Conversion between Analogue and Digital Measures

By R. H. Tizard

This paper is the text of an introductory lecture given by the author at the Symposium held by The British Computer Society in London on 16 December 1959. The lecture was intended to give a background of the principles and standard techniques used in conversion between analogue and digital measures, to precede specialist reports on new developments by other contributors to the Symposium. A bibliography is appended.

Conversion between Analogue and Digital Measures

Analogue to digital conversion and its inverse are concerned with measures of physical quantities. The reasons for wanting digital measures are:

(i) Once measures are in digital form it is possible to preserve them without error in all operations of conversion, transmission, recording, storage, and so on.
(ii) So that they shall be suitable for automatic digital computation.
(iii) So that numbers may be recorded and displayed for human inspection.

Physical measurements require the use of an instrument or "transducer" at the site of measurement, to convert the physical quantity measured into some other quantity which is more convenient for subsequent operations. If a digital measure is required, it would be ideal for such a transducer to have a digital output, but this is so difficult in the case of most physical measurements, such as of temperature or pressure, that it is usual to accept an analogue output from the transducer, and to use an analogue/digital converter at a subsequent stage.

The conversion from one analogue form of measure, or signal, to another, usually so that the two are linearly related, is now a well-practised art and can be done cheaply. A few common examples are conversion from temperature to mechanical movement, temperature to voltage, pressure to frequency, and mechanical movement to voltage. The conversion to digital form being more difficult and more expensive, the measure is usually converted to the most convenient analogue form in the first place; the commonest of these are voltage, usually D.C., and mechanical displacement or rotation. It is also common for one or more stages of analogue conversion to take place in the analogue/digital converter itself.

In practically all cases the measure which is to be converted is originally in a continuous form, continuous, that is, both in amplitude and in time. Since a digital signal can have only a finite number of amplitudes, depending on the number of digits employed, the signal is necessarily quantized before or in the course of conversion. This implies that the signal is also sampled. The output of the analogue/digital converter may change every time the input changes by one quantum, but in most cases the analogue signal is sampled at regular pre-determined intervals. The processes involved in digitizing can then be divided into:

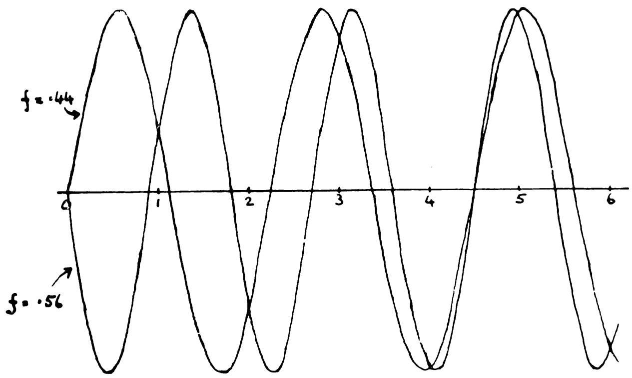

Fig. 1.—Sampled sine waves.

(i) sampling;
(ii) quantizing;
(iii) coding.

The theory of sampling, and of re-constituting a sampled signal, is now well established and the effects in a particular case can be calculated. A general point of great importance concerns the well-known principle that two samples in every cycle are sufficient to determine a sine-wave. This means that samples taken at time intervals T are sufficient to characterize any signal containing no components of frequency higher than $\frac{1}{2T}$.

It is sometimes erroneously believed that, from samples taken at intervals T of any signal, one can re-constitute all components of that signal up to frequencies $\frac{1}{2T}$. The fallacy in this is that if a sine-wave of frequency higher than $\frac{1}{2T}$ is so sampled, the results will be identical with those obtained by sampling a lower frequency. To be precise, a frequency of $\frac{1}{2T} + \alpha$ gives the same results as a frequency of $\frac{1}{2T} - \alpha$, as can be seen in Fig. 1. A frequency of $\frac{1}{T}$, i.e. a sine-wave sampled once per cycle, obviously gives the same output as a constant (D.C.) signal. For this reason any signal which is to be sampled must be pre-filtered so as to exclude all frequencies higher than $\frac{1}{2T}$.

