Analysis of Reliability Models for Interconnecting MIMD Systems

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When a computing system operates with Multiple-Instruction Multiple Data streams (MIMD), the overall system reliability is defined as the probability that one path, at least, exists between each pair of system nodes.

In this paper the reliability models of MIMD systems are classified as combinatorial, state-transition, links-enumeration and spanning tree models.

These models are analysed in such a unified manner that seven network configurations are considered. These configurations are: star, tree, ring, dual-bus hypercube, normal hypercube, multiple global buses and near-neighbour mesh. Hence the corresponding structured Algol-like algorithms are given.

The reliability evaluation of every interconnection is presented, taking into account the configuration size. Moreover, a comparison of models and configurations is emphasised to help rank them for MIMD operational conditions.

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1. INTRODUCTION

The fast development of Very Large Scale Integration (VLSI) technology has provided chips of millions of gate equivalents. Accordingly, single-chip processors, controllers and memory units could be manufactured and successfully implemented. A processor chip plus the corresponding memory and I/O chips might form a computing node. A collection of computing nodes can be built up and distributed in a single cabinet to provide a network computer (not a computer network).

Conceptually, a processing system of this type is able to execute concurrently (because of the multiplicity of system nodes) Multiple Instructions with Multiple Data (MIMD) streams. The nodes constitute a tightly coupled configuration so that the system processors can cooperate together to carry out the shared load. Typically, they communicate by sharing memory elements and exchanging messages.[1]

Recently various MIMD systems[1,2,3] have been implemented in different process-control and data-processing applications. In these applications the input load is divided into modules which are distributed on the system nodes to be processed concurrently, i.e. in parallel rather than sequential as in the traditional Von Neumann machines. By increasing the parallelism, higher computational speeds and minimum execution time could be achieved. Significant speed improvements will be cost-effective only if the following apply.

(1) Parallelism is applied on all computations.

(2) Operating systems, as well as other system programs, and I/O resources facilitate maximum operational concurrency.

(3) The system attains a sufficiently high reliability as a network computer.[1,4] This is achieved by improving the availability of interconnections that provide the nodes required for task execution. This item is the essential concern of the paper.

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2. MIMD INTERCONNECTIONS

Various MIMD systems have been successfully used in different applications. The MIMD system (Fig. 1), consists of a set of computing nodes that are connected together by a number of branches to form a network computer. Thus the entire task is divided into subtasks that are assigned to the available nodes. Many interconnections, e.g. star, tree, ring, etc. may take place between the system nodes.[4] Naturally, these interconnections will yield configurations with different reliability characteristics. Many papers have been devoted to the reliability evaluation of MIMD systems. These papers can be classified as qualitative studies,[4] investigations for the applicability of the graph-theoretic approach[5,17] or an explanation of a particular mathematical technique.[6,7] Actually several critical questions concerning the MIMD system reliability have not been answered yet. Some of these questions are the target of this paper. Here four methods are used to determine the reliability of the basic seven configurations of MIMD systems (Fig. 1). These methods are based on the following models. (1) The combinatorial model.[8] (2) The state-transition (Markov) model.[7] (3) The links-enumeration model.[9] (4) The spanning-tree model.[5,6]

