Conference Paper

The Use of Computers for Economic Planning in the Petroleum Chemical Industry

by G. S. Galer

INTRODUCTION

The Royal Dutch/Shell Group, in common with other large concerns, is currently making extensive use of the new mathematical aids to management. The manufacture of chemicals from petroleum, in which the Group is closely concerned in many parts of the world, holds a considerable potential for the profitable exploitation of these techniques, especially for the solution of complex economic problems.

The two economic problems which most frequently occur, and which are particularly amenable to mathematical analysis, are those of joint supply and of the allocation of scarce resources between alternative uses. The classical illustration of the principle of joint supply is the sheep, which supplies meat and wool in proportions which are, in the short term, inflexible. In our case, we have chemical plants producing two or three products in ratios which, due to the properties of the chemical reactions, can be varied only within certain strictly defined limits. Since these ratios do not necessarily coincide with the ratios of the market potentials for the products, we have to decide, knowing the prices at which we can sell the products, the set of ratios at which we must operate the plant, and the proportion of the market potential which we must satisfy, in order to function most economically. The problem of allocation between alternative uses is, of course, basic to all economics. In our case we may well, in the short term, sometimes meet limitations of base material or plant capacity, which will pose us this problem.

A TYPICAL PLANT COMPLEX

In order to describe these problems more clearly, we will consider a typical plant complex, as illustrated in Fig. 1. In this case a single base material is processed on four different plants to give a large number of final products. The first plant may at a given moment be used to make any one of three groups of products, each group consisting of the α-, β- and γ-forms of the same chemical. Plants "2" and "4" are used for two other product groups, the β- and γ-forms of product L being used as intermediates for the four different products produced on plant "3." In practice β- and γ-L will be alternatives for this purpose. Finally, α- and β-products from plant "1" are blended with β-E and γ-E to make another product, Blend D, while β-E and γ-E are also blended with α-L to make Blend F. α-, β- and γ-products from plants "1," "2" and "4" will, at a given moment, be emerging at fixed ratios, but these may be altered to a certain extent in the slightly longer term by appropriately adjusting the plant. In addition, several of the processes may be carried out with different catalysts, which will give different yields, final product ratios, and operating costs. The blended products must meet certain specifications, and there are in general many different combinations of the possible constituents which will adequately do this.

There is thus a high degree of flexibility in the plant system: for not only may plants be used for different purposes and at different ratios, but a given set of product requirements may generally be manufactured in many different ways. It is sometimes advantageous to make a surplus of a product in order to meet the requirement for a related product. But clearly such wasteful production should be minimized, and only undertaken when it is profitable. The programming of production on such a plant complex is so difficult, however, that the normal hand programmer can only draw up a program that is feasible, without having time to worry unduly about its economics. In the case where there is a shortage of plant capacity or base material, the problem becomes virtually insoluble without some kind of mathematical analysis.

THE ECONOMIC PROBLEM

The main economic problem which arises is, therefore, the following.

In a given period, the operating possibilities of the plant complex are known, and it is known that certain sales potentials exist for the products. How should available resources be deployed in order that the company shall operate to the best advantage? This, of course, is the classical economic problem of allocation between alternative uses, with the difference, however, that base material and plant capacity are not always "scarce resources." Even when there is an abundance of base material, however, the previous description will have made it clear that, by varying plant ratios, etc., there are many different ways of meeting a given sales potential. In this case, therefore, it is important to find that operating program which will yield the lowest possible manufacturing cost. In this case, too, it may be profitable to fall short on some sales potentials rather than produce unsaleable surpluses of related products.

