however, if a stop is made in the middle of the operation of reading in weighting information from paper tape.

10. Conclusion and Acknowledgement

Precise details of the language have been omitted in this paper in some places. These omissions have been made, not only to simplify the explanations given of the structure and usage of the language, but also because it is anticipated that later versions of the scheme on machines other than Pegasus will differ materially in some details, though not in outline, from the version being developed now. Several interesting points have arisen in producing a Pegasus program for providing Autostat facilities; these will be dealt with in a later paper. Use of the scheme will also be illustrated by reference both to a large-scale readership survey, and to costing and sales analyses. Work on both types of application, and on extension of the scheme to handle retail audit procedures, is currently in progress.

It is a pleasure for the authors to acknowledge their indebtedness to the British Market Research Bureau and, in particular, to Dr. J. A. P. Treasure and Mr. J. Fothergill of that firm for assisting in the initiation of this work and for encouraging its development.

Reference

CLARKE, B., and FELTON, G. E. (1959). "The Pegasus Autocode," *The Computer Journal*, Vol. 1, No. 4, p. 192.

Correspondence

The Editor,
The Computer Journal.

Sir,

May I comment on the paper by K. T. Boyd on "Simultaneous Equations and Linear Programming" in your April issue?

The use of the Simplex method for the solution of simultaneous equations and inversion of matrices was first suggested by Orden (1); practical applications were made on the Ferranti Mk. I computer at Manchester University in 1953 (2). Here are some details of the method used.

The machine starts by computing the row sums

$$\sum_j a_{ij} = R_i$$

of the given matrix (a_{ij}) and then reverses the sign of all the rows for which this sum is negative, keeping a record of the sign reversals.

This results in a modified matrix (a'_{ij}) with non-negative row-sums R'. For this matrix, the machine computes the column sums

$$\sum_i a'_{ij} = K_j$$

and then solves the linear programming problem:
Maximize

$$z = \sum_j K_j x_j \tag{1}$$

subject to

$$\sum_j a'_{ij} x_j \leqslant R'_i. \tag{2}$$

The set of variables

$$x_1 = x_2 = \ldots = 1 \tag{3}$$

satisfies relations (2) as equations; there are no slacks. That it constitutes an optimum solution can be seen as follows. Put

$$\sum_i R'_i = \sum_j K_j = \sum_{i,j} a'_{ij} = S.$$

Summation of (2) over i shows that

$$\sum_{i,j} a'_{ij} x_j = \sum_j K_j x_j \leqslant S$$

or

$$z \leqslant S$$

i.e. that S is an upper bound for z.

But this upper bound is reached when (3) is substituted in (1), proving that (3) is indeed the optimum solution.

In practice, the "contracted" version of the simplex method was used so that matrix (a'_{ij}) was replaced by its inverse, apart from certain permutations of rows and columns. The correct order of rows and columns was restored during printing out; at the same time, the sign of those columns, for which corresponding rows in the original matrix had undergone a change in sign, was reversed. The result was the required inverse of the given matrix.

The programme was used successfully for the inversion of matrices too badly conditioned to be inverted by other programmes available at the time, but it is not known whether this was due to some peculiarity of the simplex method or to the fact that an unusually large number of digits was employed in the computation.

Yours faithfully,

D. G. Prinz.

Ferranti Limited,
West Gorton,
Manchester, 12.

13 June 1960.

REFERENCES

1. ORDEN, ALEX. "Application of the Simplex Method to a Variety of Matrix Problems," *Symposium on Linear Inequalities and Programming*, Washington D.C., 14–16 June 1951.

2. PRINZ, D. G. "Some Experiences on the Manchester Computer with the Simplex Method," *Linear Programming Conference*, London, 4 May 1954.