An Aid to Pattern Recognition

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A system is described that uses the characteristics of associative memory to recognize patterns. It operates by storing related and concurrent pattern-generated signals so that the reoccurrence of one will cause the related signals to be synchronously regenerated. Investigations, using a digital computer, have shown that a correct regenerated signal can be obtained when using a signal which is only similar to the original. Also, the system can be made to give an identical regenerated signal from a number of different associated signals. A case is also put forward to suggest that the system could display a form of artificial intelligence.

INTRODUCTION

The system to be described could assist in overcoming two particular difficulties often encountered in pattern recognition.

The first is that in a real world situation, the data given by a pattern on which recognition is to be attempted may be incomplete or contain spurious or incorrect information. By using a form of feedback, in which the output is continually related to the input, a 'partial' input pattern is converted into a correct reference pattern, which can then be analysed either by conventional techniques, or by using another modified system to directly drive a desired form of output.

The second concerns the volume of data to be analysed to obtain a good fit which, using sequential techniques, can be embarrassingly time-consuming. It has been suggested that this processing time can be reduced by using parallel processing channels. This mode of operation is a fundamental feature of the system to be

The system bears a superficial resemblance to some earlier systems, in particular, the Perceptron.^{1,2} However, it differs from these by combining four fundamental concepts. (i) Related but different concurrent signals are stored in such a way that the subsequent reoccurrence of all or part of one will cause the other(s) to be synchronously regenerated. (ii) The mode of operation is such that 'feedback' between the reoccurrent and regenerated signals makes the system insensitive to input errors or variations. (iii) The system uses a pattern generated time/sequence array of input pulses. (iv) Finally, the system incorporates a dynamic control system, using information from the present and recent past, to control the mode of operation.

SYSTEM DESCRIPTION

In its most simple form, the system could be used, for example, as shown in Fig. 1. Here a reference generator is used to produce the reference signal to be associated with pattern 'H'. This same reference signal is also easily recognized by the recognition system. The reference signal is fed into the reference section at the same time as the detector array scans pattern 'H' and feeds the signal

so obtained into the detector system. The reference signal is also fed to the recognition system by connections similar to those between the reference and detector sections. If required, a number of different patterns may be recorded against the same reference signal (e.g. different type faces; an object seen from different directions; or different voices). The reference generator is disconnected. Now, when the detector scans a previously 'known' or 'near known' pattern, the reference signal is reconstructed by the reference section, and analysed by the recognition system.

As the reference signal is reconstructed by the detector inputs as they are received, the addition of the associative memory system will not add to the total pattern recognition time.

In Fig. 1 the associative memory system consists of interconnected 'sections', into which the desired inputs are fed. A schematic of the simplest possible system,

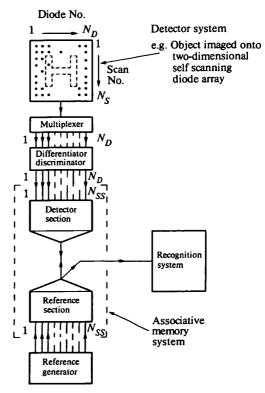


Figure 1. Schematic of system application (example).

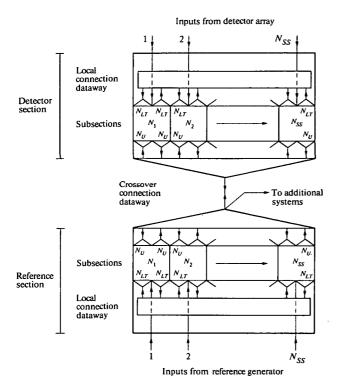


Figure 2. Simplest associative memory system.

consisting of just two sections with inputs is shown in Fig. 2. The concurrent phenomena are coded by a number of detectors, each of which feeds pulses to a separate sub-section within the section. These pulses cause the subsections to trigger and send pulses down a series of delay lines, termed the 'local' or 'crossover' connections. A schematic of a subsection is shown in Fig. 3. Each subsection is multiply connected to all other subsections within its section by the local connections, and to each subsection in the other section by the crossover connections. The mode of operation of the system allows the local and crossover connections to be made randomly. This also applies to the connection of inputs. Each subsection is divided into units, one for each crossover connection. A schematic of a unit is shown in Fig. 4, the connection terminals being some of those shown collectively in Fig. 3. Information is stored

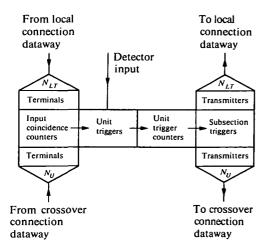


Figure 3. Schematic of a subsection.