The effect of quantizing can only be analysed statistically, but in most practical cases the quantum is small compared with the errors expected in the analogue signal. In some types of converter the quantizing and sampling are interrelated, as will be shown subsequently. The effect may be unimportant, but if not, it can be avoided by use of a "holding" circuit in which the sampled value, in analogue form, is maintained constant during the time of conversion.

There are many methods of digitizing, and the factors affecting the choice for any particular application are:

(i) form of input signal;
(ii) required form of output, e.g. binary or decimal;
(iii) accuracy;
(iv) speed;
(v) cost.

These are closely interrelated; for instance, in certain types of analogue/digital converter, accuracy and speed are inversely proportional. They are also very largely affected by one important factor, namely whether it is, or is not, possible to use a single converter to digitize a number of analogue signals. Such use fits in very easily with the sampling characteristic of analogue/digital converters, for by using a higher-speed converter it is possible to multiplex it, or time-share it, with many input signals. Thus a converter which accepts and digitizes samples at intervals T may deal with N signals, provided each one contains no frequency higher than $\frac{1}{2NT}$. It is, of course, possible to vary this by sampling some signals more frequently than others, at the expense of higher complexity in the equipment. Sharing is also possible by means other than direct sequential sampling.

Reasons for conversion from digital to analogue form are:

(i) when the output of a digital system is required in analogue form for controlling a process;
(ii) for graphical display of the output of a digital computer;
(iii) where the final form of signal is necessarily analogue, as in speech transmission.

Again the analogue signal is almost always required to be continuous in amplitude and time, and very similar considerations of filtering apply as in the sampling of an analogue signal for digitizing.

Before considering examples of analogue/digital and digital/analogue converters, it is useful to make a general distinction of principle between two types. To construct an analogue/digital converter, for example, one may use a direct method, or one may use a digital/analogue converter in a feedback arrangement. This is illustrated in Fig. 2, where A and D indicate analogue, and digital signals respectively. In the feedback case, the control signal alters the digital input to the converter in a logical manner to bring its analogue output to coincidence with the analogue signal to be converted. Reciprocal schemes apply, as shown, to the digital/analogue converter. (Note that the blocks do not have the same significance as in block diagrams of servo systems.) Since the distinction is not always clear-cut, it is useful to make the following definition:

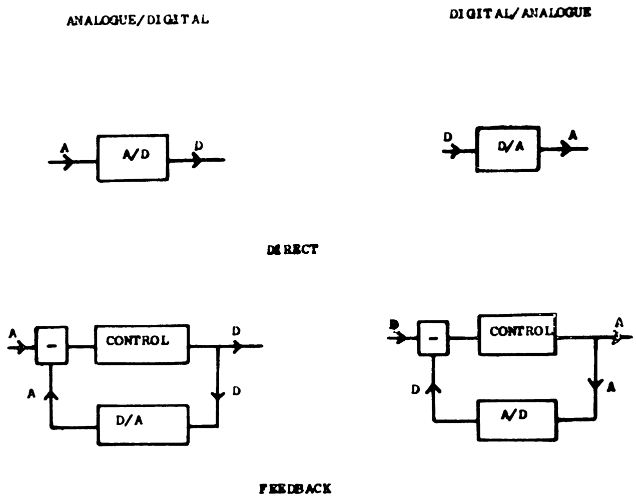

Fig. 2.—Principal types of converters.

A converter is of the indirect or feedback type if it generates a signal of the same form as, and for comparison with, the input signal; otherwise it is of the direct type.

This and other principal characteristics of converters will be brought out in describing particular examples.

The clearest practical distinction is between mechanical displacement, or rotation, converters on the one hand, and voltage or current converters on the other. It is of course possible to digitize a voltage by using a servo to convert it to mechanical displacement, and then digitizing. Conversely, it is possible to digitize a displacement by using a potentiometer to convert it to a voltage. Such methods should not be allowed to confuse the distinction.

Analogue to Digital Conversion—Mechanical Displacement

The two common methods of digitizing mechanical displacement are the *coded-plate* and the *counting* method. In the first a plate is marked with a pattern of "black" and "white" marks as shown, for example, in Fig. 3. The plate is attached to the moving member whose displacement is to be digitized, and a fixed reading head senses the black and white marks. The sensing may be done by photocells, in which case the marks are literally black and white, or opaque and transparent, or it may be by electrical contact, using insulating and conducting marks, or by magnetic or other means. The pattern changes for every quantum of displacement, and by using sufficient rows of marks it is possible to have a unique pattern of marks for every possible displacement of the plate. If the digital signals are required only for transmission there are no restrictions on the pattern or code, but if required for digital display or computing, it must be numerical in nature, usually either binary or decimal, or be capable of simple conversion to a numerical form.

