1. INTRODUCTION

There will be many applications for large databases that cannot be performed in an acceptable response time by current database systems. Since one way of improving performance significantly is through parallelism, the importance of parallel processing in database systems has been increasingly recognized. There have been several proposals in the past for parallel processing of database systems. These include two stage processing models, parallel query processing strategy for complex objects and machine architectures for parallel processing of database systems.

A multikey search query has many qualified records because it specifies some of the attributes and the rest of them are unspecified. Thus, data distribution among the parallel devices is important to enhance access concurrency for multikey search queries. Though there are some heuristics on data distribution for multikey accesses, range specification in a query has not been considered in those methods. In this paper we investigate optimal data distribution for multiattribute range queries in parallel processing file systems. We show for various types of multiattribute range queries that there are inherent limitations to achieve optimal distribution. The results show that optimal distribution does not exist in many cases even for files with two fields. We give sufficient conditions for the nonexistence of perfect optimal distribution for certain types of multiattribute range queries. We have also developed data distribution methods for several useful multiattribute range queries. Sufficient conditions for optimal distribution by these proposed data distribution methods are given. It will be shown that the proposed distribution methods are perfect optimal for certain types of multiattribute range queries, and strict optimal for a large class of multiattribute range queries.

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2. DEFINITIONS AND TERMINOLOGY

In this section we describe basic notations as well as relevant definitions and assumptions. In section 3 we show the inherent limitations for optimal data distribution for some types of range queries. Section 4 and 5 describe optimal data distribution methods for range queries. The optimality conditions for these methods are also given. Section 6 contains the concluding remarks.
where $S$ and $T$ are $t_1 + 1, \ldots, t_b = F_t$ for range queries.

**Definition.** A range query for files with $n$ fields is denoted by $\{A_1, A_2, \ldots, A_n\}$, where for each $i = 1, \ldots, n$, $A_i = *$, or $r(u, v)$, or $w$, where $u, v, w \in f_i$ and $u < v$.

**Definition.** When the number of range specified fields is $\alpha$ in a query $q$, query $q$ is called type $\alpha$ range query.

**Example 1.** Let $q_1$, $q_2$, and $q_3$ be queries in the file (DEPT, AGE, STATE) such that $q_1 = \{\text{Math,} r(20, 27), \text{Ohio}\}$, $q_2 = \{*, r(20, 27), *\}$ and $q_3 = \{\text{r(Math, Physics),} r(20, 27), *\}$. Therefore, $q_1$, $q_2$, and $q_3$ are type 1 range queries, and $q_1$ is type 2 range query. By the above definition a partial match query such as $\{\text{Math,} *, \text{Ohio}\}$ is a type 0 range query.

**Definition.** Let $R(q)$ be the set of buckets which satisfy the qualification for a range query $q$. The distribution method is called strict optimal for a range query $q$ in a given file system if each device has no more than $|R(q)|/M$ buckets. When the distribution method is strict optimal for all queries in type 0 through type $\alpha$ range queries in a given file system, it is called perfect optimal for type ($0-\alpha$) range queries in that file system.

**Definition.** Let a file consist of $n$ fields, and $S_i = 1, \ldots, n$, be a subset of $f_i$. Then, $M(S_1, \ldots, S_n)$ denotes a set of devices at which bucket $(b_1, \ldots, b_n)$ is stored, where $b_i \in S_i$. For convenience, when $S_i = \{s_i\}$, i.e., $S_i$ has a single element, we will use $S_i$ instead of $\{s_i\}$.

**Theorem 1.** Let a file consists of $n$ fields ($n \geq 2$) and the given number of devices is $M$, there does not exist a perfect optimal distribution method which satisfies the following conditions for certain types of range queries. We give optimal data distribution methods for various types of range queries in sections 4 and 5.

3. INHERENT LIMITATIONS OF PERFECT OPTIMAL DISTRIBUTION FOR RANGE QUERIES

In this section we show that perfect optimal distribution for certain types of range queries does not exist inherently in many cases.

**Theorem 1.** When a file consists of $n$ fields ($n \geq 2$) and the given number of devices is $M$, there does not exist a perfect optimal distribution method for range queries if (i) there are at least two fields $i$ and $j$, and two integers $a$ and $b$ such that $2 \leq a < F_i$, $3 \leq b < F_j$ and $ab = M$, or (ii) there are at least two fields $i$ and $j$ such that $F_i \geq 3$, $F_j \geq 3$ and $M = 4$.

