Scheduling algorithms for concurrent execution

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The paper introduces a notation for describing algorithms in a pseudo-mathematical form. It uses ideas introduced by various higher level programming languages, and by extensions proposed for concurrent processing. This notation is shown to be suitable to describe both sequential and concurrent scheduling schemes for numerical quadrature and matrix inversion. Additionally, pseudo-programs are developed for concurrent calculation of a scalar product of two vectors and of linear recurrence relations, as needed for a not too well known integration method.

(Received September 1973)

1. Introduction

Real time, online computer control requires speeds above those normally available on digital computers.

One approach to accelerating the computations would be to undertake them on a parallel processing system (pps hereafter) which includes a number of processors working concurrently on banks of common memory through multiplexers. The associated operating system keeps pointers on 'executive points' of programs and queues the processors. Whenever a processor is 'freed', it enters a queue; whenever a 'job' is waiting, the first processor in the queue is allocated to do it.

The imminent availability of such systems (Lorin, 1972; Comtre Corporation, 1975) raises the questions:

(*a*) what language should be used to describe algorithms as scheduled on a pps?

(*b*) which algorithms are best suited for execution on a pps and how should they be scheduled?

The aim of this paper is to develop a notation suitable for programming a pps and apply it to the scheduling of two problems: quadrature and matrix inversion.

2. Notation

Various languages are used to express the programming of an algorithm. When dealing with concurrent execution of an algorithm, these languages are supplemented by certain extensions (Anderson, 1965; Dijkstra, 1968; Hansen, 1973). The same approach was applied by the author to other problems (Wallach, 1974) for a particular system (Wallach, 1977). The present paper though will use a different notation because of the following considerations.

An actual program includes declarations, temporary variables and (to be efficient) probably a few 'tricks'. All this obscures the idea behind the algorithm. On the other hand, the number of basic, new ideas introduced by higher level (sequential or concurrent) languages and essential to express numerical algorithms is limited. We will therefore adhere to the old-fashioned way of expressing algorithms in a mathematical notation with its Greek letters, powerful operators and interspersed sentences in English to which we add the following:

(*a*) The assignment operator is ':=' whereas '=' will be used as a (Boolean) relational operator. Vectors are assigned by positions, e.g. $(a, b, c) := (x, y, z)$ is equivalent to $a := x$, $b := y$ and $c := z$.

(*b*) A Boolean expression is enclosed in braces.

(*c*) A label is the customary identifier, followed by a colon.

(*d*) A 'goto' operator is denoted by the '$\gg$' symbol.

(*e*) Instructions to be executed sequentially are separated by semicolons, concurrently by commas and both may be completed by one of the delimiters to be yet described. Semicolons may be read 'followed by', commas as 'concurrently with'. In this paper, S will stand for a single instruction, a block of instructions, or even for a complete subroutine.

(*f*) Any block of instructions which is to be treated as a single one is enclosed in a pair of square brackets. In the case of a concurrent block e.g. $[S_1, S_2, \ldots, S_n]$ it was pointed out (Hansen, 1973) that the instructions S_i, $i = 1, \ldots, n$ should be disjoint or noninteractive. Inside of a block there should be no mixing of semicolons and commas, since this would require either parenthesisation or setting up of precedences for the two modes of execution.

(*g*) If a block is preceded by symbol '#' and a list of values, it is to be performed for all of them. The four possibilities:

$$\#i := 1; 2; \ldots; n[S_i; S_{n+i}] \quad (1)$$

$$\#i := 1, 2, \ldots, n[S_i; S_{n+i}] \quad (2)$$

$$\#i := 1; 2; \ldots; n[S_i, S_{n+i}] \quad (3)$$

$$\#i := 1, 2, \ldots, n[S_i, S_{n+i}] \quad (4)$$

correspond to the following execution sequences:

$$[S_1; S_{n+1}; S_2; S_{n+2}; \ldots; S_n; S_{2n}] \quad (5)$$

$$[[S_1; S_{n+1}], [S_2; S_{n+2}], \ldots, [S_n; S_{2n}]] \quad (6)$$

$$[[S_1, S_{n+1}]; [S_2, S_{n+2}]; \ldots; [S_n, S_{2n}]] \quad (7)$$

$$[S_1, S_2, S_3, \ldots, S_n, S_{n+1}, \ldots, S_{2n}] \quad (8)$$

respectively. The 'step' of the index value i is always implied by its initial two values; the loop is completed upon $i > n$.