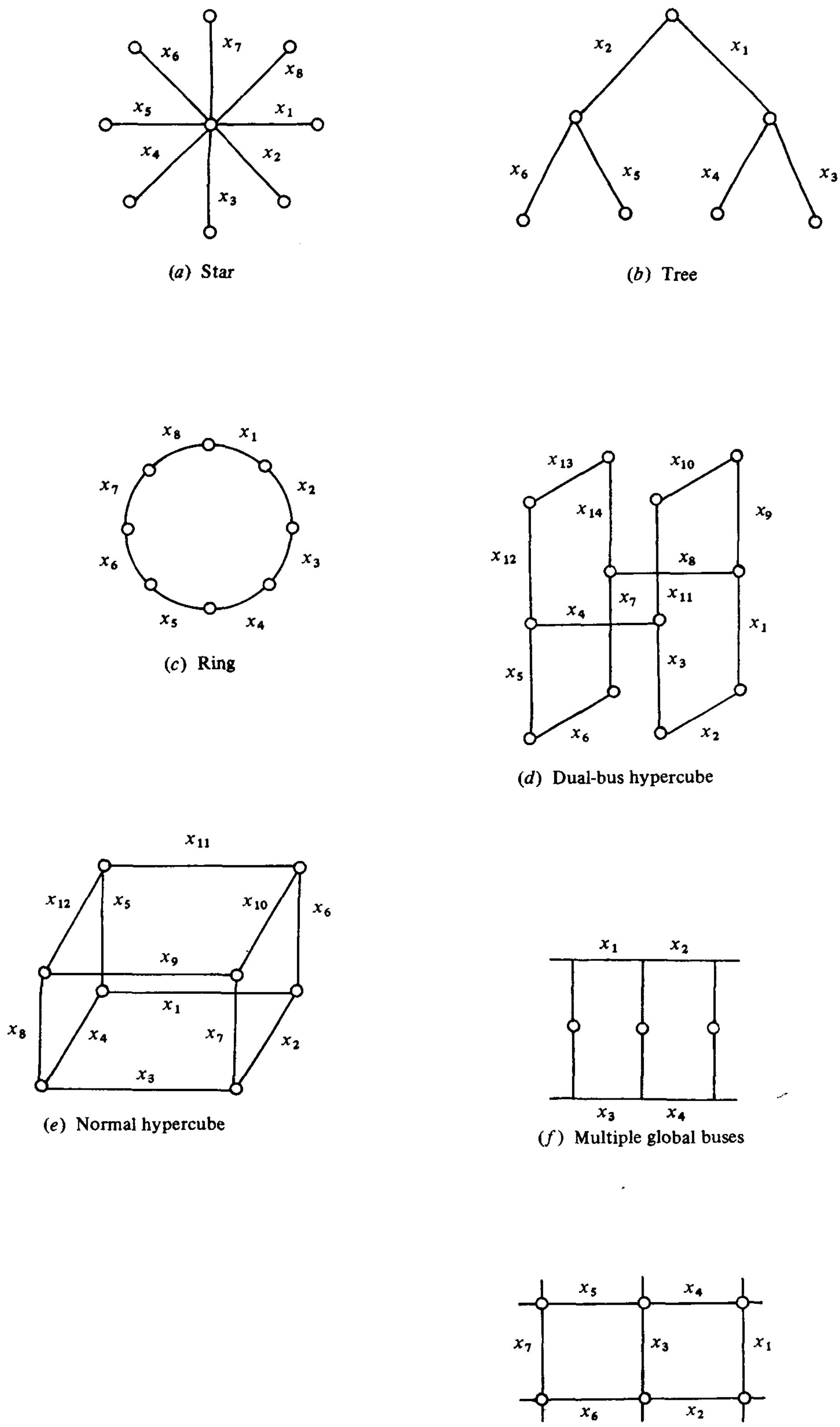
The first three models are presented together in a unified approach to calculate the configuration reliability, whilst the fourth model is given separately. For every configuration the effect of increasing the network nodes on the system reliability has been investigated. The seven configurations are classified into two groups according to the following rule.

if all minimum cut sets of a configuration have the same number of elements
then the configuration is in Group 1.
else the configuration is in Group 2.

Consequently, the two groups will be:

(Group 1) star, tree and ring

(Group 2) dual-bus hypercube, normal hypercube, multiple global buses and near-neighbour mesh.



(*a*) Star

(*b*) Tree

(*c*) Ring

(*d*) Dual-bus hypercube

(*e*) Normal hypercube

(*f*) Multiple global buses

(*g*) Near-neighbour mesh

Fig. 1. Configurations of MIMD systems.

For the two groups the corresponding forms, Algol-like procedures are given in order to provide a means of estimating the time and space requirements of every reliability model.

Several significant results are obtained. These results are of practical use in many applications, namely, the

(1) Any node in the configuration consists of a processing unit, a memory and input/output devices.[4]

(2) Every node has perfectly reliable hardware, software and communication facilities.

(3) All the network branches permit bidirectional communication.

(4) Each interconnection includes s-independent branches.[9] The probability of any branch to be in a good state (fault-free) is p.

(5) Any branch has only two states, i.e. it is either in a good state or a failure state.

(6) The MIMD network is in a good state if and only if any node can be reached from any other node in the system.[2]

(7) The system is in a failure state if and only if any node is isolated.

(8) The system repair is ignored.

Thus the system reliability and its availability will be synonyms.

Actually, assumptions (6) and (7) represent the essential difference between MIMD systems and other distributed computing systems in which information flows from a given source to a predefined sink.[10,11] Throughout this work the following notation is used: R, system reliability; P, probability of a branch being in a good state; m, total number of branches in a graph; n, total number of nodes in a graph; r, minimum number of branches for a system to be in a good state.

