A Revised Theory of Action and Time based on Intervals and Points

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This paper examines J. F. Allen's interval-based theory of action and time and the corresponding revisions suggested by A. P. Galton which have been proposed to accommodate the representation of facts concerning continuous change. Agreeing with Galton's argument that Allen's system needs revisions by means of diversifying the temporal ontology to include points, we show that Galton's determination to define time points in terms of the 'meeting places' of time intervals does not, as it stands, axiomatize points on the same footing as intervals, and hence that some problems still remain in these revisions. It is shown that it is necessary to revise the fundamental axioms about time itself so as to extend the abstract concept of time elements to include both intervals and points, and to extend the temporal relations between intervals to address points as well. We provide here a further revised theory which overcomes the problems in Allen's and Galton's systems. The revised system utilizes a new axiomatization of time, given previously by the authors, as the underlying temporal basis. A diversification of the range of properties/ occurrences over intervals and points is also proposed which may replace both Allen's and Galton's results.

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1. INTRODUCTION

The theory of temporal logic is an integral concern of philosophical inquiry reasoning with statements which have some temporal aspect, and has been recognized be of relevance to several distinct areas in computer science since the early 70s. In particular, researchers have found that understanding and treatment of time plays an increasingly important role in the domain of artificial intelligence. An early attempt at mechanizing part of the understanding of time within an artificial intelligence context is Bruce's *Chronos model* (6), for the analysis of tenses, time relations, and other references to time in natural language. Kahn and Gorry continued the work of Bruce in the development of their *time specialist* (12), endowed with the capacity to order temporal facts in three major ways: (i) relating events to *dates*; (ii) relating events to special *reference events*; (iii) relating events together into *before–after chains*. The time specialist can answer different types of questions such as: Did event X happen at time T? When did event X happen? What happened at time T. Gabbay (8) has presented an executable temporal logic called US logic which is comprised of first-order classical logic with the addition of the modal operators *since* and *until*, together with a fixed-point operator. The logic is based on the natural numbers as the flow of time and can be used for specification and the control of process behaviour in time. It is shown that US logic is fully expressive for a historical data model, in the same sense that first-order logic is for a non-temporal data model.

However, the most influential work as regards the application of temporal logics to artificial intelligence is that of McDermott and Dean (7,16), and of Allen and Hayes (2, 3, 4). McDermott's point-based first order temporal logic is proposed to provide a versatile 'common-sense' theory for temporal reasoning: reasoning about causality, reasoning about continuous change, and planning actions. In order to deal with incomplete relative temporal information such as '*Event A is before or after event B*', Allen introduces his temporal system as a framework for the naive treatment of two major subareas of artificial intelligence: natural language processing and problem solving. Instead of adopting time points, Allen takes intervals as the primitive temporal quantity, as being the natural means of human reference to time. Based on Bruce's 7 relationships [see (6)] between intervals, Allen introduced 9 (mutually exclusive) basic binary relations between any two intervals (1), extended later to 13 (2): EQUAL, BEFORE, MEETS, OVERLAPS, STARTS, STARTED-BY, DURING, CONTAINS, FINISHES, FINISHED-BY, OVERLAPPED-BY, MET-BY, AFTER. However, whereas Bruce's time intervals are constructed out of points, and hence, the relations over intervals are derived from the order between their greatest lower bounding point and least upper bounding point, Allen takes intervals as primitive objects and defines the temporal relations between intervals as primitive predicates as well. However, as shown in (4, 5), all Allen's 13 temporal relations may be indeed formally defined in terms of a single primitive relation: 'MEETS'.

