Time-domain matrix evaluation of a digital servomechanism using semi-hybrid computational techniques

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This article describes the simulation of a digital type of sampled data system, using a Solartron 247 analogue computer with digital mode control facility. A method for interfacing, off-line, to a Honeywell DDP 516 digital computer is also described, thus enabling the time-domain matrix technique to be investigated as a means of predicting the closed-loop response of the system from the open-loop response.

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Introduction

A typical digital control system normally contains a combination of digital and linear/non-linear elements. The main features of such a system can be simulated on an analogue computer which incorporates logic and automatic mode control facilities, provided that a small number of quantisation levels are acceptable to limit the setting up complexity. For the purpose of this work the number of quantisation bits was restricted to 6, giving 64 discrete levels. A purpose built interface system was available, permitting the taping of data from the computer model in such a form that 'off-line' calculations could be performed on a small readily accessible digital computer.

The need for this simulation stemmed from the author's research interest in the field of digital control, using the time-domain matrix technique as outlined by Dorf (1962). Aspects of the Dorf theory which are currently under investigation include (*a*) reduction of the number of matrix coefficients, (*b*) the effect of course quantisation, (*c*) compensation techniques and (*d*) application to non-linear systems. This article is particularly concerned with item (*a*).

The basis of Dorf's technique

A linear system, preceded by an ideal sampler, is shown in **Fig. 1.**

Let $g(t)$ be the impulse response of the time invariant system $G(s)$. Let $y(nT)$ be the response of $G(s)$, n sampling intervals after the application of the input signal $x(t)$, T being the sampling interval. Then, assuming an ideal sampler, the convolution summation can be written

$$y(nT) = \sum_{k=0}^{k=n} g(nT - kT)\, x(kT)\,. \quad k = 0, 1, 2, \text{- -}\, n$$

or in matrix form

$$Y = G\,.\,X$$

where $Y =$

$$\begin{bmatrix} y_0 \\ y_T \\ y_{2T} \\ \vdots \\ y_{nT} \end{bmatrix} ; \; X = \begin{bmatrix} x_0 \\ x_T \\ x_{2T} \\ \vdots \\ x_{nT} \end{bmatrix} \text{ and } G = \begin{bmatrix} g_0 & 0 & 0 & \cdots \\ g_T & g_0 & 0 & \cdots \\ g_{2T} & g_T & g_0 & \cdots \\ \vdots & \vdots & \vdots & \\ g_{nT} & g_{(n-1)T} & g_{(n-2)T} & \cdots \end{bmatrix}$$

The matrix G is referred to as the system transfer matrix.

This technique is easily extended to closed-loop systems. Thus, the systems shown in **Figs. 2** and **3** have identical transfer matrices, assuming synchronised samplers in the latter case.

$$G = G_0\,.\,[1 + G_0]^{-1}$$

In practice, when using a digital processor in a control loop, the theoretically ideal sampler is replaced by a sampler plus zero order hold circuit. The sampled waveform of **Fig. 4** is then replaced by that of **Fig. 5.**

To apply Dorf's technique when the sample and hold periods, T, are not short relative to the smallest time constant of the continuous waveform, the 'hold' levels may be regarded as a series of pulses as shown in **Fig. 6,** using a larger scale for clarity.

Effectively, a fictitious 'inner' sampling period, τ, is created. Then, provided τ is small compared with the reciprocal of the highest frequency present in $x(t)$, the first pulse spanning the time interval t_1 to t_2 may be treated as an impulse of strength $x_1\tau$ occurring at t_1. Successive pulses are regarded as impulses, of appropriate strength, occurring at t_2, t_3 and so on.

By making τ short relative to T, the accuracy of using the Dorf technique is improved, but only at the expense of larger matrix calculations, with the consequent strain on minicomputer storage capacity. Therefore the opportunity was taken of carrying out some calculations with fewer impulses and larger values of τ to see what simplifications could be made, whilst still achieving acceptable results. Sample graphs are included in a later section.

Basic digital control system features

For the purpose of this work a control system of the type used to control satellite tracking aerial positions was assumed, in which the error actuating signal is derived as the difference between a digitised reference signal and a digital shaft position encoder. The arrangement for such a system, which was fully described by Marshall and others (1962), is shown in **Fig. 7.** Damping feedback was not included for these initial computer studies, but is being included in a future exercise.