Our experience has shown us that linear programming is a powerful tool in the solution of such problems. The rest of this paper is based upon a particular example of this approach; an exercise in mathematical planning

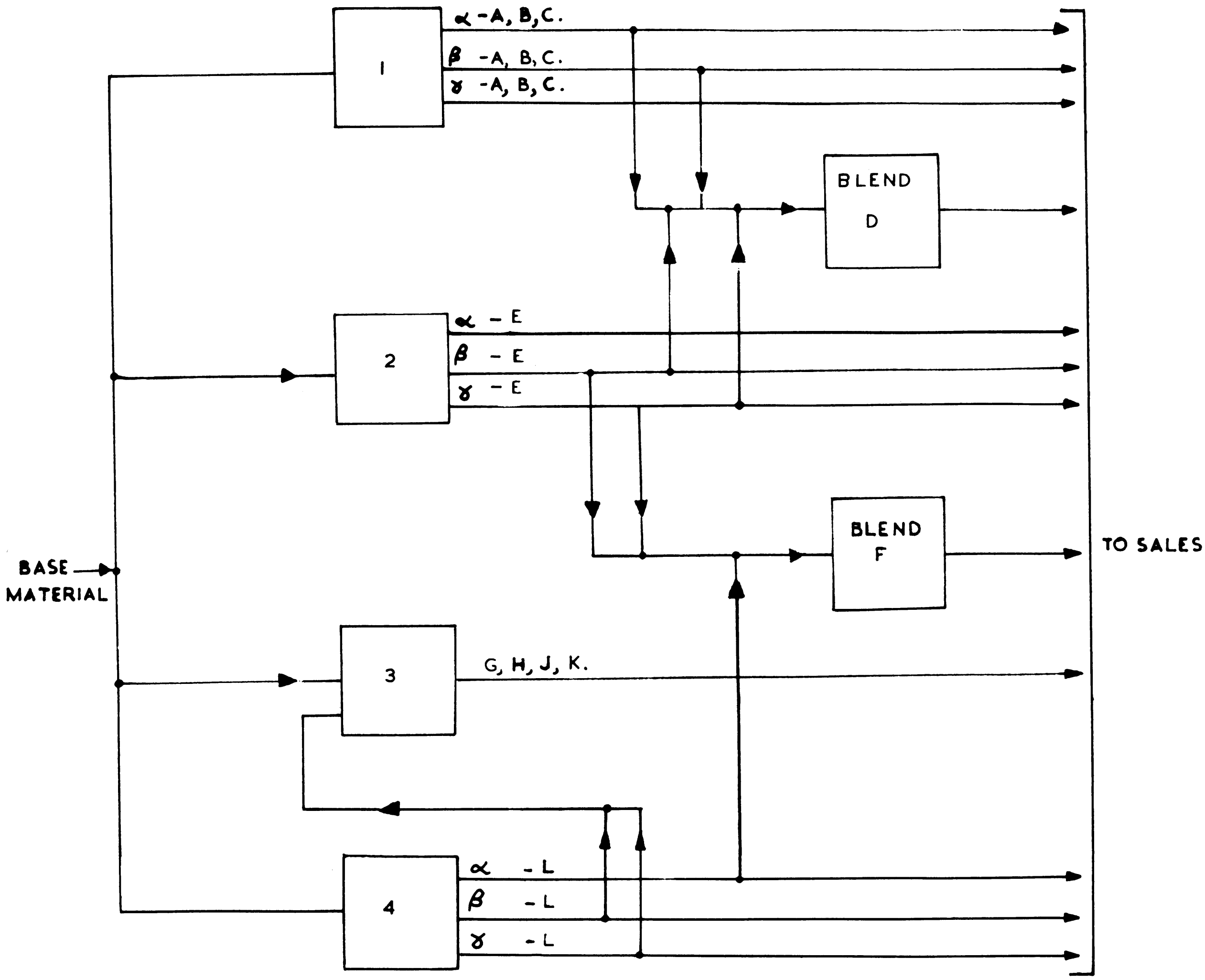

FIG. 1.—Flow sheet of typical petrochemical plant complex

which began in 1955 and which has now reached the stage when it is applied and accepted as a matter of routine.