**proof.** We will give the proof for (i) only. The proof for (ii) is straightforward based on that of (i). Now, it is sufficient to prove the theorem for files with only two fields. Assume perfect optimal distribution for type ($0-2$) range queries exists. Let $S = \{s_1, s_2, \ldots, s_k\}$ be a subset of $f_i$, where $a \geq 2$ and $s_1 > 0$, $s_a = s_1 + 1, \ldots, s_k = s_{a - 1} = 1, \ldots, t_b = t_{b - 1} + 1$ such that $ab = M$. In other words, $S$ and $T$ are a and $b$ consecutive values in field $i$ and $j$, respectively, where $|S \times T| = M$. Clearly, the sets $M(s_1, T), \ldots, M(s_{a - 1}, T)$ are mutually disjoint and $M(S, T) = Z_M$ (otherwise, the distribution is not strict optimal for query $q_i = \{s_1, s_a, r(t_1, t_2)\}$). Since the sets $M(s_1, T), \ldots, M(s_{a - 1}, T)$ are also mutually disjoint, and the size of cartesian product of these sets is $M$, $M(s_1, T) = M(s_2, T)$. We will show a contradiction for either case when the size of $T$ is even, or odd.

**Case 1.** $b$ is odd: Let $T = \{t_1, \ldots, t_{b - 1}\}$ and $T_0 = \{t_b, t_{b - 1}\}$. The sets $M(s_1, T_0)$ and $M(s_{a - 1}, T_0)$ are disjoint (otherwise, the distribution is not strict optimal for query $q_i = \{s_1, s_a, r(t_1, t_{b - 1})\}$). Thus, $M(s_1, T_0)$ is equal to $M(s_2, T_0)$, and $M(s_{a - 1}, T_0)$ is equal to $M(s_0, T_0)$. Now, let us consider range query $q_i = \{s_1, s_2, r(t_1, t_{b - 2})\}$. Suppose the bucket $\{s_1, s_2, r(t_1, t_{b - 2})\}$ is stored at device $m_0$, and the bucket $\{s_0, s_2, r(t_1, t_{b - 2})\}$ is stored at device $m_1$. Since $t_{b - 2} \in T$, and $M(s_1, T_0)$, $m_0$ is an element of $M(s_1, T_0)$, similarly, $m_1$ is an element of $M(s_1, T_0)$. Thus, device $m_0$ and $m_1$ have at least two qualified buckets for query $q_i$. However, the devices in

$$U_{k=1}^{a-1} M(s_k, t_{b-2+1}, \ldots, t_b)$$

do not have any qualified bucket for query $q_i$. This contradicts the assumption.

**Case 2.** $b$ is even: Let $T = \{t_1, \ldots, t_{b - 1}\}$ and $T_0 = \{t_{b - 1}, t_b\}$, where $b = b + 1/2$. The sets $M(s_1, T_0)$ and $M(s_{a - 1}, T_0)$ are disjoint (otherwise, the distribution is not strict optimal for query $q_i = \{s_1, s_a, r(t_1, t_{b - 1})\}$). However, $M(s_1, T_0) = M(s_2, T_0)$, and $M(s_{a - 1}, T_0)$ in $M(s_0, T_0)$ cannot be satisfied at the same time because $|T_0| > |T|/2$. This is a contradiction. □

**Theorem 2.** When a file consists of $n (n \geq 3)$ fields and the given number of devices is $M$, there does not exist a data distribution which is strict optimal for any range query that has at most two unspecified fields and any range query that has at most one unspecified field if (i) there are at least three fields $i, j, k$ such that $F_i \geq F_j < F_k$, and $M < F_k F_i \leq F_k F_j \leq 2M$, and $F_i \geq F_j + F_k/M$, and (ii) there are integers $b$ and $c$ such that $bF_i c = F_k M$.

**proof.** It is sufficient to prove the theorem for files with only three fields. Suppose the theorem is not true. Then, we have a distribution which is strict optimal for any query in the theorem. Let $cF_i c = M$ and $bF_i c = M$. This implies that $c < F_j < b < F_i$. Suppose bucket $\{0, 0, 0\}$ is stored at device $m_0$, bucket $\{0, 0, 1\}$ is stored at $m_{a - 1}$, and bucket $\{0, 0, F_i - 1\}$ is stored at $m_{a - 1}$.