Fig. 3 is an example of a linear displacement type coded for decimal read-out. Digits 2 and 3 are coded by a normal binary code for the values 0 to 9, using four bits each. Digits 1 and 4 use only two bits because in the particular application they are limited, by the range and the resolution respectively, to only three values each. The scale is read by scanning across it vertically with a spot of light smaller than the width of each column, obtaining a signal from a photocell corresponding to the sequence of blacks and whites in the column. If the scale is in such a position that the spot overlaps two columns an erroneous result may be obtained. For instance, if the scan is between the columns corresponding to 2·547 and 2·550, a readout from the first few rows might be bwbb, which would be interpreted as 2·557; on another part of the scale an error of similar nature but much larger might occur due to misreading in Digit 1 place. Three ways have been used to avoid this, and the figure illustrates one of them. Before scanning the code proper the spot of light passes across a guard ring as shown. If a pulse is obtained from the photocell the spot is moved half a column width to one side and the scan completed. Provided the scan is accurate and the spot width not greater than half that of the column, no error can occur. This leads to a very simple system but it requires high accuracy of scan and the plate must be stationary, or practically so, during the scan.

A method, using a similar principle, which overcomes these disadvantages makes use of the fact that with a binary code the frequency of change from black to white in each row is half that in its adjacent less significant row. Taking a change of one in the least significant digit as unity, the spot, instead of being moved by $\frac{1}{2}$ for all rows, as in the first method, is moved by $\frac{1}{2}$ after the first row, 1 after the second, 2 after the third, and so on. For each row the spot is moved either towards the higher or towards the lower numbers according to whether the previous row had been read as 0 or 1. In practice this is an awkward arrangement with a moving spot, and the system as described is mainly used with an electrically conducting plate with pick-up brushes, or with an optical plate flooded with light and using separate photocells for each row. An analogous system uses two rows for each digit, each row being displaced to left or to right. The scan is then in a straight line, each digit being read from one or other of its pair of rows, according to the reading of the previous digit. By these systems the allowable tolerances can be greatly

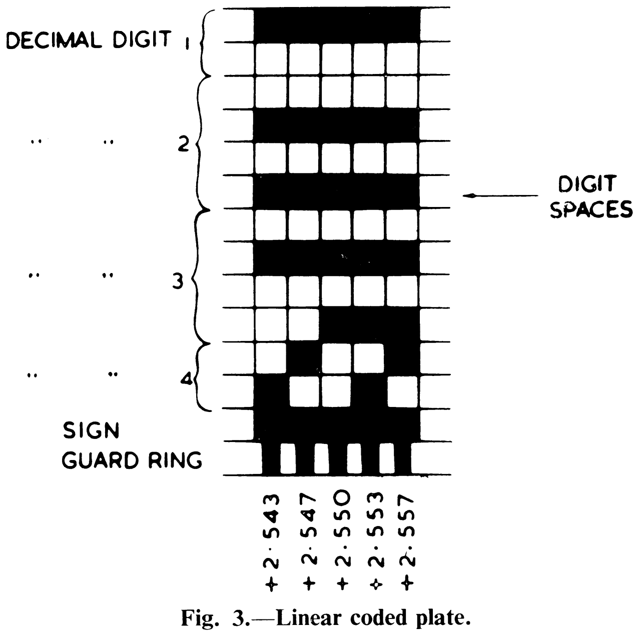

Fig. 3.—Linear coded plate.

increased, and the accuracy of conversion depends only on the accuracy of the least significant row. Against this, the number of photocells or brushes or code rows is doubled, and extra circuitry is needed to perform the switching.