THE MODEL

(i) *The Market*

Within the limits set by production possibilities, the demand curves for our petrochemical products show a slight downward slope only, and can for all practical purposes be assumed to run horizontally as in Fig. 2. In other words, it is assumed that, due to a low degree of substitutability for most products, a reduction of price from normal levels will not affect the volume of business. If it were otherwise, a non-linear demand function would have to be used in the mathematical representation of the problem. This is, in principle, possible computationally, but would raise many additional complications, going beyond the scope of this paper. In fact, we assume that the almost constant demand gives us an upper limit to the amount we can sell at a fixed price. This linearizes the revenue part of the objective function.

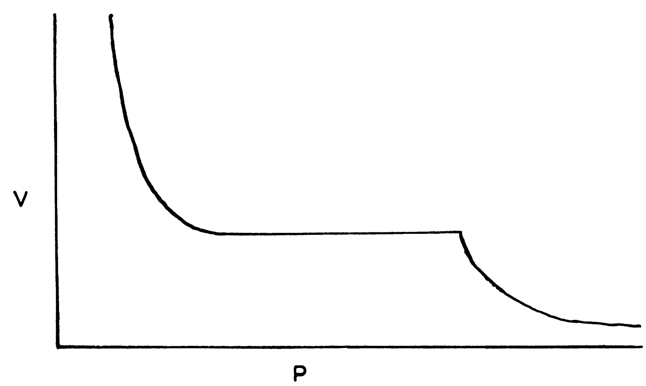

FIG. 2.—Demand curve

(ii) *The Plant*

The economic objective is so to operate the plant complex that production is expanded as long as marginal revenue exceeds marginal cost. In more simple terms, an operation is worth undertaking as long as the revenue it produces covers the marginal cost of the operation and also contributes something towards fixed costs and profit.

The objective function must therefore be based upon variable costs only. Fortunately, the variable cost curves in this case are linear, at least within the region of normal operation, and a completely linear objective function can therefore be formed.

Fortunately, too, it has also been found that all the production relationships are sufficiently linear for our purposes. It has therefore been possible to describe the whole system in linear programming terms. The best way of explaining the construction of the model will be to consider a much less complex problem of a similar type. Consider a hypothetical plant, as illustrated in Fig. 3, producing three products at once with yields p, q, r (where $p + q + r < 1$, allowing for some loss). It costs £t per ton to obtain and process the base material, and up to l, m tons, respectively, of the first two products may be sold at £a, £b per ton. In addition, the second and third products may be blended in the ratio $h : k$, and up to n tons of this sold at £c per ton. The capacity of the plant is C tons of base material in the time period being considered.

What is the best marketing and manufacturing plan for the plant?

Let variables be allocated to the input and sales flows, as shown. The situation then gives rise to the following linear programming problem:—

Maximize: $ax_1 + bx_2 + cx_3 - tX$

Subject to:

$$\left.\begin{aligned} x_1 &\leqslant l \\ x_2 &\leqslant m \\ x_3 &\leqslant n \end{aligned}\right\} \text{Sales}$$

$$\left.\begin{aligned} x_1 &\leqslant pX \\ x_2 + \frac{h}{h+k}x_3 &\leqslant qX \\ \frac{k}{h+k}x_3 &\leqslant rX \end{aligned}\right\} \text{Conversion}$$

$$X \leqslant C \quad \text{Capacity.}$$

The meaning of the sales and capacity inequalities is obvious: the conversion inequalities state that sales and usage of a product cannot exceed the amount made. They cannot be framed as equalities, since it may sometimes be profitable to produce an unsaleable surplus of a product in order to sell another product which is produced with it.