Clearly, $m_0, m_1, \ldots, m_{a - 1}$ are all different. We will show that buckets $\{0, c, b\}$ and $\{0, 0, b\}$ should be stored at device $m_0$. At first, $M(0, 0, f_i) = M(0, c, f_i)$ (otherwise, the distribution is not strict optimal for at least one of the range queries $q_1 = [0, r(0, c - 1), *]$, $q_2 = [0, r(1, 0), *]$). Thus, the possible set of devices at which bucket $\{0, c, b\}$ can be stored is $\{m_0, \ldots, m_{a - 1}\}$. Bucket $\{0, c, b\}$ cannot be stored at any one of the devices $m_0, \ldots, m_{a - 1}$. Otherwise, for range query $q_i = [0, *, r(1, 1), *]$, at least one of the devices $m_0, \ldots, m_{a - 1}$ has two or more qualified buckets while there exist a device which does not have any qualified bucket. Bucket $\{0, c, b\}$ cannot also be stored at any one of the devices $m_{a - 1}, \ldots, m_{a - 1}$. Otherwise, the distribution is not strict optimal for range query $q_i = [0, *, r(1, 1), *]$. Thus, bundle $\{0, c, b\}$ should be stored at device $m_0$. Now, in order for the distribution to be strict optimal for queries $q_i = [0, c, 1, 0, *]$ and $q_i = [r(1, c), 0, *]$, $M(0, c, f_i)$ should be the same as $M(0, 0, f_i)$. Thus, the possible set of devices at which bucket $\{0, c, b\}$ can
be stored is \( \{m_0, \ldots, m_{F-1}\} \). Bucket \( \langle c, 0, b \rangle \) cannot be stored at any one of the devices \( m_1, \ldots, m_{F-1} \). Otherwise, the distribution is not strict optimal for at least one of the queries \( q_2 = [\ast, 0, r(1, b)] \) and \( q_3 = [\ast, 0, r(b, c)] \). Thus, bucket \( \langle c, 0, b \rangle \) should be stored at device \( m_b \). However, this bucket distribution is not strict optimal for query \( q_4 = [\ast, r(0, c), b] \) because device \( m_b \) has at least two qualified buckets for \( q_4 \) while device \( m_b \) does not have any qualified bucket for \( q_4 \). This is a contradiction.

It has been shown in Ref. 5 that when each field size and the given number of devices are power of 2, perfect optimal distribution for type 0 range queries is always possible if the number of fields whose sizes are less than the given number of devices, is no greater than four. The data distribution methods for type 0 range queries are also given in that paper. In Ref. 11, it has been shown that perfect optimal distribution does not exist for binary cartesian product files with \( n \) fields, if \( M \geq 4 \) and \( n \geq \log_2 M + 2 \). This implies that perfect optimal distribution for type 0 range queries is not always possible for files with four or more fields.

We have shown that there are inherent limitations to achieve perfect optimal distribution for range queries even for a file consisting of small number of fields. In the following sections, we present Fieldwise EXclusive-or (FX) distribution methods for range queries. These methods were first investigated in Ref. 5, where objective applications are type 0 range queries (i.e., partial match queries). In order to present FX distribution methods for range queries it is necessary to describe definitions and terminology for FX distribution methods.

4. DEFINITIONS AND TERMINOLOGY FOR FX DISTRIBUTION METHODS

This section gives definitions and terminology for FX distribution methods. For convenience we assume, from now on, that each field size and the given number of devices \( M \) are power of 2.

**Definition.** \((a_{m-1}\ldots a_0)_n\) is a binary notation of an integer, where \( a_i \) is a binary digit.

**Definition.** \([+]\) denotes exclusive-or operation between two bits. We will use the same notation \([+]\) to denote exclusive-or operation between integers and sets of integers as follows. When \( X = (a_{m-1}\ldots a_0)_b \) and \( Y = (b_{m-1}\ldots b_0)_b \) are two integers, \( X[+]Y = (a_{m-1}[+][b_{m-1}\ldots a_0[+]b_0)_b \). If \( X \) is an integer and \( Y = (y_1, \ldots, y_t) \) is a set of integers, \( X[+]Y \) is defined as \( \{X[+]y_1 \mid y_1 \in Y\} \). If both \( X = (x_1, \ldots, x_t) \) and \( Y = (y_1, \ldots, y_t) \) are sets of integers, \( X[+]Y \) is defined as \( \{x_1[+]y_1 \mid x_1 \in X, y_1 \in Y\} \).