(*h*) A conditional expression uses arrows to denote 'then' (McCarthy, 1960) and dots to denote 'else', with the 'else' matching the last 'if'. Hence $\{p_1\} \rightarrow \gg l.\ \{p_2\} \rightarrow a := a + b$; means (in ALGOL 60); **if** p_1 **then goto** l **else if** p_2 **then** $a := a + b$. Note, that **if** p **then** (**if** q **then** $S1$ **else** $S2$) corresponds to $\{p\} \rightarrow \{q\} \rightarrow S1\ .\ S2$; but that **if** p **then** (**if** q **then** $S1$) **else** $S2$ must be written: $\{p\} \rightarrow [\{q\} \rightarrow S1].S2$;

(*i*) Functions will be defined through a modified lambda notation (Carnap, 1958), e.g.

$$\psi := (\lambda x, y, z)[c := x^2 + y^2; d := 5; e := x*d + z - d] \\ => (c, e) \quad (9)$$

where the parantheses hold the parameters, function block and results respectively. Three dots inside the last pair of parantheses means that all results are required. A function is 'called' by appearing in an expression, e.g. $\psi(3, 4, 5)$ for ψ of equation (9).

(*j*) A locking (Boolean) element b may be associated with a statement S as in

$$\perp b; S\ . \quad (10)$$

If b is found 'reset', it is first 'set', then S is executed, and finally b reset again. If b was found 'set', S cannot be

executed at this time. Locking operations on the same element b must exclude each other in time.

(k) The 'wait' operator @ as in

$$@\{B\}; S \tag{11}$$

delays the execution of S until (boolean) condition B is satisfied.

We note that all eleven operators above have been defined as machine independent. The sequential operators correspond to those used on present day computers. Ways to implement parallel operators (comma, lock and wait) on multiprocessor configurations have been suggested previously. Locking may be effected be a 'test-and-set' instruction (Lorin, 1972), a modified interrupt (Carver Hill, 1973), as P and V operations on semaphores (Dijkstra, 1968) or through the WAIT, POST, ENQ and DEC macros (of OS/360). The same WAIT macro may serve as a slightly modified wait operator. A parallel block may require interleaving of data in memory and accessing it through multiplexers (Wallach, 1974). The lock and wait operators are useful in system programming (Hansen, 1973), but seldom for numerical work. All operators connected with parallelism are tailored for use on a MIMD-type of multiprocessor (Stone, 1973).

This completes the notation; its merits will be made clear by the algorithms of the following sections.

3. Quadrature methods

In some cases (e.g. online control of a plant modelled as a linear, time invariant system), evaluation of an integral

$$y = \int_a^z \phi(x) \cdot dx \tag{12}$$

takes too long on present day computers. Due to the complexity of $\phi(x)$, even methods which use a minimum number of function evaluations (Wallach, 1969) would be too slow. Hence, the need for a concurrent solution on a pps.

A method (Adams, 1970) specifically designed for parallel evaluation of y used special graphs to describe the algorithm. In the notation of Section 2, it can be defined succinctly and clearly by the use of just three functions.

ρ sums two areas (s_1, s_2) to yield s_3 and checks convergence

$$\begin{aligned}\rho := (&\lambda s_1, s_2, s, \varepsilon)\\ &[s_3 := s_1 + s_2; b := \{|s - s_3| > \varepsilon * s_3\}\\ &] => (s_3, b);\end{aligned} \tag{13}$$

v computes a midpoint o, the function f_o at o and the areas on its two sides:

$$\begin{aligned}v := (&\lambda a, f_a, z, f_z)\\ &[[o := a + 0{\cdot}5*|z - a|, r := 0{\cdot}25*|z - a|];\\ &f_o = \phi(x = o);\\ &[s_4 := r*(f_a + f_o), s_5 := r*(f_o + f_z)]\\ &] => (o, f_o, s_4, s_5);\end{aligned} \tag{14}$$

τ is a recursive function (i.e. it calls itself) which yields the integral in the form of the area computed by ρ if convergence was obtained, but calls itself twice if it was not obtained. It is in this last case that the algorithm 'forks' into parallel paths.