The third method uses a special, non-arithmetic, type of code, by which ambiguities can be avoided even with an unmovable scan or a single row of brushes. The ambiguities described above arise in those cases where in moving from one code to the next more than one binary digit changes. For instance, in going from 2·547 to 2·550 in Fig. 3 the second row changes from black to white and the third from white to black. If a code can be devised to change by only one bit each time, the error can be avoided. Such codes exist and are called *C.P.* (*cyclic permuting*) codes, special forms being known as *reflected*, or *Gray* codes. The straightforward reflected code, which may be converted easily to arithmetic binary, is shown for the first ten numbers in the following table, in which digits which have changed from the preceding number are underlined. The principle

TABLE 1

BINARY	C.P.
0000	0000
0001	0001
0010	0011
0011	0010
0100	0110
0101	0111
0110	0101
0111	0100
1000	1100
1001	1101

may be extended to a decimal read-out by using a C.P. decimal, as shown in the following table, and coding each decimal digit with a suitable C.P. binary code. The

TABLE 2

DECIMAL	C.P. DECIMAL
25	25
26	26
27	27
28	28
29	29
30	39
31	38
32	37
33	36
34	35
35	34
36	33
37	32
38	31
39	30
40	40
41	41

disadvantage of the C.P. system, besides the circuitry required for decoding, is that since a change of number is indicated by a change in *any* digit place, every row in the plate must have the same accuracy.

Fig. 4 shows a complete coded disc having 720 codes; i.e. reading to $\frac{1}{2}°$. Such discs have been made with up to 16,000 codes, and probably more, the limit being set by accuracy of positioning of the codes and the read-out device, centring of the disc, and so on.

Where it is desired to digitize more than one turn of a shaft a system of geared discs can be used, rather on the principle of the coarse/fine synchro system. Ambiguities and errors due to the gearing can be eliminated by the methods of displaced read-out mentioned for plates or single discs.

Coded plates can also of course be graduated according to a non-linear law, so that the digitized output is a function of the analogue input such as the logarithm, sine or cosine.

To sum up, the coded plate provides a direct analogue/digital converter for mechanical displacement. Its resolution, in the optical type, may be 0·001 in. Hence the accuracy may be 1 part in 10,000 on a disc of reasonable size, whilst the accuracy of geared discs and linear plates depends only on the maximum displacement. With parallel circuitry the speed of read-out is limited only by the circuit response, but of course a servo used with a mechanical digitizer to digitize some such quantity as voltage will be very slow compared with an electronic digitizer. The shaft may be moving during read-out, but the limits of speed, which are easily calculable, vary considerably with the particular method used.

The second method of mechanical digitizing is by counting. Suppose a disc or drum with equidistant marks round its circumference is attached to a shaft.

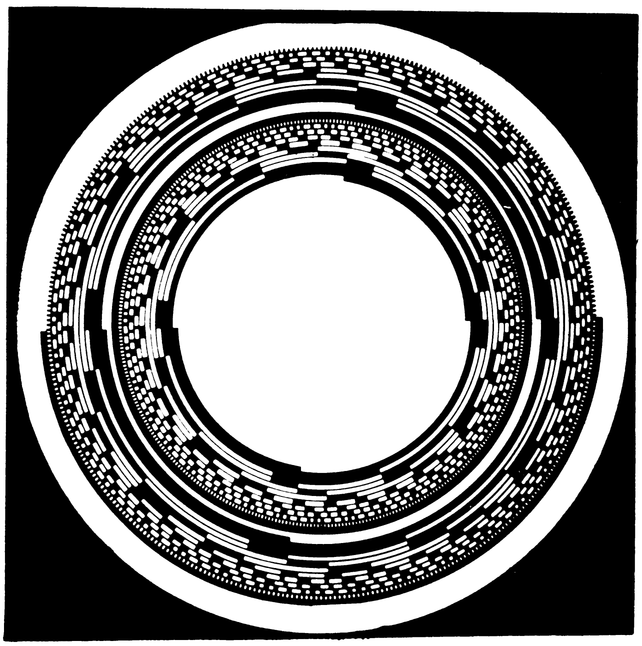

Fig. 4.—Circular coded plate.

Then the angular position of the shaft may be measured in digital terms either by counting, at the moment of digitizing, the number of marks between two reference points, one fixed and one moving with the disc, or by keeping a continuous count of the marks which move past a fixed point, adding to the count when they move past in one direction, and subtracting from it when they move past in the other. I will call these "de novo" counting and "continuous" counting respectively. By either method the marks and the means of detecting them may be of any of the normal types, e.g. electrical contacts, optical systems with photocells, or magnetic markings and pick-ups, and a non-linear law may always be incorporated by having the marks suitably spaced instead of equidistant.