In practice the analogue-to-digital converter and the shaft encoder give rise to quantisation effects, whilst the digital subtractor and digital-to-analogue converter effectively sample and hold the quantised error signal. For ease of simulation on the analogue computer, the basic scheme of Fig. 7 was modified as shown in **Fig. 8.**

The quantiser is in effect an analogue-to-digital converter followed by a digital-to-analogue converter, operating on instantaneous input levels, without 'hold' facilities, but enabling quantisation errors to be introduced. It will be noted that quantisation errors due to both input and encoder digitisation have been combined into one unit. For the particular investigation covered by this article the quantiser was included in the analogue simulation, quantisation effects being allowed for by writing them in as part of the off-line digital program routine.

System simulation

The motor transfer function used was $G(s) = \dfrac{1}{s(1 + s\,100)}$ which, to economise on amplifiers, was simulated as shown in **Fig. 9.** It will be noted that a large time constant was adopted.

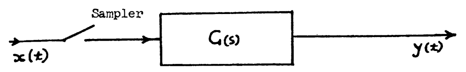

Fig. 1

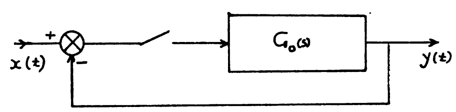

Fig. 2

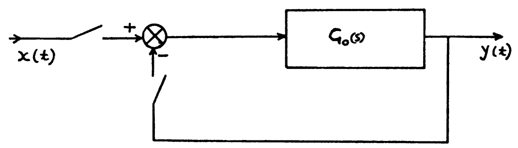

Fig. 3

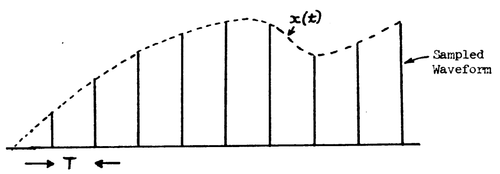

Fig. 4

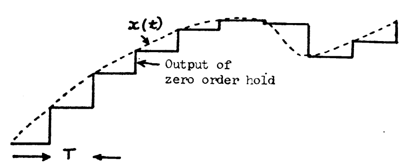

Fig. 5

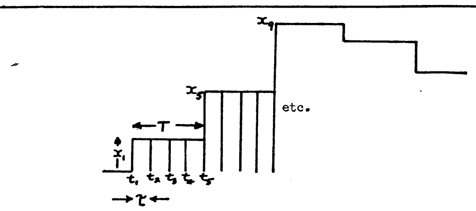

Fig. 6

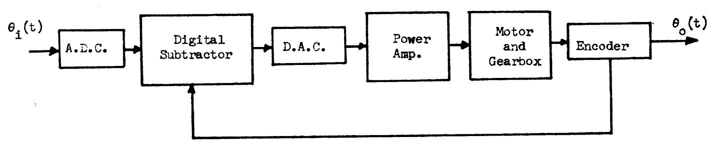

Fig. 7

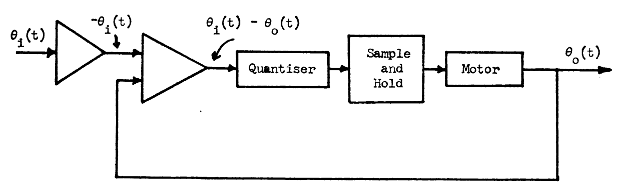

Fig. 8

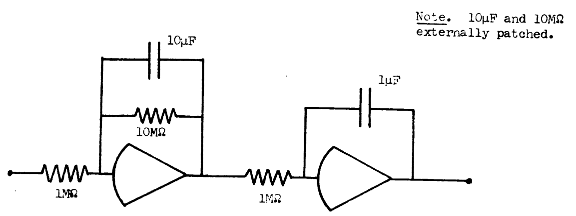

Fig. 9

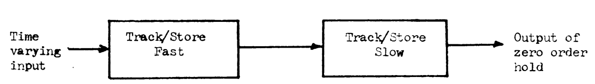

Fig. 10

This decision was dictated by the relatively slow punching rate capability of the off-line data punch, as explained in a later section.