The main objective of Allen's approach that takes time intervals as primitive rather than as any structure based on points is to bypass the problems posed by modelling intervals in terms of their ending-points. These are embraced by the question of whether these ending-points are in the interval or not. If intervals are all closed then adjacent intervals have ending-points in common. Hence, if adjacent intervals correspond to states of truth and falsehood of some property, there is a point at which the property is both true and false. Similarly, if intervals are all open, there will be points at which the truth or falsity of a property will be undefined. The solution in which intervals are all taken as semi-open (e.g. as in Maiocchi's *TSOS* where all intervals are closed at their left ending-points, and open at their right ending-points), so that they sit conveniently next to one another, seems arbitrary and unsatisfactory (2, 13). Firstly, this approach insists that every interval contains only a single ending-point, leaving the choice of which end of intervals should be open being arbitrary. Hence, any imposed rules such as left-closed & right-open would seem unjustifiable and artificial to some extent. This problem has been discussed by Allen repeatedly (1, 2, 3) and is cited as one of the prime motivations for adopting intervals as primitive time elements in his temporal logic. Secondly, the approach that takes all intervals as semi-open may offer a solution to some practical applications, but not to others. For instance, in the example modelling the process of a ball thrown vertically into the air, described in a previous paper (14), a single type (e.g. left-closed & right-open) is not adequate for the modelling purpose. In fact, for general treatment, four types of intervals, i.e., closed, open, left-closed & right-open, and left-open & right-closed, may need to be addressed (11).

However, as Galton shows in his critical examination of Allen's interval logic, Allen's theory of time is not adequate, as it stands, for reasoning about continuous change (10). Galton identifies the source of the problem in Allen's determination to base his theory on time intervals only, either excluding time points entirely, or relegating them to a subsidiary status within the theory. Galton suggests that the solution is to treat time points and time intervals on an equal footing. In addition, in order to accommodate the representation of facts concerning continuous change, Galton makes a significant distinction between two kinds of property, called *states of position* and *states of motion*, with respect to the logic of their temporal incidence, and diversifies the range of predicates specifying temporal location. This series of revisions is in fact a suggestive extension of Allen's theory of action and time that satisfies many temporal reasoning tasks in the domain of artificial intelligence. However, in Galton's revised theory, time points, and the corresponding relations between points and intervals, are, in fact, defined in terms of the 'meeting places' of time intervals (10). That is, time points are still relegated to a subsidiary status, and are not axiomatized on the same footing as intervals. We shall show that some problems still remain in Galton's revised theory, which may be avoided by adopting revised axioms about time itself so as to extend the abstract concept of time elements to include both intervals and points, and to extend the temporal relations between intervals to address points as well.

The objective of this paper is to provide a revised theory of properties and occurrences based on a new axiomatization of time, given previously by the authors. In this new time axiomatization, as Galton suggests, both points and intervals are characterized as primitive time elements on the same footing (14). In section 2, a brief presentation of the time theory assumed in Allen's logic is given. Section 3 illuminates some problems with Allen's approach to properties. We review Galton's revised theory related to properties in section 4, and point out its limitation and some remaining problems in section 5. Diversifications of the range of properties and occurrences over intervals and points are presented in section 6 and section 7, respectively, which may replace both Allen's and Galton's results. In section 8, by means of some critical examples, we illuminate the expressive power of the new system. Finally, section 9 concludes the paper.

2. TIME THEORY ASSUMED IN ALLEN'S LOGIC

Allen's temporal logic is specified as a framework to characterize properties and occurrences (3). Instead of adopting time points (or states which are associated with time points), Allen takes intervals as the primitive temporal quantity, as being the natural means of human reference to time.

In order to express temporal order over time intervals, Allen defines as primitive a set of 13 (mutually exclusive) basic binary relations between any two intervals (2): EQUAL, BEFORE, MEETS, OVERLAPS, STARTS, STARTED-BY, DURING, CONTAINS, FINISHES, FINISHED-BY, OVERLAPPED-BY, MET-BY, AFTER. However, these relationships were later formally defined in terms of the single primitive relation 'MEETS', which is axiomatized by means of a series of axioms that characterize the following assumptions, respectively (5):

- The 'place' where two intervals meet is unique and closely associated with the intervals;
- The meeting places are totally ordered, implying the linearity of the time system.
- Every interval has at least one neighbouring interval preceding it, and another succeeding.
- The time interval between any two meeting places is unique.
- If two meeting places are separated by a sequence of intervals, then there is an interval which connects these two meeting places.