4.1 The Basic FX distribution method

Let \( f_1 \times f_2 \times \ldots \times f_n \) be a set of all buckets. The Basic FX distribution method allocates bucket \( \langle J_1, \ldots, J_n \rangle \) into device \( T_m([+]) \), where \( T_m : N \rightarrow Z_m \) is a function which returns only the rightmost log \( M \) bits of domain values, and \( J_\ell \in f_\ell \) for \( \ell = 1, \ldots, n \). Note that \([+]|_m(J_\ell)\) is a shorthand notation for performing exclusive-or operation between sets of integers \( J_1, \ldots, J_m \).

**Example 2.** Table 1 shows the bucket distribution by the Basic FX distribution method, where \( f_1 = \{0, 1, 2, 3\} \), \( f_2 = \{0, 1, 2, 3\} \) and \( M = 4 \). In this figure, binary numbers are used for field values and decimal numbers are used for Device No. We can verify that the distribution in Table 1 is perfect optimal for type (0-1) range queries but is not perfect optimal for type (0-2) range queries. In fact, by Theorem 1 there does not exist a perfect optimal distribution for type (0-2) range queries for this file system.

<table>
<thead>
<tr>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>Device No.</th>
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<td>001</td>
<td>1</td>
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<td>010</td>
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<td>011</td>
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<td>011</td>
<td>011</td>
<td>0</td>
</tr>
</tbody>
</table>

4.2 Extended FX distribution methods

Extended FX distribution methods are defined by using the same principle of the Basic FX distribution method along with various field transformation techniques.

Let \( f_1 \times f_2 \times \ldots \times f_n \) be a set of all buckets. Extended FX distribution methods allocate bucket \( \langle J_1, \ldots, J_n \rangle \) into device \( T_m([+], f_\ell(J_\ell)) \), where:

(i) if \( |f_\ell| \geq M \), \( X_\ell \) is the identity function,

(ii) if \( |f_\ell| < M \), \( X_\ell \) is an element of the set of injective (one-to-one) functions whose domains are \( f_\ell \) and ranges are \( Z_m \).

\( X_\ell \) is called a field transformation function.

When \( X_\ell \) is the identity function for all \( \ell = 1, \ldots, n \), Extended FX distribution methods reduce to the Basic FX distribution method. Thus, the Basic FX distribution method is a special case of Extended FX distribution methods. From now on, we will call FX distribution methods instead of Extended FX distribution methods.

Several field transformation functions have been proposed and shown to be effective for type 0 range queries in Ref. 5. Thus, in this paper we will not discuss the cases when only type 0 range queries are involved.

5. FX DISTRIBUTION METHODS FOR RANGE QUERIES

In section 5.1 we describe the conditions for optimal distribution for various types of range queries when the Basic FX distribution method is used. In section 5.2 we will present field transformation functions developed for range queries. Following facts have been proved in Refs 5 and 10, and will be used frequently in proving theorems in this section.

**Fact 1.** For any type 0 range query which has at least one unspecified field whose size is greater than or equal
to M, all the devices have equal number of qualified buckets by the Basic FX distribution method.

**Fact 2.** The Basic FX distribution method is strict optimal for any type 0 range query which has at most one unspecified field.

**Fact 3.** For a nonnegative integer \( L \), let \( L = aw + b \), where \( w \) is a power of 2, and \( a, b \) are some nonnegative integers such that \( 0 \leq b \leq w - 1 \). When \( Z_{aw} = \{0, 1, \ldots, w-1\} \), \( Z_{aw+b} \) \( L \) = \( \{aw, aw+1, \ldots, (a+1)w-1\} \).

5.1 Optimal distribution for range queries by the Basic FX distribution method

This section gives conditions for optimal distribution for range queries by using the Basic FX distribution method.

**Theorem 3.** The Basic FX distribution method is strict optimal for any range query, for which there exists at least one unspecified field whose size is greater than or equal to the given number of devices M.

**Proof.** Since any range query is a set of type 0 range queries, and by Fact 1 we know that equal number of qualified buckets are distributed in all the devices for each type 0 range query, the proof immediately follows.