3. RELIABILITY OF GROUP 1 CONFIGURATIONS USING AN r-OUT-OF-m APPROACH

The r-out-of-m approach[7,9] represents a powerful means for solving many reliability problems. It assumes a graph of m branches, r of which ($r \leqslant m$) must be fault-free in order that the system can work successfully. The system reliability R is given by:

$$R = \sum_{\gamma=s}^{w} R_\gamma \tag{1}$$

The value of R_γ is obtained from:

$$R_\gamma = BP^{\mu-\gamma}(1-P)^\gamma \tag{2}$$

The values of the model parameters B, γ, μ, ρ and w can be obtained systematically, for Group 1 configurations (star, tree and ring), using one of the following three methods.

(1) The combinatorial method is concerned with determining the maximum number of failed branches a, whilst the system is satisfying assumption 6 (Section 2). In this case it could be found that

$$B = \binom{m}{i}, \; i = 0, 1, 2, \ldots, a$$

is the number of combinations of workable subgraphs.[8,12] Also $\gamma = i$, $\mu = m$, $\rho = 0$ and $w = a$.

(2) The state-transition (markovian) method is based on calculating both failure and repair rates.[7] Here we assume a system without repair in which the m branches have identical reliability characteristics. The method counts the states in which the system is working at different numbers of branches. The number of these states yields B such that:

$$B = \binom{m}{K}, \quad K = r, r+1, \ldots, m-1, m;$$

where $r = n-1$. Accordingly, $\gamma = m-K$, $\mu = m$, $\rho = r$ and $w = m$.

(3) The links enumeration method enumerates all links[9,14,18] that make the system satisfy assumption (6). The number of link sets yields B such that:

$$B = \binom{m}{e}, \quad e = 0, 1, 2, \ldots, m.$$

In this case $\gamma = m-e$, $\mu = m$, $\rho = 0$ and $w = m$.

Equations (1) and (2) can be directly computed by means of the following procedure.

```
Procedure ROMG1 (r, p, m).
  begin
    R:= 0;
  read p, m, r;
        k:= r;
        x:= m!;
        s:= p^k;
        d:= (1-p)^(m-k);
    while k <= m do
      begin
        y:= m-k;
        z:= y!;
        b:= k!;
        f:= x/(b*z);
        R_k:= f*S*d;
        R:= R+R_k;
        s:= s*p;
        d:= d/(1-p);
        k:= k+1
      end
    write R
  end
```

4. RELIABILITY OF GROUP 2 CONFIGURATIONS USING AN r-OUT-OF-m APPROACH

The equations of Group 1 configurations can be also employed to compute the reliability of Group 2 configurations. The equations, generally, depend on calculating the number of subgraphs of the m branches taken i at a time. Actually, for Group 2 configurations, not all these subgraphs represent links. Therefore, a means should be sought of finding out the exact number of links before using the above three models (Section 3).

For Group 2 interconnections a matrix of cut sets[5] is employed to get the corresponding cut sets of every model. In this matrix the columns represent the graph branches. The row branches, together with any one of the column branches may (or may not) express a cut set. The cross $\times$ indicates a cut set (system failure) whilst empty entries denote the system success even if the corresponding branches are cut. The mark $\times$ must not be duplicated for the same cut set, e.g. if $x_1 x_3$ is counted as a cut set the entry $x_3 x_1$ should be left empty. The cut-sets matrices of Group 2 configurations are given in the following.

4.1. Dual-bus hypercube

For a dual-bus hypercube with $n = 12$ and $m = 14$ (Fig. 1*d*), the following two matrices are obtained.

if $i = 2$ in the combinatorial model
or $k = 12$ in the state transition model
or $e = 12$ in the links enumeration model
then the corresponding cut sets are expressed by the matrix DC*1

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
x_1		X	X											
x_2			X											
x_3														
x_4								X						
x_5						X	X							
x_6							X							
x_7														
x_8														
x_9										X	X			
x_{10}											X			
x_{11}														
x_{12}													X	X
x_{13}														X
x_{14}														

*DC**1 Matrix (it includes 13 cut sets)