Taking the case of de novo counting, suppose the shaft to be digitized carries an insulating disc marked with a single conducting mark as a reference point (Fig. 5). A second shaft, concentric with the first, carries three brushes A, B and C which make contact respectively with a fixed disc containing alternate insulating and conducting segments used as counting marks, the reference mark on the moving disc, and a fixed reference mark. When the shaft is to be digitized the second shaft is rotated rapidly through one revolution. Contact of brush C starts a counter connected to brush A, and contact of brush B stops it. The resulting count will be a digital measure of the angular displacement of the first shaft from the point where brushes B and C make contact simultaneously. If this shaft is turning it is effectively sampled at a time dependent on its rotation, a factor which may have to be taken into account—clearly its rate of turn must be low compared with that of the second shaft. The same method may, of course, be used for linear displacement, but the mechanical complication of the reading equipment is greater.

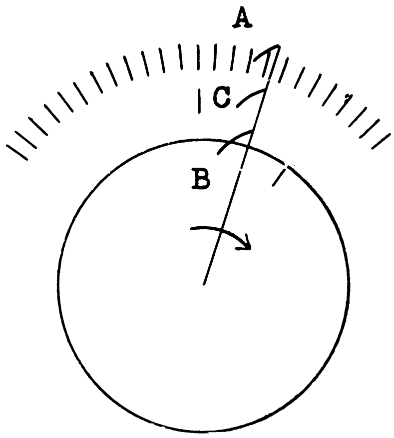

Fig. 5.—Shaft digitizing by de novo counting.

In the continuous counting method the only requirement is a disc with a single row of counting marks, carried by the shaft to be digitized, and a fixed detecting head. It is, however, necessary to detect the direction of motion of the marks past the detecting head. This is done by having two contacts, or photocells, or whatever type of detection is used, effectively separated by a distance of approximately one-quarter of the mark-spacing. Suppose that the marks consist of alternate transparent and opaque areas of equal width, as in Fig. 6, and that photocells A and B detect the passage of these past slits spaced as shown, then if a indicates that photocell A is illuminated, and $\bar{a}$ that photocell A is dark, and similarly for B, passage of the marks from left to right results in the sequence $a\bar{b}$: $\overline{ab}$: ab: $\bar{a}b$, and from right to left ab: $\bar{a}b$: $\overline{ab}$: $a\bar{b}$. A circuit of Eccles–Jordan type may be arranged to give a pulse out on one line every time $a\bar{b}$ is changed to $\bar{a}b$ via $\overline{ab}$ and on another every time $\bar{a}b$ is changed to $a\bar{b}$ via $\overline{ab}$ (the state of the circuit at $\overline{ab}$ being unchanged from its previous state), and these are fed to the positive and negative inputs of a reversible counter. Since the phases in the sequence each occupy one-quarter of the total cycle it will be seen that there is a "backlash" requiring at least this amount of movement between the output of any two pulses; hence high frequency vibration of a mark past the detecting head cannot cause errors. This backlash can be made greater by variation of the transparent and opaque widths and spacing of the detectors.

Comparing the two methods, the single-shot counting requires a considerably more complicated pick-up unit, but the counter is simpler and may be multiplexed amongst a number of shafts. It is a slow method since mechanical movement is involved, and for the same reason its reliability is not so good. The continuous counting necessitates a separate reversible counter for each shaft, and a sensing circuit for direction of movement. Theoretically, the continuous counting method involves the possibility of a continuing error if a single count is missed, but in practice this possibility can be made virtually non-existent, and there is the corresponding advantage that zero-setting is very simple.

By either method the resolution using ordinary optical markings may be to less than 0·001 in., whilst by the use of optical gratings resolutions of 1×10^{-5} in. or less are easily obtained. Read-out in binary or decimal or any other arithmetic system is obtained quite easily by using the appropriate type of counter, and by duplication of the counter read-out may be obtained simultaneously in, say, both binary and decimal.