**Corollary 3.1.** The Basic FX distribution method is strict optimal for any range query, for which there exists at least one range specified field such that the specified range size is an integral multiple of the given number of devices M.

**Theorem 4.** The Basic FX distribution method is strict optimal for any range query, for which there is at most one range specified field and all the other fields are specified as single values.

The proof is almost the same as that of Fact 2 which is given in Ref. 5.

**Theorem 5.** When all the field sizes are greater than or equal to the given number of devices M, the Basic FX distribution method is perfect optimal for type (0-1) range queries.

**Proof.** When all the fields are specified as a single value, the proof is trivial. When at least one field is unspecified, the proof follows by Theorem 3. The only remaining case is when one field is specified as a range and all the other fields are specified as single values. The proof for this case follows by Theorem 4.

Theorems 3, 4 and 5 show general characteristics of exclusive-or operation for optimal distribution. However, the Basic FX distribution method does not give optimal distribution for many types of range queries. The following proposition gives the conditions for optimal distribution for these cases.

**Proposition 1.** Let \( q(f) = \{i_1, i_2, \ldots, i_k\} \) be the set of range specified or unspecified fields for a range query \( q \). Let \( S_i \) be the set of elements in the range when field \( i \) is range specified, or be \( f_i \) when field \( i \) is unspecified. Then, FX distribution methods are strict optimal for a range query \( q \), if there exists a set of fields \( \{i_1, i_2, \ldots, i_k\} \subseteq q(f) \) such that \( |S_{i_1} \times \cdots \times S_{i_k}| = \frac{M}{|f_1| \times \cdots \times |f_k|} \) is an integral multiple of M.

The proof for type 0 range queries is given in Refs 5 and 10. The proof of the proposition for type 1 or more range queries is straightforward from that of type 0 range queries.

The proposition says that optimal distribution for a subset of fields gives strict optimal distribution for many range queries in which those fields are unspecified or range specified. However, when the size of none of the fields is greater than or equal to M, the conditions given in Proposition 1 are not satisfied in the Basic FX distribution method. Thus, in the next section we propose field transformation techniques for the fields whose sizes are less than the given number of devices M. By the definition of field transformation function, it is easy to see that all the theorems that hold for the Basic FX distribution method also hold for FX distribution methods.

5.2 Field transformation functions for range queries

In this section we present field transformation techniques which can improve the performance over the Basic FX distribution method significantly. The following paragraph exemplifies the idea.

When \( f_1 = \{0, 1, 2, 3\} \), \( f_2 = \{0, 1, 2, 3\} \) and \( M = 8 \), the distribution by the Basic FX distribution method is not strict optimal for many type 0 and type 1 range queries (we can easily see that the distribution in Figure 1 is not strict optimal for many type 0 and type 1 range queries when \( M = 8 \)). Suppose \( X \) is an one-to-one mapping such that \( X(0) = 0 \), \( X(1) = 4 \), \( X(2) = 2 \), \( X(3) = 6 \). When each bucket \( \langle u, v \rangle \) in the file \( f_1 \times f_2 \) is stored at the device \( u + X(v) \), the distribution is strict optimal for any range query in which field 1 is not specified as a range (we do not give an example but it can be easily verified). Thus, our objective is to find such mapping, \( X \) in general, for the given file system. The following proposition gives sufficient condition for the mapping \( X \).

**Proposition 2.** Let a file consist of two fields, \( i, j \) whose sizes are less than the given number of devices M, and \( X \) be an injective function from N to \( Z_M \). Then, FX distribution method with applying function X to field \( j \) is strict optimal for any range query in which field \( i \) is not specified, if for any \( L \) consecutive values \( J_1, J_2, \ldots, J_L \) in field \( j \) such that \( L \leq M/F_j \), \( \{X(J_1), \ldots, X(J_L)\} = \{x \times F_j + c_x | x = 0, \ldots, L-1 \} \), and for all \( x, k \in N \), \( k < M/F_j \), \( 0 \leq c_x < F_j \) and \( k + 0 \ldots M-1 \).

The proof for the proposition is straightforward from Fact 3. Based on Proposition 2, we describe an idea of how to find the mapping \( X \) in the following paragraph.

