

storage is run under a virtual storage operating system. As the number of virtual partitions goes up the amount of real time wasted through paging has been observed as reported below. The effect of this wasted time is discounted by regarding the machine as having all real storage with a slower access time and consequent reduction of the ratio $x = T_A/T_S$. For a constant unit cost of 1.125 the situation is summarised below.

K	x	Wasted paging fraction	$1 - P_0$	Throughput	Throughput/cost
1	4	0	0.2	0.8	0.72
2	3	0.25	0.47	1.41	1.25
3	2	0.50	0.79	1.58	1.4
4	1	0.75	0.98	0.98	0.87

Three virtual partitions would appear to be the best operating strategy.

The desirability of spending a further 0.125 cost units to acquire a second real partition is to be investigated. It is

Reference

MORSE, P. M. (1958). *Queues, Inventories and Maintenance*, John Wiley and Sons, pp. 167.

assumed, perhaps not entirely justifiably, that the same paging characteristics will apply to the same over-commitment ratios. The position may be summarised:

K	x	Wasted paging fraction	$1 - P_0$	Throughput	Throughput/cost
2	4	0	0.38	1.54	1.23
4	3	0.25	0.79	2.37	1.93
6	2	0.50	0.99	1.98	1.58
8	1	0.75	1.0	1.0	0.8

The optimum operating point is four virtual partitions. It is interesting to compare the four virtual partitions of this example with the four real partitions of Example 2(a). The throughput in the virtual case is almost 90 per cent of the four real partitions, and the performance/cost ratio is some 3 per cent better. (In practice this small improvement would be swallowed up in increased systems overheads).

Book review

The Chebyshev Polynomials by Theodore J. Rivlin, 1974; 186 pages. (John Wiley, £8.60)

Chapter 1 (55 pages) of this book lists some elementary properties of the Chebyshev polynomials, and then considers Lagrangian interpolation with some relevant nodes, showing that no choice gives convergence, as the degree of the approximating polynomial increases, for every continuous function. Hermite interpolation at the Chebyshev zeros does achieve this, and Lagrange-Chebyshev interpolation at least converges in the mean for every continuous function. Further topics include orthogonal polynomials, the differential equations, recurrence relations and generating functions for the Chebyshev polynomials, and numerical integration and the Gauss-Chebyshev formula.

Chapter 2 (68 pages) discusses convex sets and the characterisation of the best approximation in the uniform norm, a variant in terms of extremal signatures, the Chebyshev condition for best polynomial approximation, the Haar criteria and uniqueness, and the minimax theory for an interval of the real axis. The second part of the chapter examines classes of linear functionals for which Chebyshev polynomials are extremal elements particularly in the space of polynomials, and discusses growth outside the interval, a generalisation of the Lanczos τ method for approximating e^x , and methods for determining bounds for the derivatives of a polynomial.

Chapter 3 (36 pages) treats economisation, the evaluation of a

finite Chebyshev series, Chebyshev expansions, absolute and uniform convergence of Chebyshev series, the relation between least squares and uniform errors of truncated Chebyshev expansions, bounds for the errors in terms of the coefficients and for the sizes of the coefficients, their computation by various quadrature rules, optimal properties of Chebyshev expansions, and the ellipse of convergence of Chebyshev series.

Chapter 4 (7 pages) discusses briefly the identity $T_m(T_n) = T_n(T_m) = T_{mn}$, the properties of permuting and commuting polynomials and the ergodic and mixing properties of the mapping T_n^{-1} , the sequence of mappings $T_1^{-1}, T_2^{-1}, \dots, T_n^{-1}$, and the iterates T_n^{-k} , the k -fold composition of T_n^{-1} .

This book is everywhere dense with mathematical information, analysis and over 200 relevant exercises. It aims to give the mathematical student a taste of the excitement stimulated by the Chebyshev polynomial in various areas of analysis, and to serve as leisure reading for a broader mathematical community. In both these respects it will clearly succeed. There is, however, little here for the practical man, scientist or applied numerical analyst, and one would perhaps have liked something on the 'applications of Chebyshev polynomials in kinematics', one of the topics (with number-theoretic aspects) excluded deliberately 'through ignorance'. There is certainly nothing ignorant about what has been included!

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