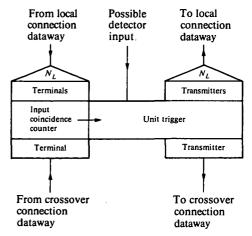


Figure 4. Schematic of a unit.

as binary bit type changes at the local and crossover terminals (terminal 'marking').

Either section (Fig. 2) operates by comparing the past time/sequence of the inputs to the other section, converted to the present by the delaying action of the crossover connections, to the past time/sequence of its own input, also converted to the present by the delaying action of the local connections. Coincidences between local and crossover pulses arising during this process are stored in the subsection receiving an input pulse. Each of these coincident events is recorded by a separate unit within this subsection. In future, the same inputs to the two sections will cause the same coincidences to occur, and these cause the subsection to trigger just as if it were receiving an input at the correct time. Control is effected by Section Potential Functions, these being determined by the recent history of the section.

The outputs to the recognition system are provided by the zero delay crossover connection of an additional unit in each of the reference section subsections.

Three simulation programmes have been written to aid the development of the system into its present form:

- (i) ROSE (Randomly Organized Storage Experiment) simulates a single section containing only subsections linked by local connections, and highlights the need to store two or more concurrent, related signals.
- (ii) AMY-1 (Associated MemorY) simulates two linked sections.
- (iii) AMY-2 simulates a two section version of the system described in this paper. It demonstrates the abovementioned permissibility of latitude in input signals, and assisted the development of the Section Potential functions to the stage to be described. The preliminary results from AMY-2 are described in a later section on AMY-2.

2.1 Basic hardware

A schematic of a simple two input system is shown in Fig. 2.

2.1.1 Input detectors. An input signal is analysed by a number of detectors working in parallel, each designed to respond to a different parameter of the signal. Each detector gives an output pulse whenever it is sufficiently stimulated. Because of the time dependent operation of

the system, this stimulation level must, in general, be caused by the rate of change of the signal parameter being monitored. The combined output from this bank of detectors is a pattern of pulses whose timed sequence structure is a unique representation of the input signal. These pulses are fed into a section, each individual detector output being connected in parallel to a number of units within its subsection.

2.1.2 The unit. A schematic of a unit is shown in Fig. 4. The construction is such that it can be triggered in a number of different ways, as detailed later in this paragraph. When a unit trigger occurs the following happen: (a) As 'seen' by the rest of the system the unit produces sets of coincident pulses at regular intervals. Each of these pulses passes along a separate local connection. One pulse from each set is delivered to each of the other subsections within the section, and each of the series of pulses delivered to a subsection is connected to a different unit within the subsection. The unit also delivers, via the single crossover connection, a delayed pulse to a unit within the second section. The terminals at which these pulses arrive have two possible states, 'marked' or 'unmarked'. Initially, all terminals are unmarked. (b) If, at the moment of trigger, pulses happen to be arriving from previously triggered subsections at any of the local connection terminals, as well as a pulse at the crossover connection terminal, the terminals can undergo a binary type change, or be 'marked'. The occurrence of marking is controlled by the subsection potential function.

A unit can be triggered in the following ways: (i) by a pulse to an input connection; (ii) by the coincident arrival of a pulse at a marked crossover terminal and sufficient pulses at marked local terminals. The required number of local pulses is determined by the unit potential function; (iii) by the coincident triggering of sufficient other units within a subsection; this number being determined by the subsection potential function.

2.1.3 The subsection. A schematic of a subsection is shown in Fig. 3. A subsection is an array of units in which, if sufficient 'units' are triggered, as in 2.1.2, then all units will synchronously trigger (see 2.1.2 (iii)).

