

$$\begin{aligned}
&= \left(x \sum_{i=0}^{\infty} p_i x^i \right) \left(\sum_{n=i+1}^{\infty} C_{n-i-1} x^{n-i-1} \right) \\
&\quad + \sum_{n=0}^{\infty} C_n x^n - \sum_{n=1}^{\infty} x p_{n-1} x^{n-1} - 1 \\
&= xP(x)C(x) + C(x) - xP(x) - 1 \\
&= \frac{C(x) - 1}{1 - x - xC(x)}
\end{aligned}$$

Using the facts that

$$1 - xC(x) = \frac{1}{C(x)}$$

and

$$xC(x)C(x) = C(x) - 1$$

after some manipulation we get

$$\begin{aligned}
P(x) &= \frac{C(x) - 1}{x^2} - \frac{2C(x)}{x} + \frac{1}{x} \\
&= \sum_{n=0}^{\infty} \frac{C_n x^n - 1}{x^2} - \frac{2C_n x^n}{x} + \frac{1}{x} \\
&= \sum_{n=2}^{\infty} C_n x^{n-2} - \sum_{n=1}^{\infty} 2C_n x^{n-1} \\
&= \sum_{n=0}^{\infty} C_{n+2} x^n - \sum_{n=0}^{\infty} 2C_{n+1} x^n \\
&= \sum_{n=0}^{\infty} (C_{n+2} - 2C_{n+1}) x^n
\end{aligned}$$

Hence

$$p_n = C_{n+2} - 2C_{n+1}$$

Solution of relation from Algorithm 3.3

This relation simplifies to

$$k_0 = 0$$

$$k_n = \sum_{i=0}^{n-1} (2k_i C_{n-i-1} + C_{i+2} C_{n-i-1}) + C_n - C_{n+1}$$

Using similar techniques to the above the generating function is found to be given by

$$\begin{aligned}
K(x) &= 2xK(x)C(x) + \frac{C(x) - x - 1}{x} C(x) + 2C(x) \\
&\quad - \frac{C(x) - 1}{x} - xC(x)C(x) - 1
\end{aligned}$$

which simplifies to

$$\begin{aligned}
K(x) &= \frac{C(x)x^{-2} - 2C(x)x^{-1} - x^{-2} + x^{-1}}{1 - 2xC(x)} \\
&= \frac{C(x) - 2xC(x) - 1 + x}{x^2(1 - 4x)^{1/2}} \\
&= \frac{1 - 4x + 2x^2}{2x^3(1 - 4x)^{1/2}} + \frac{2x - 1}{2x^3}
\end{aligned}$$

where

$$\frac{1}{(1 - 4x)^{1/2}} = \sum_{n=0}^{\infty} (-4)^n \binom{-\frac{1}{2}}{n} x^n$$

so

$$\begin{aligned}
K(x) &= \frac{1}{2} \sum_{n=0}^{\infty} (-4)^n \binom{-\frac{1}{2}}{n} x^{n-3} - 2 \sum_{n=0}^{\infty} (-4)^n \binom{-\frac{1}{2}}{n} x^{n-2} \\
&\quad + \sum_{n=0}^{\infty} (-4)^n \binom{-\frac{1}{2}}{n} x^{n-1} + \frac{2x - 1}{2x^3}
\end{aligned}$$

which eventually yields

$$\begin{aligned}
k_n &= \frac{1}{2} (-4)^{n+3} \binom{-\frac{1}{2}}{n+3} - 2(-4)^{n+2} \binom{-\frac{1}{2}}{n+2} \\
&\quad + (-4)^{n+1} \binom{-\frac{1}{2}}{n+1} \\
&= \binom{2n+6}{n+3} / 2 - 2 \binom{2n+4}{n+2} + \binom{2n+2}{n+1} \\
&= \binom{2n+2}{n-1}
\end{aligned}$$

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Corrigenda

The Editor wishes to apologise to the authors for two corrigenda which have been reported in *The Computer Journal*, Vol. 23 No. 3.

In the paper by G. R. Garside and P. E. Pintelas, 'An ALGOL 68 package for implementing graph algorithms', pp. 237-242, p. 239, column 1, l.32 should read

mode graphset = struct (int noofgraphs, ref [] graph graph)

p. 241, l.10 of Fig. 5 should begin

$S' := '$

p. 241, l.11 of Fig. 7 should begin

$\{ \text{number}$

l.29 of Fig. 7 should read

while number[head of stk[sp]] \geq number[w] do

l.32 of Fig. 7 should read

begin ccp[cp plus 1] := stk[sp]; sp minus 1 end;

p. 242, column 1, l.11 $N^2p/2$ should be replaced by $N^2p/2$.

In the paper by R. K. Lutz, 'An algorithm for the real time analysis of digitised images', pp. 262-269, the diagrams include the use of the terms 'image' and 'nonimage'. These should have been replaced by 'OBJECT' and 'NONOBJECT' in order to agree with the terms used in the text.