Let \( f_1 = 8 \) and \( M = 16 \). The sizes of field \( i \) in which we are interested are 2, 4 and 8. Note that when the size of field \( i \) is greater than or equal to M, by Theorem 3 and Fact 2 the distribution is strict optimal for any range query in which field \( i \) is not specified. When \( F_i = 8 \), in order to satisfy the condition of the mapping \( X \) in Proposition 2, \( X(0), X(1), \ldots, X(7) \) should be in intervals \( [0, 4), [4, 8), [8, 12) \) and \( [12, 16) \) in turn. Since the sequence of intervals also has to satisfy the case when \( F_i = 8 \), the above sequence of intervals has to be reordered such as \( [0, 4), [4, 8), [8, 12] \) and \( [12, 16] \) in turn. Similarly, we have to reorder the sequences of intervals such as \( [0, 2), [2, 4), [4, 6], [6, 8), [8, 10], [10, 12], [12, 14], [14, 16] \) in turn.

This idea can be stated as follows: when \( (a_m, \ldots, a_0)_b \) is the binary notation of \( j \in J \), where \( m = \log_b M \);
Thus, the proof follows.

Methods with the proposed field transformation functions are perfect optimal for type (0—1) range queries when $F_1 F_2 > M$, and (ii) perfect optimal for type (0—2) range queries when $F_1 F_2 \leq M$.

**Theorem 6.** When there are only two fields $i, j$ such that $F_j$ is less than the given number of devices $M$, the FX distribution method with $I$-transformation for field $i$ and $UR$-transformation for field $j$ is (i) perfect optimal for type (0—1) range queries when $F_1 F_2 > M$, and (ii) perfect optimal for type (0—2) range queries when $F_1 F_2 \leq M$.

**Proof.** When one field is range specified and the other field is specified as a single value, the proof follows from Theorem 4. When field $i$ is range specified and field $j$ is unspecified, the proof follows by Lemma 6.2. When field $j$ is range specified and field $i$ is unspecified, the proof follows by Lemma 6.1 and Proposition 2. When there is no range specification, the proof is given in Ref. 5.

**Example 4.** Let $f_1 = \{0, 1, 2, 3\}$, $f_2 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $M = 8$. Table 2 shows the bucket distribution by the FX distribution method with $I(f_1)$ and $UR(f_2)$, where $UR(f_2) = \{0, 4, 2, 6\}$. We can verify that the distribution in Table 2 is strict optimal for any type 0 and type 1 range query.

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<th>$UR(f_2)$</th>
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</tbody>
</table>

Now, it is worth considering the following question: Do there exist three field transformation functions $X_1$, $X_2$, and $X_3$ such that for any two fields whose sizes are less than $M$, FX distribution methods with any two of $X_1$, $X_2$, and $X_3$ transformation functions are perfect optimal for type (0—1) range queries? Unfortunately, there do not exist such transformation functions because otherwise, it immediately contradicts Theorem 2. This implies that it is inevitable to have some restrictions for perfect optimal distribution for type (0—1) range queries when $I$ and $UM$-transformation, and $UR$ and $UM$-transformation are considered.

**Lemma 7.1.** When there are only two fields $i$ and $k$ such that $F_k$ is less than the given number of devices $M$, the FX distribution method with $I$-transformation for field $i$ and $UM$-transformation for field $k$ is strict optimal for any range query in which field $i$ is not range specified.

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The lemma is a direct consequence of Lemma 6.1 and Proposition 2.

**Lemma 7.2.** When there are only two fields $i$ and $k$ whose sizes are less than the given number of devices $M$ and $F_i \leq F_k$, the FX distribution method with $I$-transformation for field $i$ and $UM$-transformation for field $k$ is strict optimal for any range query in which field $k$ is not range specified.

**Proof** Let $d_{i} = M/F_i$. It is not difficult to see that for any two different $K_i$ and $K_j$ which are in the same interval of size $d_i$, $UM(f_i)\{+\} K_i$ and $UM(f_j)\{+\} K_j$ are disjoint. Thus, by Proposition 3 it is sufficient to show that for any $J \in f_i^t, UM(f_i)\{+\} J = UM(f_i)\{+\} (\{+\} \alpha d_{j})$, where $\alpha$ is any nonnegative integer such that $\alpha d_{j} < F_j$. Now,

$$UM(f_i)\{+\} \alpha d_{j} = UM(f_i)\{+\} (UR^{\{+\}}(UR^{\{+\}}(\alpha))) = UM(f_i)\{+\} (UR^{\{+\}}(\alpha) \mod d_{j})$$

The first equality holds because $UR^{\{+\}}(UR^{\{+\}}(\alpha)) = UR^{\{+\}}(\alpha)$, and $UR^{\{+\}}(\alpha) \mod d_{j} = 0$ and therefore the proof follows.