If the numbers of units in each subsection are equal, then these triggers will send a full time spectrum of delayed pulses to each unit in the other subsections via the local connections. Also, if the units are sufficiently numerous, then a similar full time spectrum of delayed pulses will be sent via the crossover connections to each subsection in the other section.

2.1.4 The sets of pulses described above are produced by the delay-line properties of the local and crossover connections, the pulses being initiated by the unit trigger. More than one pulse can be simultaneously travelling down any one connection.

In order to ensure the required numbers of pulse coincidences on a unit, without having an infinite number of connections, it is necessary to define that coincidence means arrival during an interval τ . Hence, if the maximum local delay is D_L , then the number of local connections on a unit (N_L) is given by

$$N_L \ge D_L(N_{SS} - 1)/\tau \tag{1}$$

where N_{SS} is the number of subsections in a section.

Similarly, if D_C is the maximum crossover delay, then the number of units in a subsection (N_{II}) is given by

$$N_U \ge D_C N_{SS} / \tau \tag{2}$$

The number of local connections leaving a subsection (N_{LT}) is thus (N_LN_U) .

Finally, we can state that the average rate of arrival of detector pulses into a section (R) must obey the relationship

$$R \ge 1/\tau \tag{3}$$

This implies that after giving one output, the detectors experience a 'dead time' before being able to output another pulse.

2.2 Basic hardware operation mode

If one considers the response of the two input system to concurrent inputs, it will be observed that coincident with each subsection trigger will be the arrival at some units, via the local and crossover connections, of a number of pulses generated by previously triggered subsections. If the terminations on which these coincident pulses arrive are now 'marked', input patterns can be reconstructed simply by starting them and then using the multi-coincidence of the arrival of pulses at 'marked' terminals ('marked pulses') to re-trigger these units, and hence the whole subsection. The system decides between record and playback modes, and causes the initiation of pattern reconstruction on a non-input side by means of the Section Potential Functions. These are time dependent functions that within a section determine the number of coincident 'marked pulses' needed to trigger a unit (unit potential), the number of coincident unit triggers necessary to trigger a subsection, and whether terminals receiving coincident pulses are to be marked (subsection potential). They are defined as follows:

(a) Unit potential function (P_v) . This potential is controlled by the presence of triggering subsections within the section. Whilst subsections are being triggered at the average rate R, then the potential will increase at a rate

$$\frac{\mathrm{d}P_{v}}{\mathrm{d}t} \le R \tag{4}$$

If the trigger rate falls below $1/t \max$, where $t \max$ is the longest expected delay time between any two subsection triggers in a normal sequence, then

$$\frac{\mathrm{d}P_U}{\mathrm{d}t} \le -R \tag{5}$$

The value of P_U can vary between zero and $P_{U \text{max}}$ where

$$P_{U\max} \le D_L R \tag{6}$$

(b) Subsection potential function (P_{SS}) . This has a minimum value of $P_{SS\, min}$. For the simple system being described here, this is set empirically, but in larger more complex systems to be suggested later, it would be set by the numbers of units being 'coincidence' triggered in other sections adjacent to the one under consideration.

The working value of P_{SS} is the larger from $P_{SS \min}$ and the highest number of units being triggered in any of the

subsections in the section. The maximum value P_{SS} can reach when only coincidence type triggers are present is

$$P_{SSC} = D_C R \tag{7}$$

If one now requires P_{SS} to determine whether a section should be in record or playback mode, then the number of units to which an input is multiply connected must obey the relationship.

$$N_I > P_{SSC} \tag{8}$$

and the system must be unable to distinguish between these 'input' triggers and coincidence triggers.

Finally, if only inputs are to be stored, then it must be decreed that terminals can only be marked if

$$P_{SS} > P_{SSC} \tag{9}$$

The sensitivity of the system to errors in the input sequence is determined by the value of $P_{SS\min}$.

When inputs to both sections are present, as the subsection input triggers a number of units, the subsection potentials will be raised to levels which will inhibit subsection triggers caused by 'coincident marked pulses'. If the input to one section now ceases, the subsection potential in that section will decay and the memory mode will become active. The system will continue this self-sustaining memory mode until coincidences decline.

