Correspondence

The Reve's Puzzle

Dear Sir,

Professor R. F. Churchhouse mentioned to me the recent paper on the Reve's Problem.[1] I've encountered it a couple of times before but managed to avoid being ensnared: however, this time I don't seem to have been quite so lucky, and I'm writing to report the (not very earth-shattering) results.

The solution mentioned, based on the recurrence (m = no. of pegs, n = no. of discs, f = no. of moves)

$$f_m(n) = 2f_m(n-k)+f_{m-1}(k)$$

for suitable k, is to be found in several places in the literature. However, few of the authors concerned seem to be aware that it rests on an implicit assumption which may seem very plausible but has never been proved, for $m \geqslant 4$. That is, in moving a 'tower' of $n-1$ consecutive discs completely off a single peg on to a set of $k-1$ other (empty) pegs, at least one of the shortest routes leads to a position in which all the other pegs have (consecutive) towers on them.

Donald Knuth, in one of several letters to Martin Gardner on the subject, in 1979 christened this 'Frame's conjecture' and went on to remark 'I doubt if anyone will ever resolve the conjecture; it is truly difficult'. I became sufficiently interested to write a shortest-path search program based on Dijkstra's algorithm to search the associated graph exhaustively for the optimum solution, and (of course) the formula is right – at any rate for $k = 4$, $n \leqslant 10$ and $k = 5$, $n \leqslant 9$. Another mysterious phenomenon which this program uncovered is that no position is further away from the starting position than the finishing position is: as with Frame's conjecture, most people seem to feel this is 'obvious'.

Another variation of (3-pin) Hanoi which can be used to make a programming point – in this case, mutually recursive routines – is 'Cyclic Hanoi', first published by Mike Atkinson,[2] although Knuth mentions the idea to Gardner in a letter of 1975. The discs are now permitted to move only in one (cyclic) direction, i.e. from peg 1 to 2, or 2 to 3, or 3 to 1 only. It amply repays investigation.

Yet a third variation is 'Slow Hanoi' (R. Moeser), where each disc may move only to an adjacent peg, i.e. between 1 and 2, or between 2 and 3. Here the solution length is 3^n-1 and every possible position is traversed in between. Further computing diversions involve generating the ith position and ith move in optimal solutions to these puzzles efficiently in terms of the integer i (W. F. Lunnon, unpublished; also recent offerings by M. C. Er and T. R. Walsh).

As to the problem mentioned of finding $k = k(m, n)$ efficiently, we want to find an x such that, in binomial coefficient notation,

$$\binom{x+m-2}{m} < n \leqslant \binom{x+m-1}{m}$$

so that

$$k = \binom{x+m-2}{m-1}.$$

Well, if we choose x to be the nearest integer y to $(m!n)^{1/m}$, we certainly get

$$\binom{x+m-1}{m} \geqslant n;$$

and if $x = y-m+1$, certainly

$$\binom{x+m-1}{m} \leqslant n.$$

So it is only necessary to try the m values of x between $y-m+1$ and y. In fact, I suspect the correct choice is always the nearest integer to $y-\frac{1}{2}m+1$, but haven't sat down to do it in detail. (Notice that in general there are actually several possible choices of k for given m, n besides the one you give.)

If all we require is the number of moves (rather than the solution path), a simpler algorithm suffices. Write down the series

$$\underbrace{1,}_{\binom{m-3}{m-3}} \quad \underbrace{2,\ldots,2,}_{\binom{m-2}{m-3}} \quad \underbrace{4,\ldots,4,}_{\binom{m-1}{m-3}} \quad \underbrace{8,\ldots,8,}_{\binom{m-0}{m-3}} \quad 16,\ldots$$

and sum the first n terms of it, to get $f_m(n)$. For instance, for $n = 19$, $m = 4$, $f_4(19) = 1+2+2+4+4+4+4+8+8+8+8 = 49$. From this formula the results about possible values of k can easily be proved.

Yours faithfully

W. F. LUNNON

References

1. J. S. Rohl and T. D. Gedeon, The Reve's puzzle, *The Computer Journal* **29** (2), 187–188 (1986).
2. M. Atkinson, The Cyclic Towers of Hanoi, *Information Processing Letters* **13**, 118–119 (1981).

Measuring and Describing Users' Attitudes as an Essential Constituent of Systems Analysis and Design

Dear Sir,

It is certainly valuable to investigate the attitudes to computerisation of potential users of the system in question.[1] However, it is possible, and is probably common, to place too great emphasis on the effect of such prior attitudes in determining the outcome of computerisation in an organisation. My own research, dealing with the introduction of computerised client record systems in social services departments, showed that it is all too easy to blame users' attitudes when the faults actually lie with system designers.

In discussing reasons for the failure of several early social services systems the relevant literature (mainly written by the staff responsible for system design and/or implementation) often blamed the 'attitudes' of social workers. Their hostility, it was claimed, could lead to 'sabotage' of the system, through a combination of misuse (e.g. bad form completion) and non-use (e.g. failure to update records).

My own research included detailed case studies of computerisation in two social services departments. I found very little evidence to suggest active hostility to computerisation by social workers. True, few were enthusiastic, but most were certainly willing to give it a try. The reason why, despite this willingness, performance was frequently poor, was largely irrelevant to attitudes: unsatisfactory use of the system usually resulted from inappropriate system design, lack of self-explanatoriness of the system, and inadequate guidance in its use.

One example, showing dramatically the effect of poor form design, may be of interest. I asked social workers to complete, for a mock case, a copy of their department's case-review form, as they normally would do for subsequent entry to the computer. Amongst the items which should have been entered on the form were cessation of attendance by a child at a day nursery, a code representing the nature of the problem, and the date of the next review planned for the case. Table 1 shows the proportion of social workers in each department who entered all the information on the form, and did so in a manner which would have enabled the clerical staff subsequently to amend the computer records correctly. The remarkable differences between the two departments, on different items, were very clearly due to the design of the two forms, and could not be put down to unwillingness on the part of individual social workers.

Table 1. Proportions of social workers, from each department, who entered various items of information correctly on their review form

	Department A %	Department B %
Day nursery cessation	24	84
Nature of problem (code)	80	5
Date of next review	92	92

Yours faithfully,

D. DU FEU (Dr)
Medical Computing and Statistics Unit
Medical School
Teviot Place
Edinburgh EH8 9AG

Reference

1. A. P. Jagodzinski and D. D. Clarke, A review of methods for measuring and describing users' attitudes as an essential constituent of systems analysis and design. *The Computer Journal*, **29** (2), 97–102 (1986).