In the present version of the method, the size of the grid is not chosen adaptively. Experiments with differing grid sizes show, as might be expected, that local improvements in accuracy can continue to be obtained (as the grid size is decreased) in rough areas of the surface long after the smoother areas are well approximated. This suggests the desirability of allowing adaptive local refinement of the initial grid in circumstances where the opportunity to make additional function and gradient evaluations exists. The nature of the seamed element we have introduced permits quite easily a local halving of grid size any number of times suggested by the approximation criterion in use. The computational implementation of this technique is a future task for us.

The existence of a well defined smooth contour line is guaranteed by the implicit function theorem; this theorem breaks down in regions where the function is locally constant, and the computation must follow the mathematics in failing to produce proper contours at the corresponding levels. In practice, contours will start to display anomalous behaviour as such levels are approached. The particular form which this behaviour takes in our case is a tendency for such contours to follow the seam lines in the elements. **Fig. 7** is an artificial example to illustrate this, suggested to us by Sabin. On a 2×2 grid of unit size elements, zero value and gradient are imposed at all eight peripheral grid points, and the value and gradient at the centre are $(1; 4, -4)$. The unlabelled contours are at $\pm 0{\cdot}0001$; they coalesce visually into a close approximation to the non-anomalous part of the zero contour internal to the large square, but follow seam lines closely in an octagonal shape round the edge. This effect carries over in less extreme form to other contours at low positive and negative levels. **Fig. 8** is a practical example where this sort of effect is visible. The function is a non-negative probability density estimate (constructed by Silverman to investigate metallurgical data collected by Bowyer), which approaches zero closely away from its mode. The oscillations visible clearly in the lowest level contour appear to be associated with the position and size of the 5×5 grid of elements and we believe that they arise for the reason explained above. The use of local adaptive grid refinement in such cases is a particularly attractive possibility.

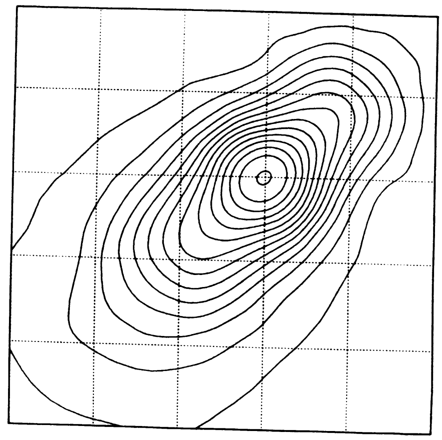

Fig. 8 A practical example of the effect shown in Fig. 7

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