If the system has been quiet for some time, the unit potential will be at zero, and at this level a unit may be triggered by just a pulse to a marked crossover connection. An input to one section will thus cause 'possibly correct' unit triggers in the other. These numbers increase until the most 'probably correct' subsection(s) trigger, causing the unit potential to start increasing. The section will now settle into reproducing only the correct pre-recorded pattern as the multi-coincident criteria defined by the potential functions become increasingly discriminating. inhibiting incorrect triggers. The high discrimination possible with this system depends on a reasonably low density of 'marked' terminals, i.e. on the system exhibiting a high degree of redundancy. The 'storage capacity' of a section depends upon the total number of crossover connections associated with it, and is thus proportional to $(N_{SS})^2$. If one considers the way in which 'marked' terminals are determined, we see the only requirements are that: (i) each unit receives, via the local connections, a full spectrum of time pulses from each of the other subsections in the section; (ii) each subsection receives, via the crossover connections, a full spectrum of time pulses from each subsection in the other section. There is no requirement that any particular connection be connected to any particular terminal, only that the above simple criteria be met. Thus, if the total numbers of units in each subsection are increased by an appropriate factor, the local and crossover connections can be made in a truly random manner.

2.3 AMY-2

This is a digital analog designed to demonstrate the basic characteristics of the system described in the previous sections. The parameters describing the system being modelled are given in Table 1. Typical outputs are shown in Figs 5, 6 and 7.

All the figures display the ability of the system to

Table 1. Parameters used in AMY-2

(1)	Data—a sequence of randomly selected subsections.		
(2)	Average Rate of subsection trigger	(<i>R</i>)	1 time unit ⁻¹
(3)			2
٠,	No. of subsections per section	(N_{SS})	8
(5)	No. of units per subsection	(N_{ν})	56
(6)	No. of input connections per subsection	(N ₁)	9 .
(7)	Maximum local Delay Time	(D_{ι})	7 time units
(8)	Maximum crossover delay time	(D_c)	7 time units
(9)	Coincidence time interval	(τ)	1 time unit
(10)	Maximum value of unit potential function	(P _{Umax})	Various
(11)		(P_{SSmin})	Various
(12)	Rate of change of unit potential function	$d(P_{\upsilon})/dt$	1 unit/time unit

initiate a correct reconstruction in a non input section. The delay before reconstruction begins is a consequence of $P_{SS \min}$.

The sensitivity of the system to errors in the input is such that it will ignore up to $(P_{SSC} - P_{SS\,min})$ errors during D_C time units (Fig. 5).

Figures 6 and 7 demonstrate the superimposition and reconstruction of a number of patterns; Figure 6 shows associated pairs and Fig. 7 a number of different inputs associated to a single reference.

In these 'multi-overlay' runs, observations were made of the storage limitations of the demonstration system. Figure 8 shows the times taken for the output to stabilize for various numbers of overlays (25 represents non-stabilization).

CONSTS ARE :-	Ni = 9 PUmax = 5 PSSmin = 4
CYCLE NO 1 I/P 1 (St 1) O/P 1 I/P 2 (St 1) O/P 2	163366435152146483236547272622 163366435152146483236547272622 351724835633477671446523612538 351724835633477671446523612538
CYCLE NO 2 I/P 1 (St 2) O/F 1 I/P 2 (St 2) O/P 2	163366435152141483236547272622 163366435152141483236547272622 9090909090909090909090909090 909024835633477671446523612538
CYCLE NO 3 I/P 1 (St 2) 0/P 1 I/P 2 (St 2) 0/P 2	163366435152141283236547272622 163366435152141283236547272622 00000000000000000000000000 000024835633477671446523612538
CYCLE NO 4 I/P 1 (St 2) O/P 1 I/P 2 (St 2) O/P 2	163366435152141233236547272622 163366435152141233236547272622 9996969696969696969696969 999924835633477671446523612538
CYCLE NO 5 I/P 1 (St 2) O/P 1 I/P 2 (St 2) O/P 2	163366435152141234236547272622 163366435152141234236547272622 00000000000000000000000000000000

Figure 5. Effect of errors in the input pattern causing reconstruction.