Our model is a more complicated example of this type, with the occasional addition of a restriction on raw material availability. The model varies from time to time, but consists generally of about 90 sales equations, 45 conversion equations and 15 to 20 capacity equations. There are about 180 non-basis variables. A few more points of detail about the model are as follows:—

(*a*) As has already been said, the percentage ratios $x : y : z$ (e.g. 60 : 30 : 10) in which α-, β- and γ-products are produced are fixed at any moment, but may be varied in the longer term by adjusting the plant. The points by which possible ratios for a given plant may be described all lie in the plane $x + y + z = 100$, but do not in general lie in a straight line. This difficulty has been surmounted by allowing for several individual sets of ratios, thus defining a convex set of points as a feasible region for each plant. In practice, an optimum solution almost invariably contains not more than two (the same two) extreme points of this set, and the model has therefore been simplified so that only two sets of operating ratios are allowed for each plant.*

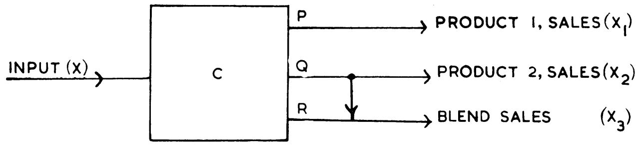

FIG. 3.—Typical production—marketing situation for a single plant

(*b*) The capacity equations are more complicated than those given in the example, since most plants are used for at least two products, and rates of reaction will differ according to which product is being made. This often makes it convenient to express plant capacity in terms of operating days rather than tons of product. For example, if in a certain period a plant can operate for T days, and if it will process q_i tons per day of base material to make product i, then the capacity equation will read:

$$\sum_{i=1}^{n} \frac{x_i}{q_i} \leqslant T$$

where the x_i ($i = 1, \ldots, n$) are quantities of material processed to each of the n different products.

(*c*) Most products are sold in two different markets, home and export. Thus there are two different variables, sales inequalities and economic returns for almost every product. The returns are known as "naked ex-tank values," and are calculated by deducting direct marketing costs from average prices. The quantity which we maximize is known as the "marginal profit."

USE OF THE MODEL

This model has gone through various stages of development since 1955, and is now regularly used for the formation of optimum marketing and manufacturing plans. It has provided valuable information on profitability for management, and has shown how marginal profit could be substantially increased by a re-framing of marketing plans.

Our chemical planning for production is carried out on two levels: a seven-year survey, and a five-quarter operating program. Linear programming is at present used within this framework in the following ways:—

* If both ratios are selected in the optimum solution, we interpret this to mean that the plant will operate for a certain length of time on one ratio, and for the rest of the time on the other.

(i) To produce truly optimum plans, by considering the balance of sales potentials and profit among all products, and showing which products must be made in which quantities, and by which route, in order to maximize profit. Very often it will be discovered that sales of a by-product are unprofitable beyond a certain margin or, conversely, that it is profitable to produce a surplus of a by-product in order to fill the market for the related main product. These plans have been used chiefly to set up annual budgets. They also give valuable information on total and marginal profitability and on the effects of small movements from the optimum.

(ii) Parallel with this, the model is used to produce the regular five-quarterly operating program. Since for commercial reasons it is not always possible to manufacture to an optimum program, we introduce new constraints when using the model for this purpose, turning the sales inequalities into equalities. This removes a great deal of flexibility from the model, but still produces worthwhile results. What is particularly interesting about this use, however, is that it enables one to keep a check on the cost of "policy" decisions. This is achieved by producing an unrestricted solution at the same time as the restricted operational program, and comparing the economic results. This difference can sometimes be quite substantial.

(iii) The model has also been used in the day-to-day program of economic work which the company maintains. Once an optimum solution has been obtained, only a short length of time is necessary to follow through the effects of changes in the initial constraints or objective function: in this way we can vary market potentials, plant capacities or yield factors, costs and prices, and thus rapidly reach new optima.

COMPUTATION

Most of our work has been carried out on the Ferranti Mark I* computer at the Royal Dutch/Shell laboratories in Amsterdam. Some other Mark I* computers have also been used, and some work has been done on the I.B.M. 704 machines in Paris and elsewhere.