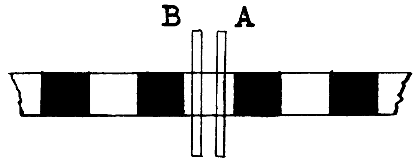

Fig. 6.—Sense detection in continuous counting.

Digital to Analogue Conversion—Mechanical Displacements

The main types of analogue to digital converters described above are of the direct type. By contrast, direct conversion from digital to analogue is rare, and the usual method is by feedback using a digital servo, or by conversion to an electrical signal followed by an analogue servo. However, the method of direct addition of displacements corresponding to binary digit positions has been used. In one version a shaft is extended by a spring so that its length contains gaps of 1, 2 and 4 units in length. These gaps are closed by electromagnets operated when the corresponding binary digits are 1. There are clearly severe practical limits on the number of digits which can be analogized in this way.

A similar method has been used to give a visual indication of a digital value similar in appearance to that of a dial and pointer instrument. Fig. 7 shows the principle. Eight slits are spaced by equal intervals Δ in a fixed plate, F. Three plates A, B and C having slits as shown are placed on top of F, and each may be displaced by a distance $\frac{1}{2}\Delta$ to the right. If the number to be indicated is 101, plates A and C will be so displaced, and it will be seen that a light shining on all four plates will be allowed to pass only through the slit five of F.

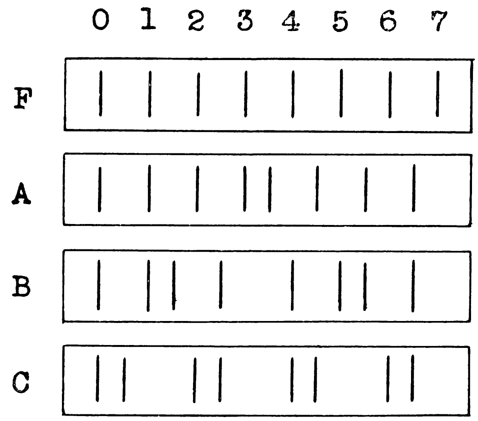

Fig. 7.—Analogue indication of a digital value.

Conversion of Electrical Signals

By contrast with the mechanical case, direct conversion from an analogue electrical signal to digital form is difficult, whilst for digital/analogue conversion a direct method is generally used. The latter will therefore be considered first.

Digital to Analogue Conversion—Electrical Signals

Confining attention to D.C. signals the principles are the same whether we consider voltages, currents, or resistance values, and in practice the choice is often marginal since voltage and current can be easily converted into each other, and resistance into either, after digital to analogue conversion. The usual principle is to switch analogue signals into some form of adding network in such a way that each switch is associated with a binary digit, and adds to the final signal an amount corresponding to that digit. The very simplest form consists of a chain of resistors in series of values 1, 2, 4, 8, etc., each of which is short-circuited by the normally-closed contacts of an associated relay (Fig. 8). The occurrence of a 1 in the corresponding digit position causes the relay to operate, and so insert the resistance into the chain. After operation of the relays, therefore, the total resistance is proportional to the digital value. In practice a voltage or current output is usually required, and the simple resistance chain must become a potentiometer or attenuator, involving two pairs of contacts per relay. The use of relays limits the speed and the reliability of such devices, and it is not easy to replace the relays with electronic switches because each is at a different voltage level. For electronic switching it is better to use methods of adding appropriate voltages or currents into a network, the simplest form being to supply voltages in binary steps to the input of an analogue summing amplifier.

Many variants of such schemes are possible, and one which will be taken as an example uses the switching of constant currents into a ladder network. Fig. 9 shows the circuit. The sources of constant current are the lines through resistances R, which are high compared with the ladder resistances r, from the supply line at $+V$ (the $-V$ supply line and its associated resistances should be ignored for the present). Current is switched into the network by changing the voltages at the points X_0 from a negative to a high positive value. The stages are numbered 0–5 from right to left. If current i enters at stage 5 it is attenuated by a half in passing through each stage to the right, and eventually contributes $2^{-5}ir$ to the output voltage. Similarly all the currents add in the load resistance after attenuation by 2^{-n}, so that the output voltage is proportional to the digital input. The speed of such a device is limited only by a single switching time, since it is parallel in operation, and by the stray capacitances and inductances of the circuit. Accuracy depends on the constancy of supply voltage and resistance values and the diode characteristics; whilst one part in 1,000 is a reasonable figure, higher accuracies have been claimed but are difficult to obtain and maintain.