AN AID TO PATTERN RECOGNITION

CONSTS ARE :-	Ni = 9 PUmax = 5 PSSmin = 4
CYCLE NO 1 I/P 1 (St 1) O/P 1 I/P 2 (St 1) O/P 2	163366435152146483236547200000 163366435152146483236547200000 351724835633477671446523600000 351724835633477671446523600000
CYCLE NO 2 I/P 1 (St 2) O/P 1 I/P 2 (St 2) O/P 2	284634671843245454428826500000 284634671843245454428826500000 132737272773351588538325800000 132737272773351588538325800000
CYCLE NO 3 I/P 1 (St 3) O/P 1 I/P 2 (St 3) O/P 2	134666788264541567775778200000 134666788264541567775778200000 148717852668322778843166800000 148717852668322778843166800000
CYCLE NO 4 I/F 1 (St 4) O/F 1 I/F 2 (St 4) O/F 2	142548743567128678764812400000 142548743567128678764812400000 151732526786588414725247500000 151732526786588414725247500000
CYCLE NO 5 I/F 1 (St 5) O/F 1 I/F 2 (St 5) O/F 2	141482884412673663241165600000 141482884412673663241165600000 166251527537352612431271380000 166251527537352612431271300000
CYCLE NO 6 I/F 1 (St 1) O/F 1 I/F 2 (St 6) O/F 2	163366435152146483236547200000 163366435152146483236547200000 0000000000000000000000000000000
CYCLE NO 7 I/F 1 (St 2) O/F 1 I/F 2 (St 6) O/F 2	. 284634671843245454428826500000 284634671843245454428826500000 0000000000000000000000000000000
CYCLE NO 8	•
I/F 1 (St 3) 0/F 1 I/P 2 (St 6) 0/P 2	134666788264541567775778200000 134666788264541567775778200000 000000000000000000000000000000
CYCLE NO 9 I/F 1 (St 4) O/F 1 I/F 2 (St 6) O/F 2	142548743567128678764812400000 142548743567128678764812400000 000000000000000000000000000000 0000
CYCLE NO 10 I/F 1 (St 5) O/F 1 I/F 2 (St 6) O/F 2	141482884412673663241165600000 141482884412673663241165600000 00000000000000000000000000000

Figure 6. Superimposition and reconstruction of five associated input pattern pairs.

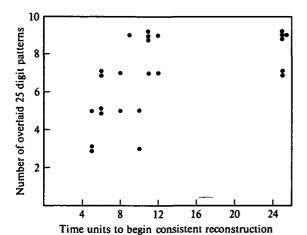


Figure 8. Storage capacity of system detailed in Table 1.

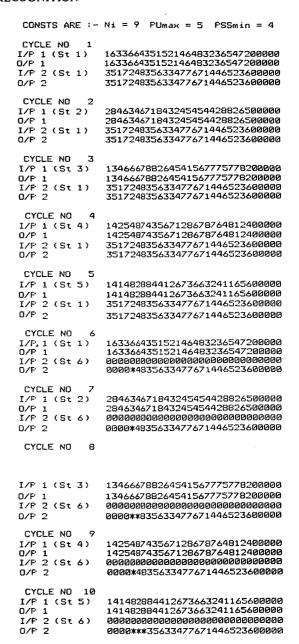


Figure 7. Superimposition and reconstruction of five different input patterns associated with a single input pattern.

A way of characterizing this limit is to specify that at least one of the crossover connections involved in the storage of a digit is exclusively used in the storage of that digit. Thus, if the number of crossover connections aiding the 're-selection' of a digit is N, then the system would be expected to begin to act erratically when the fraction of crossover connections that are marked approaches (N-1)/N. It can be shown that the number N_m of crossover connections marked by the input of n series is given by:

$$N_m \approx N_U (1 - e^{-\alpha n}) \tag{10}$$

where α = Average fraction of crossover connections marked by a single input.

By inserting the parameters for the demonstration system, we obtain $n \approx 5.9$.

