{M-1}(N - C_{M-1} + 1) - \Sigma' \sum_{C_M=C_{M-1}}^{C_M=N} C_M$$

$$= \Sigma' N(N - C_{M-1} + 1) - \Sigma' \sum_{C_M=C_{M-1}}^{C_M=N} C_M$$

$$= \Sigma' \sum_{C_M=C_{M-1}}^{C_M=N} (N - C_M)$$

Thus, Average $(C_M - C_{M-1})$ = Average $(N - C_M)$ which means that the average final seek distance equals the average number of untraversed cylinders at the end of the file; similarly, by symmetry the average initial seek distance equals the average number of untraversed cylinders at the start of the file. This approach can be followed for progressively reduced file sizes to prove that, in the average case, the M hit cylinders are equally spaced over the file at cylinders

$$\left(\frac{N-1}{M+1}\right), 2\left(\frac{N-1}{M+1}\right), 3\left(\frac{N-1}{M+1}\right), \ldots, M\left(\frac{N-1}{M+1}\right)$$

of the file. Thus, the total seek time for a single scan of the file can be estimated as the time for a single return seek of $(M-1)\left(\frac{N-1}{M+1}\right)$ cylinders plus the time for $(M-1)$ seeks of $\left(\frac{N-1}{M+1}\right)$ cylinders, rounded appropriately.

This simple estimating method probably improves current practice but yields a relatively crude estimate. Several colleagues are currently applying probability theory to yield finer estimates.

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