Thus:

$$\binom{m}{i}=\binom{14}{2}-13,\quad \binom{m}{k}=\binom{14}{12}-13 \quad \textit{and}$$

$$\binom{m}{e}=\binom{14}{12}-13$$

The next step is to find the matrix $DC^{*}2$:

if $i=3$ in the combinatorial model
or $k=11$ in the state transition model
or $e=11$ in the links enumeration model
then the cut sets are expressed by $DC^{*}2$.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
x_1, x_2			X	X	X	X	X	X	X	X	X	X	X	X
x_2, x_3				X	X	X	X	X	X	X	X	X	X	X
x_3, x_4	X							X			X			
x_4, x_5					X	X	X	X				X		
x_5, x_6	X	X	X				X	X	X	X	X	X	X	X
x_6, x_7	X	X	X	X				X	X	X	X	X	X	X
x_7, x_8				X	X									X
x_8, x_9	X	X	X	X						X	X			
x_9, x_{10}	X	X	X	X	X	X	X				X	X	X	X
x_{10}, x_{11}	X	X	X	X	X	X	X	X				X	X	X
x_{11}, x_{12}									X				X	X
x_{12}, x_{13}	X	X	X	X	X	X	X	X	X	X				X
x_{13}, x_{14}	X	X	X	X	X	X	X	X	X	X	X			
x_{14}, x_1			X									X		
x_{14}, x_2												X		

*DC**2 Matrix (it includes 111 cut sets)

Accordingly:

$$\binom{m}{i}=\binom{14}{3}-111,\quad \binom{m}{k}=\binom{14}{11}-111 \quad \text{and}$$

$$\binom{m}{e}=\binom{14}{11}-111$$

For this configuration no more matrices can be constructed because no links may exist upon cutting four branches.

4.2. Normal hypercube

For a normal hypercube with $n=8$ and $m=12$ (Fig. 1*e*), the cut-sets matrices are given as follows.

if $i=3$ in the combinatorial model
or $k=9$ in the state transition model
or $e=9$ in the links enumeration model
then *the matrix $NC^{*}1$ is obtained.*

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
x_1, x_2						X						
x_2, x_3							X					
x_3, x_4								X				
x_4, x_5	X											
x_5, x_6												
x_6, x_7												
x_7, x_8												
x_8, x_9												X
x_{10}, x_{11}							X					
x_{11}, x_{12}						X						
x_{12}, x_1					X							
x_{12}, x_2												
x_9, x_{10}												

*NC**1 Matrix (8 cut sets)

Now,

if $i=4$ in the combinatorial model
or $k=8$ in the state transition model
or $e=8$ in the links enumeration model
then the matrix $NC^{*}2$ is obtained.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
x_1, x_2, x_3						X	X					
x_2, x_3, x_4							X	X				
x_3, x_4, x_5	X							X				
x_4, x_5, x_6	X	X										
x_5, x_6, x_7								X				
x_6, x_7, x_8												
x_7, x_8, x_9										X		X
x_8, x_9, x_{10}												X
x_9, x_{10}, x_{11}						X	X					
x_{10}, x_{11}, x_{12}					X	X						
x_{11}, x_{12}, x_{13}					X							
x_{12}, x_1, x_2						X						
x_{12}, x_2, x_3							X					

*NC**2 Matrix (19 cut sets)

Consequently,

if $i=5$ in the combinatorial model
or $k=7$ in the state transition model
or $e=7$ in the links enumeration model
then the matrix $NC^{*}3$ is obtained.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
x_1, x_2, x_3, x_4					X	X	X	X				
x_2, x_3, x_4, x_5	X					X	X	X				
x_3, x_4, x_5, x_6	X	X					X	X				
x_4, x_5, x_6, x_7	X	X	X				X	X				
x_5, x_6, x_7, x_8	X	X	X	X					X	X	X	X
x_6, x_7, x_8, x_9										X	X	X
x_7, x_8, x_9, x_{10}	X	X	X	X	X	X					X	X
x_8, x_9, x_{10}, x_{11}					X	X	X					X
$x_9, x_{10}, x_{11}, x_{12}$					X	X	X	X				
$x_{10}, x_{11}, x_{12}, x_1$			X	X	X	X	X	X	X			X
x_{11}, x_{12}, x_1, x_2				X	X	X				X		
x_{12}, x_1, x_2, x_3						X	X					
x_{12}, x_2, x_3, x_4							X	X	X	X		

*NC**3 Matrix (62 cut sets)

However, after NC*3 no more matrices can be obtained for this configuration.

4.3. Multiple global buses

In this configuration (Fig. 1*f*), the number of nodes n ranges from 2 to n (max), where n (max) denotes an upper practical limit. Here a simple example in which $n = 3$ and $m = 4$ is illustrated. In this example $i = 2$ and $k = e = 2$ and the following matrix, GC*1, only can be obtained.

	x_1	x_2	x_3	x_4
x_1		X		
x_2				
x_3				X
x_4				

*GC**1 Matrix (2 cut sets)

Naturally, as n increases other matrices will be yielded.