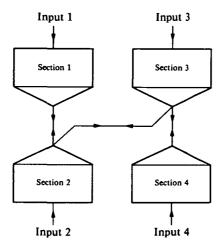


Figure 9. Multi section system.

2.4 The multi-input system

A simple three input system is shown schematically in Fig. 9. Here, the number of units in each subsection of section 2 has been doubled, and the increased number of crossover connections divided between sections 1 and 3. Three inputs will now be recorded in such a way that the reoccurrence of only one of them will cause the others to be reconstructed. A fourth input is also shown in Fig. 9. A large number of inputs can be interrelated in this way. If a particular input is so large that it requires a too complex section, it can be divided amongst a number of smaller sections.

A multi-input system can be visualized as two parallel planes; the units with their local connections being in the planes, and the crossover connections passing through a central volume, where their arrangement is randomized. A subsection is defined by that area over which a number of triggered units will cause sympathetic triggering of the whole area, and a section is defined by the maximum range of the local connections. The subsection potential function would, in this case, define a two-dimensional 'map' of the area rather than a simple potential covering a discreet section. If this potential is thought of in terms of a rubber sheet model, where the 'rubber sheet' is supported on rods as high as the number of unit triggers in each subsection, then the tension in the sheet will (a) cause it not to rest on the shorter pins, inhibiting these subsections from triggering, and also (b) cause the minimum potential $(P_{SS \min})$ in other subsections.

If the size of an input is such that it requires more subsections than are present in any section, then it can be connected to a large group of subsections, the 'section' boundaries now being defined relative to each subsection. This is possible as the whole group of subsections record and reconstruct interdependently, there being no reason why a particular subsection should not be in more than one 'section'.

TOWARDS ARTIFICIAL INTELLIGENCE

A system, such as the one described in this paper, able to associate phenomena and to recognize inputs similar to previously encountered ones, should be able to demonstrate a form of artificial intelligence, and it is of interest at this stage to consider, in general terms, how this may be achieved.

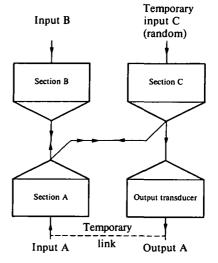


Figure 10. System output mode.

First, a way must be suggested whereby the internal patterns can be used to directly initiate a form of output. This could be achieved using a modified multi-input system. A simple two input/one output system is shown in Fig. 10. This is similar to the three input system of Fig. 9, except section C has extra units, the crossover connections of which (the output drives) activate an output transducer. The output transducer system is such that it will give an output in the form of input A, and consists of a number of transducers each connected to a group of crossover connections. The output from these individual transducers depends upon the rate of arrival of crossover pulses, the higher the rate above a threshold level, the greater the output.

The system is 'trained' to give an output which is a translation of the internal patterns simply by connecting the output directly to input A, and then feeding section C with a random input. This causes the system to 'memorize' the output relative to the same input A. Then, if input B reconstructs a previous recorded input A pattern this in turn will cause section C to make the output transducer reproduce the original input A.

Second, the system described so far is essentially passive in its response, in that an output is only initiated by an input. A self-motivating device could possibly be constructed by extending the concept suggested above, that is by providing an additional internally generated input to the system. This could be achieved by connecting the first or main system to a second smaller system. termed the drive system. Inputs to the drive system would be exclusively crossover connections from the main system, and would have similar connection characteristics as the main system inputs. These connections would be made in a random manner. Output drives from the drive system would be connected, also in random fashion, to units in the main system in a similar way to crossover connections, but the terminals would exhibit one of the following states:

State (i) Exhibiting the same characteristics as marked crossover connections. This is the initial state of the terminals.

State (ii) The terminal becomes permanently unmarked.

A terminal is changed from state (i) to state (ii) by being involved as the crossover connection in a unit trigger just

prior to or coincident with the introduction of an externally generated inhibitor command to the system.