$$\begin{aligned}\tau := (&\lambda a, f_a, z, f_z, s, \varepsilon)\\ &[(o, f_o, s_4, s_5) := v(a, f_a, z, f_z);\\ &(s_3, b) := \rho(s_4, s_5, s, \varepsilon);\\ &\{b\} \rightarrow\\ &[[u := \tau(a, f_a, o, f_o, s_4, \varepsilon),\\ &\quad v := \tau(o, f_o, z, f_z, s_5, \varepsilon)];\\ &\quad w := u + v].\\ &\quad w := s_3\end{aligned}$$

$$] => (w); \tag{15}$$

Very effective, automatic quadrature methods are based on shifting the integration interval $(a{:}z)$ to $(-1{:}1)$ with a subsequent integration of the polynomial which interpolates $\phi(x)$ at the (Chebyshev) points

$$t_j := j*\pi/n, j = 0, 1, 2, \ldots, n\ . \tag{16}$$

The basic procedure (Clenshaw and Curtis, 1960) approximates the integrand $\phi(x)$ by a series of Chebyshev polynomials $\sum c_k T_k(x)$ and produces the value of the definite integral from $F = 2(d_1 + d_3 + d_5 + \ldots)$ where the coefficients d_i result from integrating the first polynomial to get $\sum d_k T_k(x)$.

The algorithm is well known and documented, so that a similar but much less known method (Filippi, 1964; Davis and Rabinowitz, 1967) will be discussed. It approximates

$$F = \int_{-1}^{1} \phi(x)\, dx \tag{17}$$

and not $\phi(x)$ as a Chebyshev series at the interpolation points t_j.

Since no algorithm for this method has been published yet, the schedule for its concurrent execution will be commented extensively and exemplified throughout.

The number of subdivisions n should be chosen so that not all values of the sines and cosines will have to be recomputed. This is best done by choosing a value 'max' (e.g. max = 4) and starting the program through

$$k := 2^{\max - 1}; n := 5*k; m := 4;$$

(e.g. $k = 8$; $n = 40$; $m = 4$). The initial values of $cn_j = \cos(j\pi/n)$, $sn_j = \sin(j\pi/n)$ and $g_j = sn_j*\phi(cn_j)$ are computed in the block:

$$\begin{aligned}&\#j := k, 2k, \ldots, n \div 2\\ &[i := n - j; cn_j := \cos(j\pi/n); cn_i := -cn_j;\\ &\quad sn_j := sn_i := \sqrt{(1 - cn_j^2)}; g_j := sn_j*\phi(cn_j);\\ &\qquad g_i := sn_i*\phi(cn_i)];\end{aligned} \tag{18}$$

(e.g. $[cn_8 = \cos 36°; cn_{32}; sn_8; sn_{32}; g_8; g_{32}]$ concurrently with $[cn_{16}; cn_{24}; sn_{16}; sn_{24}; g_{16}; g_{24}]$).

Next, three values d_n, d_{n-1} and d_{n-2} are computed:

$$\begin{aligned}&lb{:}\ l := 1, 2, 3\\ &\{e := 2*cn_{n-l*k}; c_{-1} := c_o := 0;\\ &\#h := 1; 2; \ldots; m[c := e*c_o - c_{-1} + g_{n-h*k};\\ &\qquad c_{-1} := c_o; c_o := c];\\ &d_{n+1-l} := (2*sn_{n-l*k}*c)/((m + 1)*(m + 1 - l))\};\end{aligned} \tag{19}$$

In the above, the c's are computed from a marching recurrence relation. Since every c_h depends on c_{h-1} and c_{h-2}, and since h loops from 1 to m, it seems as if there is only one way to calculate the c's: sequentially. However, it is shown in the Appendix that even such 'inherently sequential' calculations may be successfully parallelised.

