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INTEGRAL 'PLUS' 'IF' TYPE 'THEN'
'C' THE WEIGHT FUNCTION IS SIN PX 'C'
PART1*CS2+PART2*CS1
'ELSE'
'C' THE WEIGHT FUNCTION IS COS PX 'C'
PART1*CS1-PART2*CS2
'FI';
INTEGRAL 'TIMES'X2
'END';

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N2*N'/2; [0:N2]REAL'XS;
K40;
L2: UPB+(K+2)'/2;
D[K]'LOC'[1:UPB]REAL';
D[K][1:UPB]*CHEB'COEF'(K);
((K'PLUS'1)<=N)'GOTO' L2;
'C' THE PRECEDING PART GENERATES A TRIANGULAR ARRAY
TO RETAIN ALL THE NON-ZERO COEFFICIENTS OF CHEBYSHEV
POLYNOMIALS OF ORDER 0,1,2,...,N . 'C'

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'C' XS IS (B-A)T/2 AS IN (4.2.13). ONLY HALF OF THE
NUMBER OF XS ARE CALCULATED. 'C'
XS[0]+X2; 'FUR'S'TO'N2'DO' XS[S]+X2*COS(PI*S/N);
'C' FOR USE OF THE OPEN FORMULA (4.1.21) THE PRECEDING LINE
IS REPLACED BY
'FOR'S'FROM'0'TO'N2'DO'
XS[S]+X1*COS(PI*(2.0*S+1.0)/(2.0*N+2.0)); 'C'
(OMEGA>4.0)
'FOR'I'FROM'0'TO'N'DO'(TCWT[I]+INTEG(I,OMEGA,WB); TSWT[I]+WB)
I'FOR'J'FROM'0'TO'N'DO'(TCWT[J]+INTEG2(I,OMEGA,WB); TSWT[J]+WB) );
'C' TCWT AND TSWT STORE THE VALUES OF INTEGRAL OF T+I COS OR
SIN OMEGA*T ON [-1,1] . FOR OMEGA<4 PROCEDURE INTEG,
OMEGA<4 INTEG2 IS CALLED . 'C'
CS1=COS(CAPK); CS2=SIN(CAPK);
'FOR'J'FROM'0'TO'N2'DO' CAPF[J]+F(X1+XS[J]);
'FOR'J'FROM'N2+1'TO'N'DO' CAPF[J]+F(X1-XS[N-J]);
INTEGRAL*CAPI(CAPF);
INTEGRAL
'END';

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Book review

Queueing Systems, Vol 1: Theory by Leonard Kleinrock, 1975; 417 pages. (John Wiley, £9.75).

This first volume of a two volume work is aimed at providing an acceptable basis, neither excessively theoretical nor sloppily intuitive, for its successor.

Volume 1 is divided into four parts: (a) preliminaries; (b) elementary queueing theory; (c) intermediate queueing theory; (d) advanced material. None the worse for a tendency towards avuncular jocularly and an occasional idiosyncrasy (Thusly (sic) on page 17) this is a competent presentation of a range of topics in queueing theory. The bias is towards the computer applications promised for Vol. 2, which confers a distinctive character on the book. Indeed it contains much material and discussion which has probably not been collected together before. A balance has indeed been achieved in the level of treatment between the intensely pure mathematical and the over-simplified, and this should appeal to a wide audience of under graduates and some graduates. This reviewer would not choose it as a teaching companion, but it is assured a place on his reference shelf.

The second part of the book discusses the equilibrium theory of M/M systems with one or more servers, with and without finite storage capacity (waiting room), with and without a closed population of customers. Use is made of simple general results for state dependent demand and service of the ' λ_n and μ_n ' kind. Attention is then devoted to generalisation with Erlangian service or demand, the familiar application to group arrival and bulk service being noted. Finally, there is discussion of networks of Markov queues.

This third part of the book incorporates first a treatment of M/G/1. This is acceptable except for the busy period where the author would have done better to describe immediately Prabhu's beautiful time domain analysis leading to joint probability and probability density function of number served and duration in time. There is no mention of output. We are then led to the multiserver G/M/m whose treatment is relatively cursory. In particular there is no discussion of busy period.

In the fourth part of Volume 1 we find a more extended discussion of Lindley's treatment of waiting time, of Kingmann's algebraic formalism, and of duality relations.

The volume is concluded with an Appendix on the Laplace transform and on generating functions (here called z-transforms, which is most irritating and rates a definite minus mark) and their use in the solution of differential-difference equations arising in *Queueing Theory*. A second appendix is an aide m emoire on probability.

The glossary of notation and summary list of 'important results' constitutes a definite plus mark.

Other features which merit plus marks are the following. There is a painstaking discussion of traffic intensity and utilisation factor very early. Little's formula is also introduced at a much earlier stage than is common. It is right to emphasise such widely applicable general results.

The reviewer awards negative marks for a poor selection of references, poorly displayed. It is more useful to collect a list at the end.

But the overall impression has a positive balance. Volume 1 and its promised successor deserve success.

B. W. CONOLLY (London)