Inputs to the drive system, whether caused by external stimulation or playback mode in the main system, will cause an output to all sections of the main system which is a unique representation of the total condition of that system. This total condition pattern would also be stored, and could be reconstructed either by a similar or partly identical total main system condition. Under normal circumstances, these drive pulses would help to initiate main system 'actions' when the unit potential function is low, and also be incorporated into learned patterns. When a learned pattern or output is considered unsuitable, if the drive system output terminations were modified as above by an external inhibitor command, then in future, any playback of this particular learned pattern or output, in the same main system total condition, would be disrupted by the reduction in unit trigger numbers below the minimum number required by the subsection potential function. Thus, by careful 'disciplining' over a period, a pattern of 'behaviour' could be established in this self-motivated system.

The required intelligent system could thus possibly be defined as follows: (a) A number of inputs, each input sufficiently large to require a number of sections for its connection, and each monitoring a different aspect of the systems environment. The detector arrays are such that at any one time they describe only a part of the total environmental phenomena. (b) A number of outputs, of which each can be monitored by the system inputs. The outputs are driven by the combination of a number of sections. (c) The multi-section main system is connected to a multi-section drive system.

Let us now consider the possible development of the system when starting with a completely blank memory. At this stage the outputs will be easily driven by the drive system, and the main system will start to record the 'partial' inputs against the drives. After a while, when sufficient similar operations have been recorded, the main system will begin to 'fill in' the gaps in the input signals from memory. Also there will be a progressive increase in the average subsection potential, and this will reduce the relative effect of the drive system, but not of the main system memory.

The system's 'view' of its surroundings will thus be a composite of reality and memory. Judicious 'training' during this period should produce the desired result; the system's actions are already largely determined by the design of the outputs, it only being necessary to restrict them to the correct time and conditions.

If the system is presented with an unfamiliar set of inputs, these can be expected to generate a 'composite pattern' within the system, which, if not recognized by an output system, will cause no output. However, this composite pattern now effectively becomes a system input, and can start a process that will continue either until an output based upon the most relevant pre-recorded inputs is produced, or the coincidence rate falls to a level at which the memory mode can no longer be sustained.

CONCLUSIONS

A system that exhibits the characteristics of associative memory has been described. The basic principles of this system have been demonstrated using a digital software analogue.

A description has been given of the way in which this system may assist in overcoming difficulties sometimes encountered during pattern recognition. Other possible applications are: (a) The man-machine interface: the dialogue could employ natural phenomena, e.g. speech; or artificial phenomena, e.g. implanted electrodes, handwriting, etc. (b) Robotics: The ability to simultaneously correlate numerous inputs would enable a system to communicate easily with its controllers; to have a sense of position within an environment; to co-ordinate vision and movement: to maintain balance; etc., as well as performing tasks. (c) Decision-making/event prediction (e.g. weather forecasting): by virtue of its operation, each output can be made to be a composite of all previous related inputs. (d) Data retrieval. (e) Translating. (f) Complex process control: an exact knowledge of process and control dynamics would be amassed, resulting in the best possible control.

The physical realization of a dedicated system has been shown to be relatively non-complicated, and should lend itself easily to micro-electronic techniques. The construction of a large system is greatly simplified by the inherent ability to self-calibrate input detectors and output transducers, and also by the random connections.

It has also been suggested that the full realization of the potential of this system should enable another advance to be made into the realm of artificial intelligence.

Finally, it has been observed that the system described bears a number of similarities to the brain. The more obvious are summarized: (i) The overall brain structure resembles a number of interconnected sections.^{3,4} (ii) The halves of the system are most easily visualized as planes.³ (iii) The units composing the planes resemble groups of brain cells.⁵ (iv) A unit fires when sufficiently stimulated.^{1,6-9} (v) The system has local connection circuits and remote connection circuits. 4, 10 (vi) The time taken for a pulse to reach the end of the axon of a brain cell varies from one axon to another. (vii) The connections within the system can be made randomly.1 (viii) The system units have markable terminals. 10-13 (ix) The ease with which cells can fire depends upon the amount of previous activity in that area. 5 (x) The input form consists of a time/sequence pattern of pulses from a bank of detectors.⁶ (xi) Memory recall is instant, not involving a conventional search procedure.⁶ (xii) An input is localized to an area in one of the system planes. Similarly, outputs originate from a defined area.^{3,14}

Investigations are continuing towards the development of a digital computer analogue of a system capable of giving an intelligent answer to a simple question concerning its environment.

