R be any one of these data sites. We shall deduce a contradiction.

R must lie strictly outside $\Delta P_1P_2P_3$, because $\mathscr{T}$ is a triangulation; but also it must lie strictly inside the circumcircle $\bigcirc P_1P_2P_3$, because of the extremal property of $\bigcirc P_1P_2R$. Thus it lies in the shaded region in **Fig. 3**; without loss of generality it can be assumed to lie in the part between P_2 and P_3, as shown. Since R and P_1 lie on opposite sides of P_2P_3, there must be a $\mathscr{T}$-triangle on the opposite side of P_2P_3 from P_1, and with P_2P_3 as one of its edges—the position of R prevents P_2P_3 from being a facet of the convex hull. Let P_4 be the third vertex of this triangle. P_4 cannot be R, because R is strictly inside $\bigcirc P_1P_2P_3$ and, by Lemma 1 and the assumption that $\mathscr{T}$ is locally equiangular, P_4 is outside or on $\bigcirc P_1P_2P_3$. R lies strictly outside $\Delta P_2P_3P_4$, because $\mathscr{T}$ is a triangulation. And it lies strictly inside $\bigcirc P_2P_3P_4$, because P_4 is outside or on $\bigcirc P_1P_2P_3$, whence the region strictly to the opposite side of P_2P_3 from P_1, and strictly inside $\bigcirc P_1P_2P_3$ (in which region R lies) is contained in the region strictly inside $\bigcirc P_2P_3P_4$ and also on that side of P_2P_3. A possible position for P_4 is shown in Fig. 3. What we have done is to establish one step of an inductive construction, and this can be repeated indefinitely to obtain P_5,P_6, and so on. The perpendicular distance from R to P_iP_{i+1} is monotone strictly decreasing, so there must be infinitely many distinct points P_i. This is impossible. It follows that P_3 must after all have been one of the data sites on $\bigcirc P_1P_2R$ on the positive side of P_1P_2, and the proof is complete.

These three lemmas together establish the main result, some of the implications of which have already been discussed. We conclude by investigating some other criteria of equiangularity related to local equiangularity. The algorithm which Lawson (1972) suggested for achieving local equiangularity was to start with an arbitrary (but hopefully reasonably good) triangulation and make exchanges of diagonal in convex quadrilaterals according to the max-min angle criterion until no more such exchanges are required. To avoid the trivial possibility of cycling at a degeneracy, it is desirable to make only exchanges *required* by the criterion; ambiguous cases are left undisturbed. As mentioned above, algorithms of this kind are unlikely to be competitive with the Green-Sibson algorithm for computation, but they may, as here, prove useful as theoretical tools. It is not immediately obvious that Lawson's algorithm is guaranteed to terminate. One way of establishing that it does so is to look at the circumradii of the triangles. Each Lawson exchange replaces two triangles by two different ones, and it is easily seen that the new triangles both have strictly smaller circumradii than both of the ones they replace. Thus cycling cannot occur, and since there are only finitely many triangulations, the algorithm must terminate. It does so at a locally equiangular triangulation and hence, by the theorem, at a completion of the Delaunay pretriangulation. This argument also identifies a family of globally defined objective functions optimised by locally equiangular triangulations and by those only: every symmetric strictly isotone function of the circumradii—for example, their sum or the sum of their squares—is such a function. Another criterion, superficially an attractive one, is the maximisation of the minimum angle occurring in the entire triangulation. Such a triangulation might be called *globally equiangular*. If $\mathscr{U}$ is a globally equiangular triangulation, Lawson's algorithm can be applied to it to produce a locally equiangular triangulation $\mathscr{V}$. Now the minimal angle occurring in $\mathscr{V}$ is not greater than that in $\mathscr{U}$, by global equiangularity of $\mathscr{U}$; neither is it less, because Lawson exchanges certainly cannot reduce it. Thus $\mathscr{V}$ is also globally equiangular; we conclude that the completions of the Delaunay pretriangulation are all globally equiangular. Of course, the converse is false; in many cases it will be possible to apply a few anti-Lawson exchanges to a globally equiangular triangulation without reducing the minimal angle occurring in it as a whole.

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Book reviews

Methods for Statistical Data Analysis of Multivariate Observations, by R. Gnanadesikan, 1977; 311 pages. (*Wiley*, £15·00)

The development of multivariate techniques has been rapid in recent years and the use of computers has changed the emphasis from statistically rigorous procedures of the multiple regression type to data reduction and graphical exploration techniques. This trend is reflected in this new book which begins with Factor Analysis and related techniques and proceeds to discuss dependency methods, classification and clustering before turning to the difficult question of hypothesis testing in multivariate analysis. The final chapter, oddly entitled 'Summarization and Exposure' deals with multi-response samples, including methods using multidimensional residuals and outlier methods.

This is an advanced methodological book suited to graduate statistical students and other research workers involved in multivariate analysis. There are a good number of illustrative examples which use computer graphical facilities. The bibliography is reasonably comprehensive but the references to it appear in curiously condensed form without annotation at the ends of chapters. More within text reference would have helped. The book is an up-to-date and worthwhile addition to the applied multivariate analysis literature and will prove a useful reference book.

ROBERT W. HIORNS (Oxford)

Microprocessors: Infotech State of the Art Report (*Infotech*, £110)

It may be fairly claimed that these two volumes embrace all aspects of the subject they treat. There is material on the underlying LSI technology, on microprocessors design and on microprocessor applications with particular attention to control applications. There is something on software and even a contribution on the content of a university course on the subject.

The presentation follows that of earlier state of tne art reports from Infotech. Volume 2 contains the text of twenty invited papers; Volume 1, which is of similar size, surveys the subject systematically, topic by topic, and consists of quotations from the invited papers and other relevant Infotech publications with linking editorial material. The main topic headings in this volume are: microprocessors—definitions and evaluation; integrated circuit technologies; microprocessor architecture; microprocessor applications; multi-processors and distributed intelligence; evaluation and selection; system design and development; software; manning. There is also an annotated bibliography. In Volume 2 a paper that particularly interested me was one by K. Dixon on the Ferranti F100-L 16 bit microprocessor. I am sure that anyone who is at all concerned with microprocessors will find something to interest him.

M. V. WILKES (Cambridge)