If all three d's are small, the value of the integral is approximately

$$\int_a^z \phi(x) \cdot dx \simeq (z - a)*(d_1 + d_3 + \ldots + d_r) \tag{20}$$

with $r \leq n$. Otherwise, the m and k values are changed according to

$$m := 2*m + 1; k := k \div 2; \tag{21}$$

and the new cn, sn, g are calculated not from equation (18) but through:

$$\begin{aligned}&cn_k := \sqrt{[(1 + cn_{2k})/2]}; e := n - k; cn_e := -cn_k;\\ &sn_k := sn_e := \sqrt{(1 - cn_k^2)}; g_e := sn_e*\phi(cn_e);\\ &\#j := m - 1, m - 2, \ldots, 2\\ &[\{j \text{ rem } 2 \neq 0\} \rightarrow\end{aligned}$$

$[l := j*k;\ cn_l := (cn_{l+k} + cn_{l-k})/(2*cn_K);$
$\{l < n/2\} \rightarrow sn_l := sn_{2l}/(2*cn_l).$
$sn_l := \sqrt{(1 - cn_l^2)};\ g_l := sn_l*\phi(cn_l)$ (22)
$]];$

For $m = 9$, $k = 4$, $e = 36$ this computes first

$$[cn_4; cn_{36}; sn_4; sn_{36}; g_{36}]$$

and then, concurrently $[cn_{28}; sn_{28}; g_{28}]$ with $[cn_{20}; sn_{20}; g_{20}]$ with $[cn_{12}; sn_{12}; g_{12}]$. Finally g_k(e.g. g_4) is computed and the program returns to label *lb*:

$$g_k := sn_k*\phi(cn_k);\ \gg lb; \quad (23)$$

The program exhibits so much parallelism that it should compare favourably with the previously mentioned concurrent method. This is especially true if the recurrent calculation of c_n is not done per equation (19) but in the way shown in the Appendix.

A final remark concerns the evaluation of the integral equation (12) using the Romberg method. Both in the general case (Bauer *et al*, 1963; Krasun and Prager, 1965) and in the particular case of a 'state model' (Wallach, 1969), the notation of Section 2 lead easily to models for parallel execution.

4. Matrix inversion

One of the older algorithms for inverting a matrix A (Shipley and Coleman, 1959) is based on the theory of 'diakoptics' (Kron, 1939). It consists of choosing pivots A_{ii} and substituting *all* elements A_{jk} not on the pivoting row or column by

$$A_{jk} := A_{jk} - A_{ij}*A_{ki}/A_{ii}\ . \quad (24)$$

The element A_{ii} is replaced by $c = -1/A_{ii}$ and the remaining $(2n - 2)$ elements of row i and column i are multiplied by c. The pseudo-program of this algorithm is thus:

```
#i := 1; 2; ...; n
  [c := -1/A_{i,i};
    #j := 1, 2, ..., n
      [#k := 1, 2, ..., n
        [{j ≠ i} ∧ {k ≠ i} →
          A_{j,k} := A_{j,k} + c*A_{i,j}*A_{k,i}]; ];
    #j := 1, 2, ..., n
      [A_{i,j} := c*A_{i,j}, A_{j,i} := c*A_{j,i}];
    A_{i,i} := c
  ];                                              (25)
```

This substitution requires two multiplicative operations for each of the $(n^2 - 2n + 1)$ elements. The remaining $(2n - 2)$ elements are multiplied by a constant c. Hence, the total number of multiplications is:

$$\mu = n(2(n^2 - 2n + 1) + 2n - 2) = 2n^2(n - 1) \simeq 2n^2 \quad (26)$$

This is twice the number required in Crout's algorithm (Kunz, 1957), which probably explains why this algorithm was seldom used outside the field of the electrical power industry. It is similar to the elimination method of Gauss, and the main reason why we prefer it is essentially the following. Equation (24) may be performed *in any order*, which means that processor scheduling and data storage will not be a problem (as it certainly is in other methods, e.g. those suggested for ILLIAC-IV).

It was noted (Pease, 1967) that: 'With parallel processing, the total number of operations is not significant. What matters is the number of sets of operations, where each set involves those being done in parallel'. Since the entire inner loop (of j and k) may be performed concurrently, this set equals:

$$\sigma \simeq \frac{2n^3}{p} \quad (27)$$

where p is the number of processors. Hence, σ would be only $2n$ for a system of $p = n^2$ processors. The second advantage is the small σ.