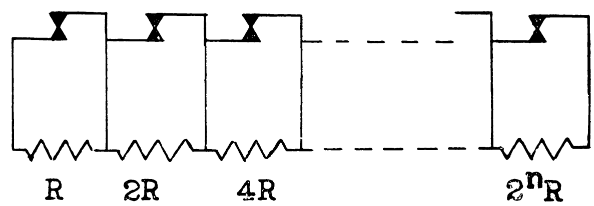

Fig. 8.—Resistance chain converter.

A very simple form of digital/analogue converter is the type known as the Shannon-Rack, which is applicable to a binary number serialized in time so that the least significant digit appears first. The principle is to feed a constant charge into a condenser at exactly each digit time, when the digit is a one. The condenser is arranged to leak at such a rate that it loses half its charge in one digit period. Thus the effect of every digit that has passed is successively halved at each digit period. The condenser voltage must be sampled at an exactly constant interval after the last digit time and this, and the difficulties of maintaining timing, leakage rate, and quantity of charge, make it a method suitable only for approximate conversion of numbers already in suitable form, such as for analogue display of numbers in a serial binary computer.

Analogue to Digital Conversion—Electrical Signals

Analogue to digital conversion by feedback may use any of the resistance network methods. The additional equipment required is a comparator and a logical control unit. Taking the ladder network of Fig. 9 again as an example, v_0 is subtracted from v_i, the signal to be digitized, to give an error voltage E. Initially no currents are switched into the network. If E is positive the current in stage O is switched in. If E remains positive this current is left in and the next stage current is switched in, but if E becomes negative the first stage current is switched off again before proceeding to the second stage. By proceeding down all the stages in sequence v_0 is eventually made equal to v_i, within one quantum, and the pattern of current switching provides the digital output. The operation may be speeded up, and a negative input voltage catered for, by duplicating the switching on a negative line $-V$ as shown. Suppose v_i, and hence the initial value of E, is positive, positive current is switched in at stage O. If the resulting v_0 is too high, giving a negative value of E, this current is not switched out but a negative current is switched in at stage 1. This procedure arrives at the correct result with only one switching decision at each stage.

Accuracy of such a device is further limited by the accuracy of comparison, and the speed is reduced, over the digital/analogue converter, by a ratio of at least the number of stages, since switching has to be done serially. Nevertheless, it can be very fast and is particularly suitable for multiplexing over many channels, such as voice transmission signals, at accuracies of about 1 in 100. It is necessary, of course, to multiplex the analogue signals themselves, which provides a further possible source of inaccuracy.

The logical control system described may be replaced by a reversible counter whose output is analogized by the ladder network. The counter is pulsed positively or negatively until coincidence of the analogue signals is reached. This method is slow unless the signal changes little between conversion times; hence it is not suitable for multiplexing.

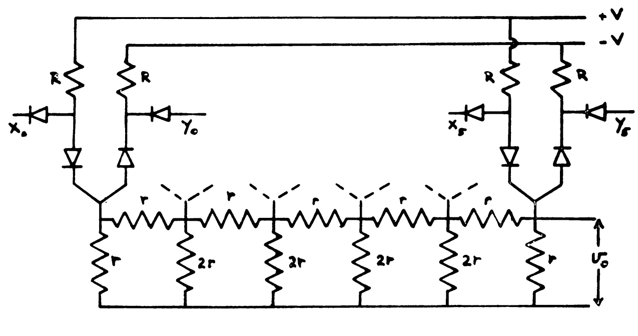

Fig. 9.—Ladder network converter.

A method of analogue/digital conversion which essentially involves intermediate conversion to another form uses a cathode-ray tube. Input voltage or current deflects the beam in, say, the X direction, and it is then scanned in the Y direction across an optical coded plate identical with those described for conversion of mechanical displacement. The system is in principle identical with the use of a mechanical servo, fitted with a coded disc, to follow a voltage input, but does not suffer from the inertia of moving parts.