4.4. Near-neighbour mesh

For this configuration (Fig. 1*g*), an example is considered in which $n = 6$ and $m = 7$. In this case $i = 2$ and $k = e = 5$, and only one matrix MC*1 is obtained as demonstrated.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_1		X				X	
x_2						X	
x_3				X	X		
x_4					X		
x_5							
x_6							
x_7							

*MC**1 Matrix (6 cut sets)

The case of $n = 6$ represents a configuration with a minimum number of nodes, because with $n < 6$ no near-neighbour mesh is possible.

In what follows procedure *ROMG*2 is presented to calculate the required system reliability. The algorithm is based on the following:

(*a*) the combinatorial model,
(*b*) the markovian model, or
(*c*) the links enumeration model.

(ii) Calculating the exact number of the configuration links using the corresponding matrices of cut sets.

For any interconnection the number of graph links will depend on the number of cut sets, C^*.

The algorithm is expressed by the following procedure.

```
Procedure ROMG2 (r, p, m, v)
 begin
   R:= 0
   read p, m, r, v                /*v is introduced to indicate the subgraphs
       k:= r−v;                     that correspond to the configuration cut
       x:= m!;                      sets */
       s:= p^k;
       d:= (1−p)^(m−k);
       while k ≤ m do
         begin
           read C*_k;
               y:= m−k;
               z:= y!;
               b:= k!;
               f:= x/(b*z);
               f:= f−C*_k;
               R_k:= f*s*d;
               R:= R+R_k;
               s:= s*p;
               d:= d/(1−p);
               k:= k+1
         end
       write R
 end
```

5. A SPANNING-TREE MODEL

This model provides a single algorithm that can be used for the configurations of both groups. The model is based on finding all the spanning trees of an interconnected network. The ith spanning tree T_i of an interconnection is a subgraph with $(n-1)$ branches and contains all the nodes n.[6,13] The first step is carried out by making use of the Cartesian product of $(n-1)$ vertex cut sets C_y, whose elements are the branches connected to any of the $(n-1)$ nodes of the interconnection. Accordingly, the resultant elements of the set of the Cartesian product will represent all the spanning trees of the graph under consideration. The second step is to interpret the Boolean algebraic statement of the first step as a function of probability expressions. A Boolean algebraic expression has a one-to-one correspondence with the probability expression, so that the Boolean terms are modified until they represent disjointing grouping S_i (disjoint). Actually, each S_i represents a system success. The details of the required computations for finding the probability expression and consequently the network reliability are expressed formally in procedure STG1G2, shown below. Among the graph spanning trees, choose a central tree T_0 to be a reference. A comparison of the ith tree T_i relative to T_0 yields the function F_i given by:

$$F_i = T_0 \cup T_1 \cup \ldots \cup T_{i-1}; \quad 1 \leqslant i \leqslant N-1 \tag{7}$$

The literals of T_i are assigned a value 1, that is substituted in any predecessor term in which they occur. Also from F_i, the sets of spanning trees N_j $(o \leqslant j \leqslant (m-n+1))$ that have the same number of F_i literals are determined. Since the system success S_i is defined as the event of having at least one workable spanning tree, S_i will be expressed by:

$$S_i = T_0 \cup T_i \cup \ldots \cup T_{N-1} \tag{8}$$

Consequently, the exclusive operator ξ[13] can be exploited to obtain the probability expression S_i (disjoint) such that:

