
Special Issue Editorial

The quantifier elimination problem for a given formal theory is that of devising an algorithm with the following specifications:

Input: A quantified formula.

Output: An equivalent quantifier-free formula.

A trivial example in the elementary theory of the real numbers is:

Input: $\forall x [x^2 + bx + c > 0]$

Output: $b^2 - 4c < 0$

Note that the output formula is equivalent to the input formula, but the quantifier is “eliminated”, hence the name *Quantifier Elimination*.

There are two main sources of motivation for tackling the quantifier elimination problem: the traditional motivation by the foundational questions of mathematics, and the more recent motivation by the computer scientific applications in various areas.

The motivation from mathematics is based on the fact that the “existence” of a quantifier elimination procedure often implies various other important properties about the theory under investigation.

On the other hand, the motivation from computer science is based on the observation that quantifier elimination provides a simple but expressive abstraction for various important application problems such as constraint logic programming, robot motion planning, geometric modeling, geometric theorem proving and discovery, termination proof of term rewriting systems, stability analysis of numerical methods for solving partial differential equations, approximation, optimization, etc.

Due to its importance to the foundations of mathematics and also to the computer scientific applications, intensive research has been done by both logicians and computer scientists. Recently the close connection between real algebraic geometry and quantifier elimination for the elementary theory of the reals became evident and various important new results have also been established by several algebraic geometers.

Obviously it is not possible to pack into one special issue the complete spectrum of the current research in quantifier elimination. Thus, it was decided to restrict the scope by two criteria:

- Algorithmic. Thus, the works addressing only the “existence” of quantifier elimination procedures were not considered. In fact, efforts were made to solicit papers dealing with “efficient” algorithms.
- Classical theories such as the elementary theories of the real numbers, the complex numbers, and the p-adics.

Under these restrictions, attempts were made to cover various aspects such as structural complexity (F. Cucker), general algorithms (J. Canny; D.P. Dubhashi), special algorithms for important fragments of the full theories, (J. Heintz, M.-F. Roy & P. Solerno; H. Hong; S. McCallum; R. Loos & V. Weispfenning; V. Chandru; J.-L. Imbert), important sub-algorithms (D. Manocha), and applications (R. Liska & S. Steinberg). Two seemingly “non-quantifier elimination” papers (J. Canny; M.J. González-López & T. Recio) have also been included since the algorithmic ideas used there are very closely related to (often the same as) those used in quantifier elimination, in particular for the elementary theory of the reals.

I hope that this issue stimulates further research in this challenging and important area, where significant results have been achieved in recent years, but much more remains to be done. In particular, I hope that this issue initiates a forum where the researchers in computer algebra, algebraic geometry, constraint logic programming, and numerical analysis interact with each other in order to further improve the practical efficiency and applicability of quantifier elimination algorithms.

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