The simplex programs written by our own programmers for the Mark I* are particularly well suited to our purpose, and, as stated above, we have made extensive use of them. They afford the possibility of making amendments to a problem once the optimum has been reached, in such a way that a new optimum is rapidly achieved. Given the basic problem: to find the values of a set of non-negative variables $x_1\ x_2\ .\ .\ .\ x_n$, which satisfy the linear equations (in matrix notation):

$$Ax = b,$$

and which maximize the linear function (again in matrix notation) cx, the following related problems can rapidly be solved by using an amendment program:

(*a*) $$Ax = b_1$$
$$\text{Maximize } cx.$$

This is achieved by adding the column $b_1 - b$ to the matrix, and solving the problem:

$$Ax = b + y(b_1 - b)$$
$$y = 1$$
$$\text{Maximize } cx.$$

(*b*) $$Ax = b$$
$$\text{Maximize } c_1x.$$

This is used to investigate the effects of different prices or costs upon an optimum plan.

(*c*) $$A_1x = b$$
$$\text{Maximize } cx.$$

Non-zero elements of the matrix A may be modified; thus the effect of changing plant yields may be investigated. Also, complete columns associated with variables in the non-basis set may be eliminated from the matrix.

Our problems normally run for 5–6½ hours on the Mark I* machine, while amendments take up to 2 hours, depending upon their number and the extent to which they alter the structure of the problem. Flexibility of operation in Amsterdam has been greatly increased recently by the acquisition of magnetic-tape units, which permit the storage of intermediate matrices, thus saving time on paper tape output, and providing a guard against computer failures.

FUTURE DEVELOPMENTS

The most promising use of mathematical planning in the chemical industry is undoubtedly for long-term investigations, and it is to these that we are now turning our attention. The Group's investment programme is going to bring many complex economic problems, most of which should be amenable to analysis. We hope that future models will continue to be linear, but some United States experience suggests that this will not be so.

We have now firmly established the value to management of the by-products of optimum plans, and we propose to extend the use of these. Particularly interesting are the marginal profitability figures. These are the criterion figures (d-row) which decide whether a solution is optimal and, if not, which column shall enter the basis. Consider, for example, the sales constraint:

$$x + \lambda = q,$$

where x and λ are non-negative variables and q a positive quantity, x carrying some positive value c in the objective function, while λ, a slack variable, has value zero. At the optimum solution to the problem containing this constraint, it may happen that x is in the basis at the level q, while λ is in the non-basis set. In this case, the

d-row value, d_j say, of the variable λ measures the cost per unit of introducing the variable λ into the basis, or the loss of profit per unit caused by a shortfall on sales. Clearly, on certain assumptions, d_j also measures the profit to be gained by selling an additional ton. Hence, by subtracting the d_j value of the slack on a sales equation, from the economic return on the product concerned, the marginal cost of a product may be obtained. This can be very valuable, especially when this technique is used to explore a range of potential production in which marginal costs are unknown. This exploration can be achieved by means of "parametric programming" solving the set of problems:

$$Ax = b + \lambda b'$$
$$\text{Maximize } cx,$$

where $\lambda, \geqslant 0$, is a parameter which is allowed to vary over a defined range. Starting from the solution to the case when $\lambda = 0$, λ is increased until the optimal basis is no longer optimal. At this point, the value of λ is recorded, an output is taken, a new optimal basis is formed and the process is repeated.

Our five-quarterly plans have brought home to us the difficulties of optimizing in the short term, when demand fluctuations from period to period may be important. It may be worthwhile eventually to attack this problem of optimizing across time-periods by linking several matrices together with equations representing transfers into and out of stock. These block triangular matrices will contain up to 1,000 equations, and should be soluble on our new Ferranti Mercury computer in London.

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Annual Prizes

It has been decided to offer two annual prizes for the best two papers published in *The Computer Journal* or *The Computer Bulletin*, one of the papers to be on a business application of computers and the other on a mathematical, scientific or engineering application or on logical design. Each prize will be of twenty guineas.

The first competition will cover those papers published between April 1959 and April 1960, inclusive. The competition will be judged by the Editorial Board, whose decision will be final and whose members will not be eligible for award.

Papers to be considered for publication in this period must be received by one of the honorary editors in their final form not later than 11 January 1960. Notes on the Submission of Papers are printed at page vii in the *Journal*, Vol. 2, No. 2 (dated July 1959).