$$S_i\,(\text{disjoint}) = T_0 \bigcup_{i=1}^{N-1} T_i \xi(F_i) \tag{9}$$

From equation (9), the system reliability R is obtained by substituting the branch's reliability, since S_i represents the success of the MIMD system.

```
Procedure STG1G2 (p, m, n, R)
  begin
    read m, n, p
      y := 1;                                  /* Cy is the yth vertex cut set that connect
                                                  any of (n-1) nodes with x's of branches,
                                                  and X_{y=1}^{n-1} represents the corresponding
                                                  Cartesian product that generates all
                                                  possible spanning trees T_0, T_1, T_2, ..., T_{N-1}
  while y <= n-1 do                               */
    begin
      T_i[x] := X_{y=1}^{n-1} C_y[x];          /* x denotes any branch in the graph while
                                                  X (X̄) expresses the logical success (failure)
        y := y+1                                  of a branch */

    end                                        /* To get F_i[X] the literals of T_i[X] are
      F_0[X] := T_0[X];                           assigned a value 1 and they are substituted
      T_i[X] := T_1[X];                           in the corresponding predecessor */
    while T_i[X] ¬ T_0[X] do
      Repeat                                   /* N represents the total number of
        F_i[X] := T_0[X] ∪ T_1[X] ∪ ... ∪ T_{i-1}[X];   spanning trees of the interconnection */
            i := i+1
                                               /* ξ is the exclusive operator. ξ(F_i[x̄]) is
        until i = N                               obtained from F_i[x] such that the X's of F_i
                                                  are changed to X̄'s and the summation is
      for i=1 to N do                             changed to multiplication */
          get ξ(F_i[X̄])

          z_0 ξ = X_{j=1}^{n-1} x_j;           /* x_j is the logical success of the jth
    for i=1 to N-1 do                             spanning tree and x̄_j denotes the product
      begin                                       of ξ(F_i[X̄]) */

        z_i := X_{j=1}^{n-1} x_j;
        z̄_i := X_{j=0}^{m-n+1} x̄_j
      end
    R := 0
    for i = 1 to N do                          /* S_i is the system success which is
      begin                                       eventually expressed as a function of X's
        S_i = z_0 ∪_{i=1}^{N-1} z_i * z̄_i;        and X̄'s */
        for g = 1 to m do
            begin                              /* X's and X̄'s of S_i are substituted by the
              X_g := p_g;                         corresponding probabilities */
              X̄_g := (1-p)_g
            end
          R := R + S_i
      end
    write R
  end
```

6. EVALUATION OF CONFIGURATIONS AND ALGORITHMS

Now a comparative study is presented in order to provide a reliability evaluation of the seven basic MIMD configurations.

The reliability of the MIMD systems has been determined and the corresponding results are reported in Table 1. The reliability characteristics of each configuration are pointed out in the following.

(1) The star and tree graphs have equal reliabilities so that:

R (star) $= R$ (tree) $= p^m$

Accordingly $\dfrac{\partial R}{\partial m}$ (star, tree) $= p^m \log p$

Since $\log p$ has a negative value, for these two configurations the system reliability decreases as the network size increases (Fig. 2). This fact represents the major restriction for implementing these networks in fault-tolerant applications.[16]

(2) For the ring, the reliability is given by:

R (ring) $= mp^{m-1} - (m-1)p^m$

and $\dfrac{\partial R}{\partial m}$ (ring) $= p^{m-1} + mp^{m-1}\log p - p^m - (m-1)p^m \log p$

Again, in this case the system reliability decreases with the increase of the number of nodes in the interconnection. As shown in Fig. 2, the rate of decrease of R (ring) with respect to n is considerably smaller than that

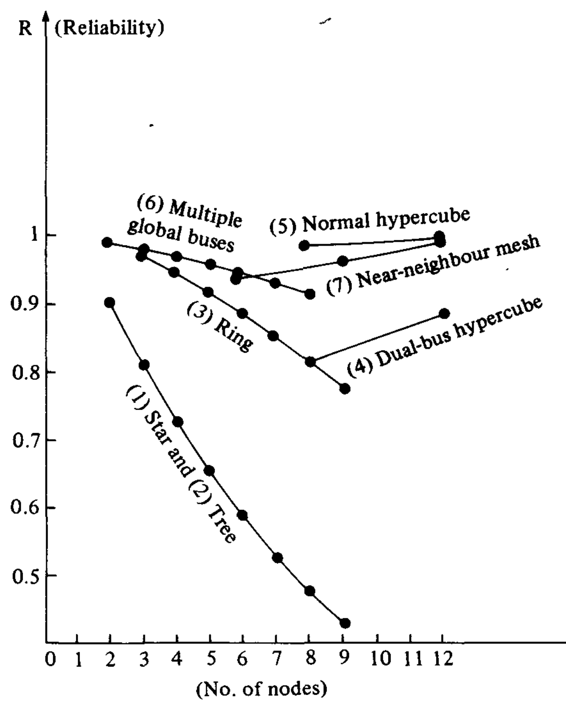


Fig. 2. Reliability characteristics ($p = 0.9$).