The sample and hold function was simulated using the track/store facility available in the non-linear unit area. Full mode control of the track/store was available in conjunction with the sequence unit. A difficulty experienced here was the incompatibility of the track and store requirements. Thus, fast tracking was required to pick up the 'instantaneous' value of the signal being sampled, together with a long storage time equivalent to a zero order hold for up to 15 seconds. The available hardware offered two alternative settings—'fast' and 'slow'—which gave tracking time constants of 1 ms and 20 ms respectively. The former value was acceptable since it offered a relatively negligible tracking time of some 10 ms, but the associated drift in the store condition was totally unacceptable, being of the order of volts at best over 15 seconds. However, the drift rate for the slow setting could be balanced down to the much more acceptable value of 10 mV/s, although the minimum associated tracking time of about 200 ms was considered to be too long.

A compromise involving two track/store units was adopted as shown in **Fig. 10.**

It will be appreciated that the drift of the first unit is not significant while it is being tracked for 200 ms by the second unit. Overall, fast tracking and reasonable freedom from drift is achieved.

The patch panel connections to give the quantiser function are shown in **Fig. 11.** Up to 6 'bits' (64 levels) of quantisation are catered for.

Soft comparators were used, set to give +32V or 0V as their outputs, a typical arrangement being as shown in **Fig. 12.**

Sequence control programme

The mode control system used is of the sequential type, linked directly to the analogue machine, and has been fully described by Bellamy and Hulton (1968). For this particular application the sequential operation also has to control the paper tape punch multiplexing routine, which is used to interface to an off-line digital computer. It will be noted that three timers are called for in the sequential mode control programme shown in **Fig. 13.** This requirement can be fulfilled because the installation has twin analogue computers, each with two timers.

The inner sampling period, τ, is achieved by using a master clock generator set to a pulse repetition rate which is a suitable higher multiple of the main program cycle time—this in turn being given approximately by the summation of the timer settings. Thus

$$T = TA2 + TB2 + TB1$$

A bistable multivibrator is included in the conditional logic to synchronise the start of the track/store sequence with a pulse from the clock generator—otherwise the punched tape coefficients could not be related to the elements of the system matrices.

With switch SWA closed, the program is held in the 'Wait' condition, both track/store units tracking zero volts. Upon opening SWA, the track/store units store zero volts at sequence (3) and the program then goes into compute at sequence (4) for the period set by timer *TA*2—initiated by sequence (6). The fast/slow track and store sequence is then initiated through sequences (7) to (12), taking about 0.2s to complete, followed by a jump at sequence (13) back to timer *TA*2, and so on. However, should SWA be closed during the above sequence, then the sequence register will subsequently ignore the 'Jump' command at sequence (13) and will continue through sequences (14) to (31), thence back to sequence (0) and on to sequence (2), at which point it reverts back to the 'wait' condition. Note that