Digital to Analogue Conversion by Intermediate Conversion to a Time Interval

In many ways the simplest type of conversion is from digital form to an analogous time interval. This is done simply by generating a constant frequency or train of pulses, and counting the pulses. The time interval is started from the zero of counting, and stopped when the count equals the input number. Such a time interval is rarely of direct use, but it may be converted to any desired physical quantity by starting a linearly rising "sawtooth" of this quantity at the start of the time interval, and stopping or sampling it at the end. Because the idea of a sawtooth voltage is so well known, this method has been used mainly for conversion to electrical form, but it can be equally applied to, say, air pressure. Alternatively, if the conversion to time intervals is carried out at a constant frequency, conversion to voltage form may be achieved by switching constant voltage into a smoothing circuit during the period of each time interval. Again, any other physical quantity may be so treated, as by switching a constant flow of liquid into a "buffering" tank.

Analogue to Digital Conversion by Intermediate Conversion to a Time Interval

The type of digital/analogue conversion described in the last section lends itself very readily to analogue/digital conversion by feedback, a comparator being the only additional equipment required. To take the example of voltage input, a linear voltage sweep is started at the same moment as a counter starts to count cycles from a constant frequency oscillator. When the sweep equals the input voltage, as indicated by the comparator, the count is stopped, and the contents of the counter are then the digital value of the input voltage. This scheme is shown in block form in Fig. 10. The accuracy of such a converter is limited by the linearity of the sweep and the errors of the comparator. Constancy of the sweep and the counting frequency are also involved, but may be compensated over the long term by automatic comparison against a reference voltage. In some cases it is possible to make the sweep non-linear deliberately, if it is desired to incorporate a non-linear law in the conversion. The ultimate speed is limited by the maximum possible frequency of counting, and is inversely proportional to the required accuracy of conversion, since the number of pulses counted must be proportional to this.

This system has the great advantage that a single sweep generator and oscillator may be used to convert any number of inputs, as shown in Fig. 11, without loss of speed. Moreover, if the loss in speed can be tolerated, a single counter may be multiplexed over a number of input channels, without the necessity of multiplex sampling of the analogue signals; it is of course necessary to provide a comparator for each input signal, but in many cases this may be a comparatively simple device. As in the case of digital/analogue conversion, this system is equally applicable to any physical quantity for which a sweep generator and a comparator can be devised.

The sampling and quantizing of the input signal are implicit in such a device, and it should be noted that if the signal is changing during the period of the sweep the exact moment of sampling will depend on its amplitude. In most practical cases the effect is negligible, but it might need consideration. The analogy with the de novo counting method for displacement digitizing will be noted.

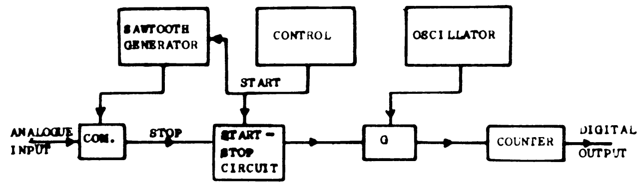

Fig. 10.—Sweep and time interval converter, single unit.

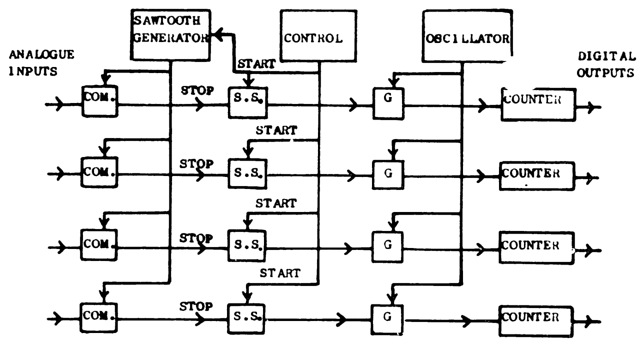

Fig. 11.—Multiplexed sweep and time interval converter.

Conclusion

Some general principles of analogue/digital and digital/analogue conversion have been discussed and a number of the more common methods briefly described. In such a short space a great deal of important detail has had to be omitted. In particular, there has been only occasional reference to the use of codes other than binary, and particularly to non-arithmetic codes such as chain codes and error-detecting or correcting codes, and there has been no mention of conversion to and from A.C. signals. It is hoped, however, that some idea has been given of the ways in which the choice of method is affected by the natures of input and output, the accuracy and speed requirements, and the possibility of using common equipment for many channels.

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