Meetings on Reliability of Digital Computer Systems

A series of discussion meetings is to be held on Wednesday and Thursday, 20 and 21 January 1960, at The Institution of Electrical Engineers, Savoy Place, London, W.C.2, which will deal with:

"Managerial and Engineering Aspects of Reliability and Maintenance of Digital Computer Systems"

The programme will be divided and, on the first day, the British Computer Society will be responsible for arranging meetings which will deal with these subjects: Maintenance and Fault Diagnosis Techniques, with reference to logging and recording procedures; Programming Strategy for Protection against Computer and Operator Errors; Management and Organization Problems.

On the second day, the discussions will be concerned with such topics as Experience of System Reliability; the Influence of Engineering Design on Reliability; and Factors affecting the Reliability of Peripheral Equipment.

The whole series of meetings is being held under the aegis of Group B, the British Group for Computation and Automatic Control, of the British Conference on Automation and Computation.

On-Line, Off-Line, or Shared-Time?

As this *Journal* goes to press, the debate among business users, on how many pieces of input and output equipment may economically be connected directly to a large and fast computer, receives a new impetus. American experience had led many to believe that, whereas medium-speed machines could have *on-line* equipment, the faster machines required *off-line* magnetic-tape converters for economic operation.

In his Presidential Address to the British Computer Society in June 1958 (reported in Vol. 1, p. 98), Dr. M. V. Wilkes had referred to the major pending technical advance in logical design—*time sharing*. During the following 15 months, several manufacturers announced new business machine developments, some being extensions of facilities on their existing computers, while others, such as Emidec 2400 were of new design.

On p. 97 of this issue, Mr. J. A. Goldsmith has reviewed the State of the Art in June 1959. In October 1959 the pace has quickened. The talk to the Society in London on 15 October, by Monsieur P. Dreyfus of Compagnie des Machines Bull, Paris, described parallel developments in France, of particular interest in the context of current British developments.

English Electric (p. ix) have recently announced that they are also building an advanced data-processing system based on an American design with off-line conversion: we expect to give details of this in later publications. In this issue of the *Journal*, AEI ask: "Will your new computer be too small in 1964?" (p. xv). Further developments on the 405M are promised by National-Elliott (p. iv). A wide choice of equipment is offered by I.C.T. (p. v). The Emidec 1100 promises to grow with the user's needs (p. vii). IBM(UK) have data processing for everybody (p. xiii). Burroughs offer a full range of equipment (p. xvi). The services available from Leo Computers Ltd. are well illustrated by Mr. A. G. Wright's concise paper (pp. 103–4). The mathematician in business is also catered for by many of the above advertisements and those on pp. xi, xii and xiv. The makers of paper-tape and card-punching equipment, of source documents, output stationery and files, are also ready to meet any business-user need.

At the end, as we go to press, we have received the first announcement of the Ferranti ORION data-processing system, a logical and major development from the wide range of machines already offered by that manufacturer. From their preliminary specification, which we have also received, it would appear that on-line working will now be economical in this fast business machine, because of special decoding circuitry and the facilities for time-sharing and parallel programming of the kind forecast by the President last year. Other manufacturers will, no doubt, be announcing further developments for 1960–61 before very long.

The potential business user can, perhaps, soon decide which system will best suit his particular requirements. The large capital investment required to obtain a comprehensive business system will require careful justification; this may be based on the promise of achieving better management control through more effective information, rather than on purely clerical economies. The equipment which is now offered gives one the opportunity to build on from existing routines (on keyboard accounting machines or punched cards) towards a quicker and more efficient management information service. Some users may prefer to do this at first with a computer service bureau, keeping in mind the ultimate objective of a comprehensive system—from basic data to final report—when laying out magnetic tapes and other storage media. But, as far as equipment is concerned, it will all soon be available; and in the United Kingdom we have a wider choice than in most other countries.

H. W. GEARING

(*Honorary Editor for Business Application Papers*)