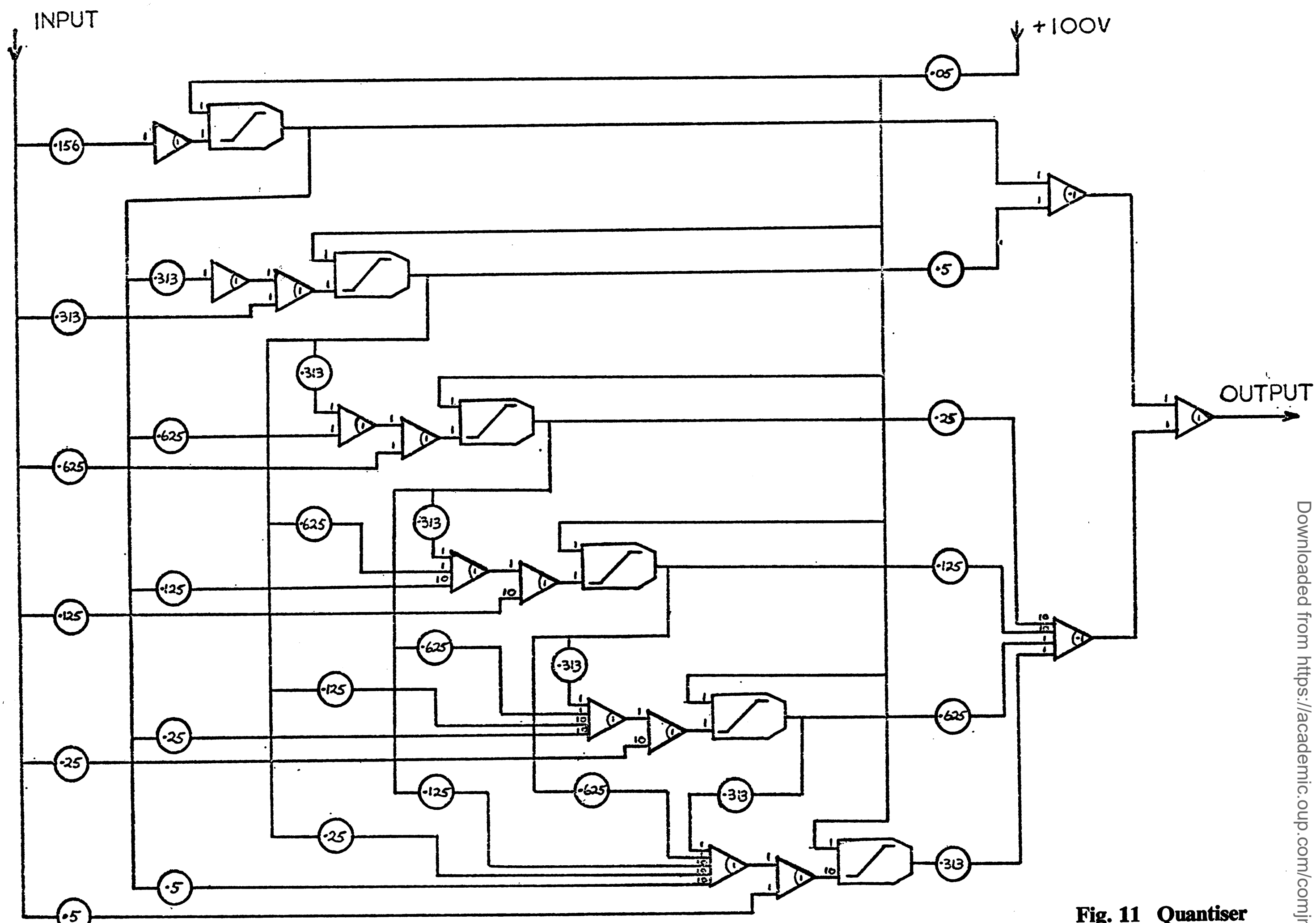

Fig. 11 Quantiser

all amplifiers have been reset at sequence (0) and the track store units put into 'track' at sequence (1).

Off-line interface to digital computer

The digital voltmeter incorporated in the analogue computer includes a binary coded decimal output facility. It was considered to be satisfactory if any voltage from 0V up to ± 99.9V, in steps of 0.1V, could be recorded on eight hole punched tape for off-line evaluation. Thus the number of tape holes to be punched, per voltage reading, is thirteen—allowing one for sign. Hence a multiplexing arrangement was built, whereby the thirteen characters per reading could be incoporated on two adjacent rows of eight holes—an additional character being punched on one of the two rows for identification purposes. The tape format is shown in **Fig. 14.**

Logic and driver equipment was built to interface in the above manner between the digital voltmeter and the punch mechanism. The maximum punch rate was 20 rows per second, equivalent to 10 voltage readings per second, the punching sequence being initiated by the sequence control and conditional logic.