Table 1. Reliability evaluation of MIMD systems

Configuration... n	Star	Tree	Ring	Dual-bus hypercube	Normal hypercube	Global bus	Near-neighbour mesh
2	p	p	—	—	—	$2p-p^2$	—
3	p^2	p^2	$3p^2-2p^3$	—	—	$4p^2-4p^3+p^4$	—
4	p^3	p^3	$4p^3-3p^4$	—	—	$8p^3-12p^4+6p^5-p^6$	$4p^3-3p^4$
5	p^4	p^4	$5p^4-4p^5$	—	—	$35p^4-113p^5+153p^6$ $-99p^7+25p^8$	—
6	p^5	p^5	$6p^5-5p^6$	—	—	—	$15p^5-23p^6+9p^7$
7	p^6	p^6	$7p^6-6p^7$	—	—	—	—
8	p^7	p^7	$8p^7-7p^8$	$8p^7-7p^8$	$730P^7-3174p^8$ $+5588p^9-4954p^{10}$ $+2202p^{11}-391p^{12}$	—	—
9	p^8	p^8	$9p^8-8p^9$	—	—	—	$413p^8-1474p^9+2006p^{10}$ $-1230p^{11}+286p^{12}$
12	p^{11}	p^{11}	$12p^{11}-11p^{12}$	$253p^{11}-681p^{12}$ $+617p^{13}-188p^{14}$	—	—	—

Table 2. Time complexity (in operations)

Method Configuration	n	Combinatorial	State transition	Links Enumeration	Spanning tree
Star	3	11	10	29	1
	4	14	13	43	2
	5	17	16	59	3
	6	20	19	77	4
	7	23	22	97	5
	8	26	25	119	6
Tree	3	11	10	29	1
	5	17	16	59	3
	7	23	22	97	5
	9	29	28	143	7
Ring	3	24	22	43	8
	4	28	26	59	14
	8	44	42	143	48
Dual-bus hypercube	8	46	45	160	48
	12	112	111	358	32392
Normal hypercube	8	138	137	284	267549
	12	310	309	628	$\approx 10^8$
Global bus	2	22	21	34	3
	3	41	40	68	13
	4	64	63	110	14
	5	91	90	160	666
	8	196	195	358	$\approx 5\times 10^6$
Near-neighbour mesh	4	30	29	68	14
	6	56	55	134	137
	9	119	118	284	≈ 86000

of R (star, tree). Therefore, in large systems the ring interconnection will be potentially better than the star and the tree.

(3) The reliability of the multiple global buses configuration slightly decreases as the number of nodes increases. Therefore the attractiveness of this interconnection is limited.

(4) The negative of values of $(\partial R/\partial m)$ for star, tree and ring networks are justified by the fact that the ratio $(m/n) \leqslant 1$ for these configurations. For the multiple global buses $(m/n = 2)$, however, any node should have two connections with the system buses.

(5) For the normal hypercube, near-neighbour mesh and dual-bus hypercube configurations the system reliability increases as the number of nodes increases (Fig. 2). This result gives way to these interconnections as candidates for fault-tolerant computation. Actually the reason for such behaviour is that the ratio $m/n > 1$ for each one of these configurations.

(6) For a fixed number of nodes n, the most reliable MIMD configuration can be directly obtained. For example, if $n = 8$ the most reliable configuration is the normal hypercube, while the reliability of the dual-bus hypercube is reduced so that R (dual-bus hypercube) = R (ring). This result indicates the lack of rigorousness in reference[4] conclusions.

Table 3. Space complexity (in RAM variables)

Method Configuration	n	Combinatorial	State transition	Links Enumeration	Spanning tree
Star	3	—	—	—	18
	4	—	—	—	19
	5	—	—	—	20
	6	—	—	—	21
	7	14	14	13	22
	8	—	—	—	23
Tree	3	—	—	—	18
	5	—	—	—	20
	7	14	14	13	22
	9	—	—	—	24
Ring	3	—	—	—	24
	4	14	14	13	28
	8	—	—	—	44
Dual-bus hypercube	8	17	17	22	44
	12	19	19	28	783
Normal hypercube	8	21	21	26	2210
	12	25	25	34	$\approx 10^5$
Global bus	2	17	17	16	20
	3	18	18	18	27
	4	19	19	20	40
	5	20	20	22	122
	8	23	23	28	$\approx 6 \times 10^3$
Near-neighbour mesh	4	17	17	18	28
	6	18	18	21	63
	9	20	20	26	≈ 1260

(7) Tables 2 and 3 indicate the time (in operations) and space (in variables) needed for calculating the reliability of each configuration.[15] The significance of these tables lies in the fact that they report the compatibility of each reliability model. They emphasise that the spanning tree model, for example, is reasonable only for the simple configurations such as stars, trees or rings. However, for the second-group interconnections the time and space requirements of this model increase in a drastic manner because of manipulating a very large number of spanning trees.