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Dear Sir.

On The Process of Bisection to Calculate the Eigenvalues of Quindiagonal Matrices

In a recent paper, W. A. Sentance and I. P. Cliff¹ concluded that for some type of quindiagonal matrices the method of bisection, proposed by D. J. Evans² to calculate the matrix eigenvalues, yields wrong numerical results.

We wish to point out that there exists an infinite set of quindiagonal real and symmetric matrices of dimension $N \times N$ ($1 \le N \le \infty$) for which all eigenvalues (or all minus one) are two-fold degenerate. This provides an example to show that the statement by D. J. Evans establishing that the zeros of $P_k(\lambda)$ strictly separate the zeros of $P_{k+1}(\lambda)$ is not correct. $(P_k(\lambda))$ are the leading principal minors of $|C - \lambda I|$ and C is any quindiagonal real symmetric matrix).

The set of aforementioned matrices correspond to the representation of the operator $RJ_z^2R^{-1}$ in the space $\{|JM\rangle$; M ranging $\}$, where $R = \exp(-i\alpha J_y)$, $J^2|JM\rangle = J(J+1)|JM\rangle$ and $J_z|JM\rangle = M|JM\rangle$. This representation can be written as

$$\begin{aligned} & \text{a}(\beta)_{m,m} = (1-\cos\beta) \, (N^2-1) \\ & + (1+3\cos\beta) \, (N+1-2m)^2 \\ & \text{a}(\beta)_{m,m+1} = 4\sin\beta \, (N-2m) \, C(m) \\ & \text{a}(\beta)_{m,m+2} = 2(1-\cos\beta) \, C(m) \, C(m+1) \\ & \text{a}(\beta)_{m,m'} = 0, \, m' > m+2 \\ & \text{where } \beta = 2\alpha, \, N = 2J+1, \, C^2(m) = (N-m)m \\ & \text{and } 1 \leq m \leq N. \end{aligned}$$

For these expressions it is easy to see that the eigenvalues of the quindiagonal real symmetric matrix are the diagonal elements when $\beta = 0$. The condition $a(\beta)_{m,m} = a(\beta)_{N+1-m,N+1-m}$ shows that there are N/2 degenerate eigenvalues for N even and (N-1)/2 for N odd.

Yours

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Dear Sir,

A Program for Generating Word Squares

Apropos of Smith and Steen's paper on generating crosswords, I would like to report on a related program for generating word squares.

The method is simple. A square diagram is filled depth first, one letter at a time in raster sequence, making letter choices in simple dictionary order, subject to the condition that every partially filled word is a prefix of some word in the vocabulary. To support the search, the vocabulary is kept in a trie, and each cell of the diagram contains pointers to the trie nodes for the across and down prefixes that end there. This data structure is quite compact: in one run 9663 seven-letter words were represented in 43,000 bytes of memory.²

An exhaustive search for nonsymmetric 4×4 word squares among a vocabulary of 816 four-letter words found 271 squares in 94 seconds on a PDP11.⁴ In other experience,

fifty-two 7×7 squares, mostly symmetric, were found among the already-mentioned 9663-word vocabulary, and 117 nonsymmetric 6×6 s were found among a 4917-word vocabulary. It took far longer to search these spaces than the much smaller space of 4×4 s—several weeks in each instance.

On 4 × 4 squares, the only case where we have comparable data, Smith and Steen's program ran three orders of magnitude more slowly. The difference is largely explained by the fact that their bit-map data structure for the vocabulary is an order of magnitude larger and consequently has to be relegated to secondary memory. It seems likely, too, that the constraints present in word squares are so severe that the simple live-prefix criterion of my program is in fact comparably effective to the tightest-constraint criterion of theirs, so the raw speed advantage of the former approach is indeed fully realized in this case. In any event, when looking for word squares, the costs of sophistication evidently far outweigh the benefits.

Yours
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- The 816 words were sampled uniformly from all four-letter words in the Merriam-Webster Collegiate Dictionary, which also provided the vocabularies for Refs 2 and 3.