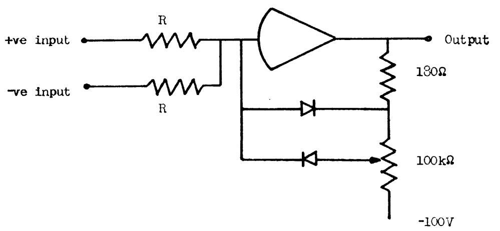

Fig. 12

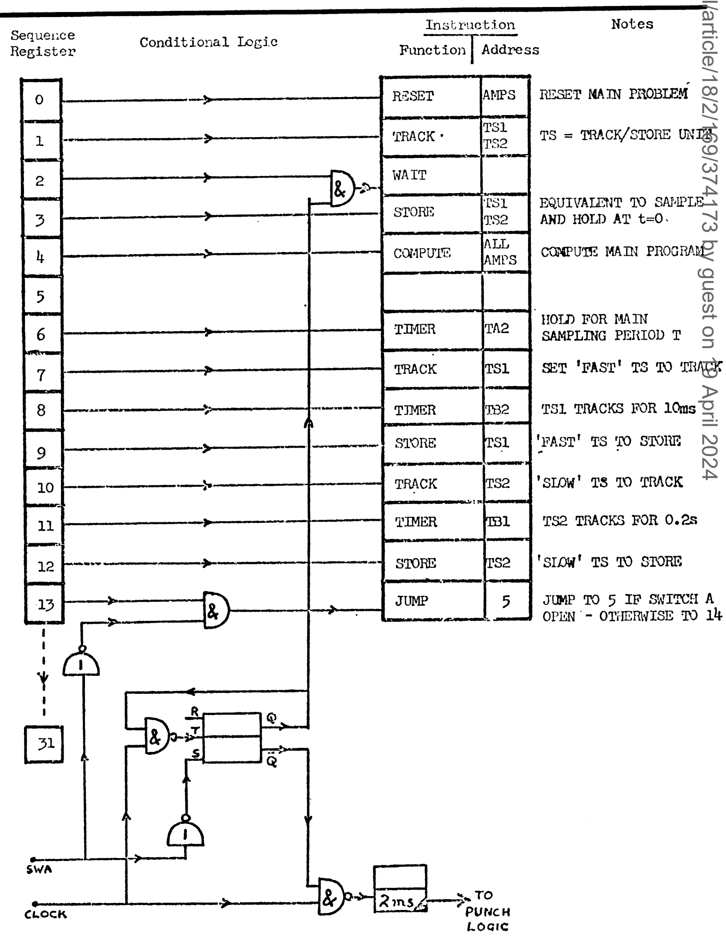

Fig. 13 Sequential-mode control program

Application of the time-domain matrix technique

The full closed-loop simulation was first tested using easily generated input waveforms, such as a step and a ramp. The system response was recorded using an X-Y plotter. Next, a punched tape was produced representing the system transfer matrix, described earlier, for the motor part of the simulation. There are various ways of obtaining the transfer matrix—the elements of which are derived from the sampled impulse response. In this particular case, because the last block of the motor simulation was an integrator, the method used was to apply a step input to the motor and to record on tape the input to the last integrator, this giving the impulse response directly.

The form of the input signal used for the earlier closed-loop response was also recorded on tape, using the same 'inner' sampling period τ.

Several such tape records were produced, using a range of values of τ. Off-line calculations of the closed-loop response were then performed on the digital computer, using a computer routine based on the Dorf technique as already outlined, the input and transfer matrix data tapes being those referred to above. The computer routine was written so as to allow for quantisation errors since, as already explained, the object of the exercise at this stage was to investigate only the variation of τ relative to T.

Experimental results

The natural period of the system studied would have been 59s with the sample and hold circuit bypassed. For the results shown in **Fig. 15,** the sampling interval—and thus the hold period—was set to $T = 15$s. Step and ramp inputs were used, with $N = T/\tau$ having integer values from 1 to 30—only $N = 2$, 6 and 30 being shown for comparison purposes. The calculated values for $N = 30$ were found to agree with the measured values to within a few per cent over the first full cycle, this being considered a satisfactory result. It was found that N could be reduced to 10 before errors of more than 5 per cent became apparent in the values calculated up to the end of the fifth sampling interval. With $N = 2$ the worst error was of the order of 70 per cent of the oscillation peak.

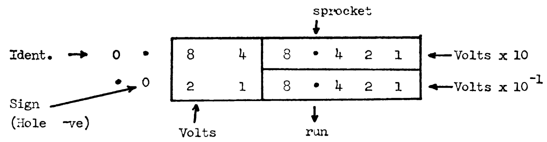

Fig. 14

Conclusion

A semi-hybrid simulation of a simple digital control system has been described. With the aid of an off-line computational facility an attempt has been made to explore the sampling rate aspect of the time-domain matrix technique. Some useful operational experience has been gained allowing further work to be attempted, with a full hybrid system, with a view to investigating the effects of coarse quantisation, and the synthesis of suitable control algorithms.