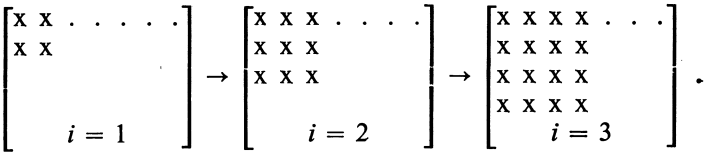


Fig. 1 Elements which change at iterations $i = 1, 2, 3$

As a fringe benefit, note that the use of a minus on the pivoted term causes a symmetric matrix to remain symmetrical throughout the entire process.

Another advantage claimed for Kron's algorithm is that it is useful in inverting sparse matrices. We will apply it to the best-known case, namely to a tridiagonal matrix which is basic to the numerical solution of elliptic, partial differential equations (Fox, 1962). We could adapt equation (25) to this case by modifying the Boolean condition for which the inner loop (of k and j) is performed. Since this point is traversed n^2 times (or by n^2 processors, once) and since, anyway, we have to assume the number of processors to be smaller than n^2, it is precisely checking of this Boolean condition that we would like to avoid. **Fig. 1** and the following pseudo-program show how this can be done.

```
#i := 1; 2; ...; n
  [c := -1/A_{ii}; l := i - 1; m := i + 1;
    #j := 1, 2, ..., n
      [#k := 1, 2, ..., n[A_{jk} := A_{jk} + c*A_{ij}*A_{ki}]
      ]; A_{mm} := A_{mm} + c*A_{im}*A_{mi};
    #j := 1, 2, ..., i
      [{j = i} →
        [A_{jm} := c*A_{jm}, A_{mj} := c*A_{mj}].
        {j ≠ i} →
        [A_{jm} := A_{jm} + c*A_{ij}A_{mi}, A_{mj} := A_{mj} + c*A_{im}A_{ji}]
    ]; #j := 1, 2, ..., l
          [A_{ij} := c*A_{ij}, A_{ji} := c*A_{ji}];
    A_{ii} := c];                                 (28)
```

For its implementation, one row and one column of zeros should be added to the original matrix. The number of sets σ will be larger than that for an adapted equation (25), but there is enough parallelism left in equation (25) to keep all processors busy most of the time. The number of operations on elements is:

$$2^2 + 3^2 + 4^2 + \ldots + n^2 = -1 + \sum_{k=1}^{n} k^2 = -1 + \frac{n(n+1)(2n+1)}{6} \simeq \frac{n^3}{3} \quad (29)$$

This is a large number, but, for large n, most of the operations will be in the $2*n$ multiplications of the inner loop (for j and k), which corresponds to equation (24).

Concurrency may also be used for inverting a reordered tridiagonal matrix or for solving lower and upper bidiagonal, as well as blockwise tridiagonal matrices (Even and Wallach, 1970).

The notation introduced in Section 2 may also be used to describe Gaussian elimination or the 'bordering' method on specially designed, parallel computer systems (Pease, 1967). For the 'bordering' method, and generally for linear algebraic problems, the ability to compute concurrently the scalar product of two vectors a and b is important. It should be noted that one method which should *not* be applied, is:

$$s := 0;\ \#i = 1, 2, \ldots, n[s := s + a_i{*}b_i]; \qquad (30)$$

The results of performing equation (30) for more than one index concurrently would depend on the timing of the computers and make the computation indeterminate. To restore determinacy, s may be accessed only after it is locked, by d, e.g.

$$\#i := 1, 2, \ldots, n[\perp d; s := s + a_i{*}b_i]; \qquad (31)$$

Since this would slow down the process considerably, the following method is to be preferred for calculating the scalar product of $a \times b$:

$$\begin{aligned}&\#i := 1, 2, \ldots, n[c_i := a_i{*}b_i];\\ &\#j := 0; 1; \ldots; (\log_2 n - 1)\\ &\quad [m := n/2^j;\\ &\qquad \#i := 1, 2, \ldots, m/2[c_i := c_i + c_{m+1-i}]\\ &\quad]; \end{aligned} \qquad (32)$$

In equation (32), the initial dimension n of the two vectors should be a power of 2, i.e. $n = 2^l$, but if it is not such, then we may set all

$$a_h := b_h := 0 \text{ for } h = 2^l - n\ . \qquad (33)$$

Equation (32) halves successively the size of vector c and leaves the result in c_1.