7. CONCLUSION

The graph-theoretic approach has been employed to compute the reliability of seven basic MIMD configurations. In order to use a unified technique, these configurations are classified as two groups according to the nature and the method of determining the corresponding minimum cut sets which play a central role in reliability calculations. Consequently, the first group includes the simple star, tree and ring interconnections, while the second group contains the normal hypercube, near-neighbour mesh, dual-bus hypercube and multiple global buses.

For small-size MIMD systems ($n < 8$), the most reliable configuration is the multiple global buses. The ring has less reliability whilst the star and tree are the least reliable interconnections. When the graph size increases ($n \geqslant 8$) the configurations could be ordered according to their reliability as follows. (1) Normal hypercube; (2) near-neighbour mesh; (3) dual-bus hypercube that has $n \geqslant 12$; (4) multiple global buses; (5) ring; (6) star and tree.

Thus the hypercube configurations represent the best choice for fault-tolerant applications. In these applications the star and tree interconnections should be excluded.

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Announcements

25–27 APRIL 1989

First Announcement and Call for Papers
ISDN in Europe, The Hague, The Netherlands

Theme

This international conference will be oriented towards an examination of the impact of Integrated Services Digital Networks (ISDNs) on all aspects of computer communications in Europe. The 3-day conference is sponsored jointly by the International Council of Computer Communication (ICCC) and the International Federation for Information Processing (IFIP), and will be hosted by the Netherlands PTT.

'ISDN in Europe' will be held at a time when both national transitional plans and European policies for the introduction of ISDN will have matured further. Thus, the conference provides a strategic venue for users of international data communications and providers of value-added services and computer network facilities, whether public or private.

'ISDN on Europe' is planned to include coverage of the following subjects.

- Business needs for ISDN
- Computer applications appropriate to ISDN
- Access to ISDN (including broadband and mobile access)
- ISDN charging structures
- Telematic and value-added services based on ISDN
- International type approval and portability of customer equipment
- Managed data networks using ISDN
- Future roles of specialised networks in the light of ISDN
- ISDN regulations – European and national policies
- ISDN evolution, including broadband plans (B-ISDN)
- Traffic patterns in ISDNs
- Security and trans-border data flow aspects
- Integration of voice and data (local- and wide-area)
- Social and economic implications of ISDNs

The conference focus will be on developments which appear to be of special interest to professional users of the future integrated networks in Europe. The conference language is English.

Organisation

'ISDN in Europe' is being planned by an Organising Committee, advised by an International Advisory Committee mainly composed of European members of the IFIP Technical Committee on Data Communications (TC6) and European ICCC governors, and assisted by a Programme Committee.
More information about the conference can be obtained from:
The Secretariat, ISDN in Europe, Ms. Marijke Newman-van Aalderen, IBM Nederland N.V., Johan Huizingalaan 265, 1066 AP Amsterdam, The Netherlands. Tel. +31 20 513 35 61.

Conference contributions

The conference sessions will include both invited and contributed papers. Prospective authors are encouraged to notify the Programme Committee of any proposed contributions to 'ISDN in Europe', by contacting the Secretariat, with a 50-word abstract. Such contributions may fall into, but are certainly not limited to, the subject categories mentioned above in the conference theme. The Programme Committee will consider all notifications in its further planning of the conference sessions.

The following time schedule is foreseen for the production of papers:
Paper submitted for review before **15 September 1988.** Notification of authors by **15 November 1988.** Deadline for camera-ready manuscripts **15 January 1989.** Publication of Conference Proceedings **April 1989.**

Review and selection of papers will be the responsibility of the Programme Committee.

Information and notification of intent

If you are contemplating attending the conference on 'ISDN in Europe' to be held in The Hague in April 1989, please write to the Secretariat. This will ensure that you are kept up to date about the detailed programme, and it will help the Organising Committee in its planning of an event which is intended to address the major changes facing providers and users of data networks and computer communications facilities throughout Europe.

JANUARY 1989

Call for papers: Voxels

British Pattern Recognition Association

There is growing interest in problems concerning the processing of three-dimensional images in which the image data is expressed in voxels (three-dimensional pixels). Processing is carried out on three-dimensional data arrays and many of the familiar algorithms, such as those for finding edges or for filtering out noise, have to be reconsidered for use in a three-dimensional domain.

The British Pattern Recognition Association would like to hold a meeting to present latest results and ideas on this topic and is inviting suggestions for speakers at a meeting to be held in January 1989.

Further details can be obtained from:
Professor M. J. B. Duff, Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT (tel.: 01-380 7010), to whom suggestions for speakers should also be sent.