Acknowledgement

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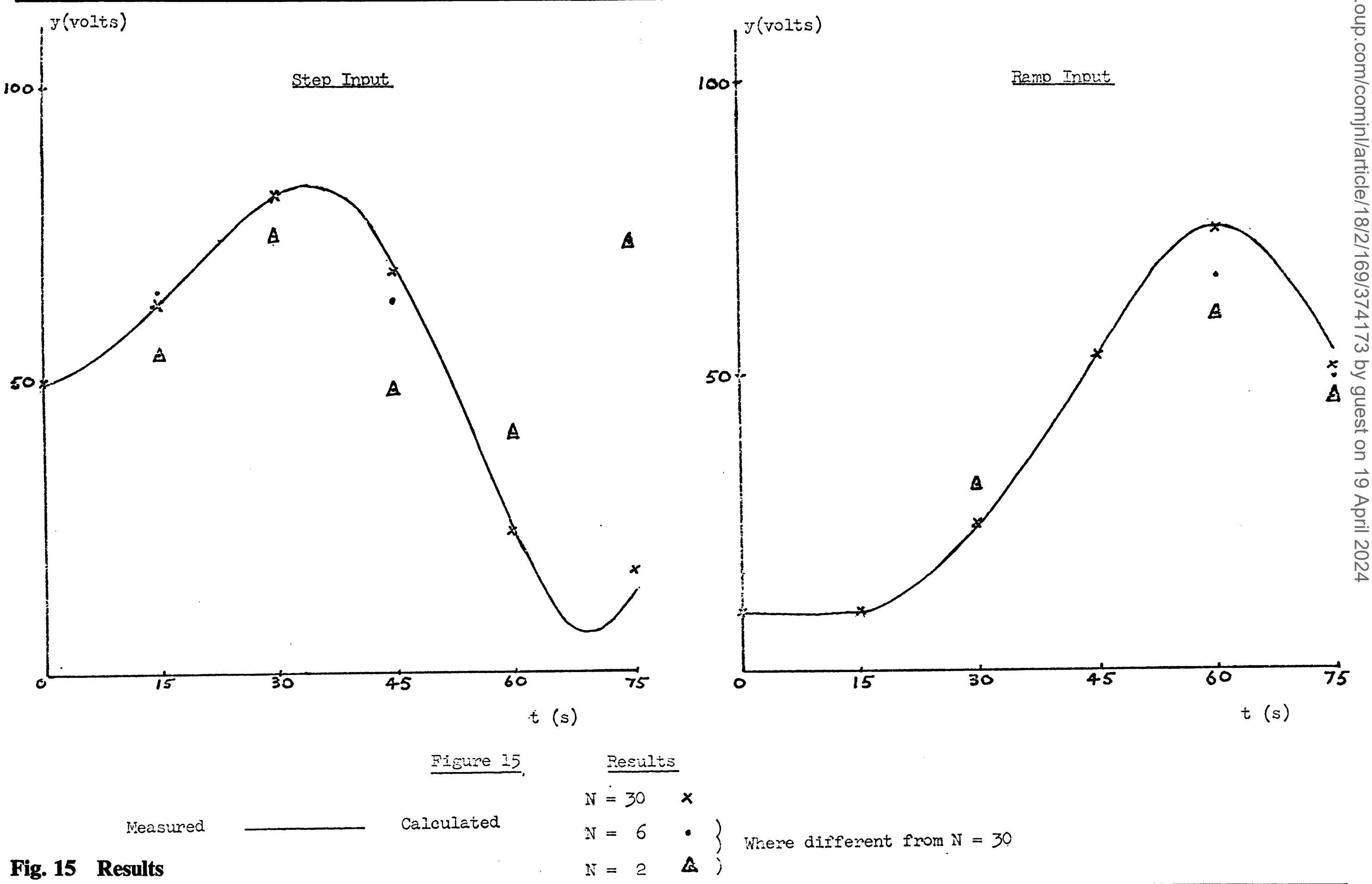

Fig. 15 Results

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