Appendix

The recurrence relation used in equation (19) has the general form:

$$c_h := pc_{h-1} + qc_{h-2} + r_h;\ h = 1, 2, \ldots, n \qquad (A\text{-}1)$$

where c_{-1}, c_o are known initial values and p, q, r_h are known constants. Element c_n is to be computed—if possible, in a parallel way.

Lemma:
For any positive integer h, c_h can be written as

$$c_h = Uc_o + Vc_{-1} + W \qquad (A\text{-}2)$$

where U, V, W are constants, i.e. c_h is a linear form in c_{-1} and c_o.

Proof (by induction):
For $h = 1$, the constants U, V, W are p, q and r_1 respectively. For $h = 2$, they are found (by insertion) to be $(p^2 + q)$, (pq) and $(pr_1 + r_2)$ respectively. Assume next that equation (A-2) holds for some $2 < i < n$. Then

$$c_{i-1} = U'c_o + V'c_{-1} + W' \qquad (A\text{-}3)$$

$$c_{i-2} = U''c_o + V''c_{-1} + W''\ . \qquad (A\text{-}4)$$

Inserting these two into equation (A-1) yields for $h = i$ equation (A-2) with

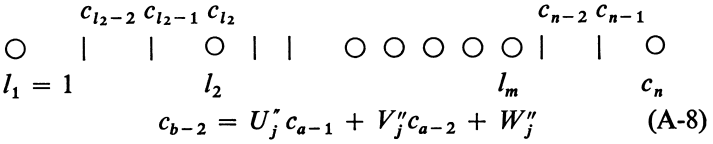
$$U = pU' + qU'' \qquad (A\text{-}5)$$

$$V = pV' + qV'' \qquad (A\text{-}6)$$

$$W = pW' + qW'' + r_k \qquad (A\text{-}7)$$

QED .

Let us split the n relations (A-1) into m contiguous blocks as depicted in the figure below. Block j starts with a c_a whose index is $a \equiv l_j$ and ends with a c_b whose index is $b \equiv l_{j+1} - 1$. By a shift of origin, and by virtue of the lemma, for this block:



$$c_{b-2} = U_j''c_{a-1} + V_j''c_{a-2} + W_j'' \qquad (A\text{-}8)$$

$$c_{b-1} = U_j'c_{a-1} + V_j'c_{a-2} + W_j' \qquad (A\text{-}9)$$

(Note that for $b = i$, $a = 1$ these equations reduce to (A-3) and (A-4).)

As is seen, the last two elements of a block are linear forms in the last two elements of the preceding block.

The algorithm for computing n concurrently is as follows: for each block l_k, assume *arbitrarily* three pairs of initial values (c^i_{a-2}, c^i_{a-1}), $i = 1, 2, 3$. For each pair a processor (concurrently with and independently of the other processors) applies equation (A-1) until the values c^i_{b-2}, c^i_{b-2} are computed. As a result three linear equations (A-8) in the unknowns U_k'', V_k'', W_k'' are obtained and another three in U_k', V_k', W_k'. A judicious choice would be the following three pairs: (1, 0), (0, 1), (0, 0) which yield:

$$\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix} * \begin{bmatrix}U_k''\\ V_k''\\ W_k''\end{bmatrix} = \begin{bmatrix}c^1_{l_{k+1}-2}\\ c^2_{l_{k+1}-2}\\ c^3_{l_{k+1}-2}\end{bmatrix} \qquad (A\text{-}10)$$

From these, the solution is

$$U_k'' = c^1_{l_{k+1}-2} - c^3_{l_{k+1}-2} \qquad (A\text{-}11)$$

$$V_k'' = c^2_{l_{k+1}-2} - c^3_{l_{k+1}-2} \qquad (A\text{-}12)$$

$$W_k'' = c^3_{l_{k+1}-2} \qquad (A\text{-}13)$$

and similarly for U_k', V_k', W_k'.

Inserting the (known) values c_o, c_{-1} into equations (A-8, A-9) yields c_{l_2-2} and c_{l_2-1}. Proceeding sequentially in **Fig. A-1** yields c_n in $(m - 1)$ steps. It should be noted that the three sweeps in each block may proceed concurrently, which increases parallelism even more.

Acknowledgements
The author wishes to thank Mr. V. Conrad, Dr. D. W. Martin and Professor M. Yoeli for their help in preparing and revising the manuscript.