**Theorem 7.** When there are only two fields $i$ and $k$ whose sizes are less than the given number of devices $M$, the FX distribution method with $I$-transformation for field $i$ and UM-transformation for field $k$ is (i) strict optimal for any range query in which field $i$ is not range specified if $F_i > F_k$, and (ii) perfect optimal for type (0-1) range queries if $F_i \leq F_k$, and (iii) perfect optimal for type (0-2) range queries if $F_i F_k \leq M$.

The theorem is a direct consequence of Lemma 7.1 and Lemma 7.2.

**Theorem 8.** When there are only two fields $i$ and $k$ whose sizes are less than the given number of devices $M$, the FX distribution method with $I$-transformation for field $i$ and UM-transformation for field $k$ is (i) strict optimal for any range query in which field $k$ is not range specified if $F_i > F_k$, and (ii) strict optimal for any type 1 range query which has at most one unspecified field if $F_i = F_k$, and (iii) perfect optimal for type (0-2) range queries if $F_i F_k \leq M$.

The proof follows.

**Example 5.** Let $f_i = \{0, 1, 2, 3\}, f_k = \{0, 1, 2, 3\}$ and $M = 8$. Table 3 shows the bucket distribution by FX distribution with $UR(f_i)$ and $UM(f_k)$. Here, $UR(f_i) = \{0, 4, 2, 6\}, UM(f_k) = \{0, 5, 2, 7\}$ and Device No $= T_{M}(UR(J_i)\{+\} UM(J_k)), J_i \in f_i, J_k \in f_k$.

We can verify that the distribution is perfect optimal for type (0-1) range queries.

**Theorem 9.** When there are only three fields $i, j, k$ such that $F_i \leq F_j \leq F_k < M$, FX distribution methods can be always (i) perfect optimal for type 0 range queries, and (ii) strict optimal for any type 1 range query which has at most one unspecified field if $F_i = F_k$, or $F_i F_k \leq M$.

**Proof** The proof for (i) has been given in Ref. 5.

Thus, let us consider only the case (ii). When $F_i \leq F_k$, let field $i$ be $I$-transformed, field $j$ be $UR$-transformed and field $k$ be $UM$-transformed. When $F_i F_k \leq M$, let field $j$ be $UR$-transformed, field $j$ be $I$-transformed and field $k$ be $UM$-transformed. Then, for both cases, the theorem is a direct consequence of Theorem 6, Theorem 7 and Theorem 8.

Note that by Theorem 2 there does not exist a data distribution method which guarantees strict optimal distribution for all the range queries in Theorem 9 (i.e., all possible type 0 range queries and all the type 1 range queries which have at most one unspecified field) for every file with three fields.

We have shown that FX distribution methods along with various combinations of field transformation functions give optimal distribution for many types of range queries. Here, it should be emphasized that the scope of optimality is increased significantly by these field transformation techniques along with Proposition 1. This is because by Proposition 1 optimal distribution for a subset of the fields guarantees strict optimal distribution for many range queries in which those fields are range specified or unspecified. Though FX distribution methods presented in this paper do not give optimal distribution in all the cases, these methods are strict optimal for a large class of range queries.

**6. CONCLUSION**

We have investigated file distribution problems for parallel processing of multiatribute range queries. Optimal distribution methods as well as the existence and nonexistence of perfect optimality have been presented. We have shown that perfect optimal distribution for multiatribute range queries does not exist in many file systems. We have given sufficient conditions for the nonexistence of perfect optimal distribution for various types of range queries. We have also presented optimal data distribution methods, called FX distribution, for several useful types of range queries. These methods are based on exclusive-or operation along...
with various field transformation techniques. We have shown that FX distribution methods are perfect optimal for certain types of multiattribute range queries, and strict optimal for a large class of multiattribute range queries. However, there are many cases for which the existence of perfect optimal distribution is not known. We are currently investigating the existence of optimal distribution for these cases by extending FX distribution methods. This will require the development of more general transformation functions.

7. REFERENCES