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Book reviews

Mathematical Foundations of Computer Science, edited by J. Becvár, 1975; 476 pages. (*Springer-Verlag*, US $16.80)

From 1 to 5 September 1975, the 4th annual symposium on the *Mathematical Foundations of Computer Science* (MFCS) took place in Mariánské Lázne, Czechoslovakia; its proceedings, edited by J. Becvár appeared as Lecture Notes in Computer Science nr. 32, Springer-Verlag. This meeting emphasised complexity theory.

The annual MCFS symposia are organised alternatively in Czechoslovakia and in Poland by the computation centres of the Academies of Science of these countries. The invited lectures and communications covered respectively constitute recent results in automata theory, complexity theory, Lindemayer systems, logic of computation and mathematical linguistics, on the one hand and the theory of data bases, methods for program proving, Petrinets and semantics of programming languages, on the other.

The Invited lectures were given by J. M. Barzdin, J. J. Bicevskis and A. A. Kalninsh, 'Construction of complete sample system for correctness testing'; P. van Emde Boas, 'Ten years of speedup'; P. Hájek, 'On logics of discovery'; M. A. Harrison, 'On models of protection in operating systems'; J. Král and J. Demner, 'Parsing as a subtask of compiling'; A. Mazurkiewicz, 'Parallel recursive program schemes'; M. Novotný, 'On some problems concerning Pawlak's machines'; A. Salomaa, 'Formal power series and growth functions of Lindenmayer systems'; P. H. Starke, 'On the representability of relations by deterministic and non-deterministic multi-tape automata'; B. A. Trakhtenbrot, 'On problems solvable by successive trials'; V. Trnková, 'Automata and categories'; I. D. Zaslavskii, 'On some models of computability of Boolean functions'.

The proceedings contain a considerable number of communications. Being a 'correctness prover' myself, I restrict myself mainly to the subject implied: *Mazurkiewicz* presentation is related to work done earlier by *H. Bekic. R. V. Freivald* contributed: Minimal Gödel numbers and their identification in the limit—recursive function theory. *I. M. Havel:* Nondeterministically recognisable sets of languages—solid work. *M. Karpinski:* Decision algorithm for Havel's branching automata—if correct, then interesting. *W. P. de Roever:* First-order reduction of call-by-name to call-by-value—explains why call-by-name leads to a theory of termination which is exponentially more complicated than call-by-value. *M. B. Trakhtenbrot:* On representation of sequential and parallel functions—provides a glimpse of Russian work on a subject introduced mainly by Vuillemin. *F. Kröger:* Formalisation of algorithmic reasoning—already known, but in the right direction, as turned out later.

These symposia constitute to my knowledge the only regular meeting place between theoretical computer scientists of Eastern Europe and the Western world, and are therefore of vital importance to our East European colleagues.

WILLEM P. DE ROEVER, (Belfast and Amsterdam)

Computing Systems Hardware, by M. Wells, 1976; 245 pages. (*Cambridge Computer Science Texts*, 6, £4·00)

This paperback is one of the new breed of books about computer techniques which attempt to short-circuit much of the conventional analysis in order to give an overall picture of a complex subject in a manageable book. When one reads this book the first impression is of astonishment that the different subjects could have been fastened so closely together and be contained in just over 200 pages. The text is intended for second year undergraduates (third year in Scotland), studying computer science or computer engineering, and there is no doubt that it will be very useful for general reading in this area.

However it is easy for someone who has struggled through the concepts of computer technology the hard way to consider the book an excellent summary. The danger is of course that like Pitman's shorthand it may be too condensed for the inexperienced reader, and it is fair to say that certain parts of the text are superficial in the sense that if the student went straight from the text to practical equipment he would be somewhat lost. A clear example is the brief way that TTL circuits are described, without reference to books like the Texas Instruments classic.

Having said this, there is a great deal to enjoy in the book, and it does approach its subject from the standpoint of *information science*. There are many different ways of looking at standard things, and some enjoyable prose. All in all a good book for use by anyone in conjunction with the services of a good lecturer. In the absence of a guide, then the reader would be prepared to buy a good dictionary of computing terms, because sometimes the specialist words come thick and fast.

F. G. HEATH (Edinburgh)