The definition of the 13 temporal relations in terms of *meets* can be given by means of the following representation (4,5):

$\text{EQUAL}(i_1,i_2) \Leftrightarrow i_1 = i_2$,
$\text{BEFORE}(i_1,i_2) \Leftrightarrow \exists i(\text{MEETS}(i_1,i) \wedge \text{MEETS}(i,i_2))$,
$\text{OVERLAPS}(i_1,i_2) \Leftrightarrow \exists i,i',i''(i_1 = i' + i \wedge i_2 = i + i'')$,
$\text{STARTS}(i_1,i_2) \Leftrightarrow \exists i(i_2 = i_1 + i)$,

$\text{DURING}(i_1,i_2) \Leftrightarrow \exists i',i''(i_2 = i' + i_1 + i'')$,
$\text{FINISHES}(i_1,i_2) \Leftrightarrow \exists i(i_2 = i + i_1)$,
$\text{AFTER}(i_1,i_2) \Leftrightarrow \text{BEFORE}(i_2,i_1)$,
$\text{OVERLAPPED-BY}(i_1,i_2) \Leftrightarrow \text{OVERLAPS}(i_2,i_1)$,
$\text{STARTED-BY}(i_1,i_2) \Leftrightarrow \text{STARTS}(i_2,i_1)$,
$\text{CONTAINS}(i_1,i_2) \Leftrightarrow \text{DURING}(i_2,i_1)$,
$\text{FINISHED-BY}(i_1,i_2) \Leftrightarrow \text{FINISHES}(i_2,i_1)$,
$\text{MET-BY}(i_1,i_2) \Leftrightarrow \text{MEETS}(i_2,i_1)$,

where '$i_1 = i_2$' means that i_1 and i_2 represent the same time interval; and '$i_1 + i_2$' represents the ordered union of two adjacent intervals, i and j, which will always imply that MEETS(i_1, i_2).

Allen bases his logic on time intervals rather than time points, advancing the argument that the only actions that humans can identify are those with temporal extent (3). According to this, nothing can be true at a point since a point is not an entity at which things happen or are true (2). However, as Galton shows in his examination of Allen's interval logic (10), Allen's temporal logic is not adequate, as it stands, for reasoning correctly about continuous change. Galton attributes the source of this failing to Allen's determination either to exclude time points entirely, or to relegate them to a subsidiary status within the theory. In their later paper (5), Allen and Hayes define a point as the 'meeting place' of intervals and propose the concept of 'moments', i.e. very short intervals to characterize the times that some 'instant-like' events occupy. The constraint imposed on time moments is that, while intervals may be decomposable, moments are definitely atomic, although they have a 'temporal extent'. Additionally, moments are not allowed to meet other moments, that is, between any two time moments, there must be an interval.

An analytical review of Allen and Hayes' formal time theory of intervals, which may be decomposable or non-decomposable, has been given by the authors in (14). A series of revisions is also proposed there to overcome the problems posed by the corresponding limitation and inadequacy of the theory.

3. PROPERTIES IN ALLEN'S THEORY OF TIME AND ACTION

As Allen points out (3), one of the most important predicates in his typed first order logic is **HOLDS**, which asserts that a property holds (i.e. is true) during a time interval. For convenience of expression, Allen introduces the derived temporal relation, 'IN', which summarises the relationships in which one interval, i_1, is a proper subinterval of another interval, i_2:

$$\text{IN}(i_1,i_2) \Leftrightarrow \text{DURING}(i_1,i_2) \vee \text{STARTS}(i_1,i_2) \vee \text{FINISHES}(i_1,i_2)$$

The crucial characteristic of the **HOLDS** predicate is then defined in terms of the following axiom

$$\text{(H.1)}\quad \mathbf{HOLDS}(pro,i) \Leftrightarrow \forall i_1[\text{IN}(i_1,i) \Rightarrow \mathbf{HOLDS}(pro,i_1)]$$

which states that a property holds over an interval iff it holds over all its proper subintervals.

Allen additionally follows this axiom with the following slightly different one:

$$\text{(H.2)}\quad \mathbf{HOLDS}(pro,i) \Leftrightarrow \forall i_1\{\text{IN}(i_1,i) \Rightarrow \exists i_2[\text{IN}(i_2,i_1) \wedge \mathbf{HOLDS}(pro,i_2)]\}$$

and shows that, under an additional assumption which states that for every interval i', there exists an interval i'', such that IN(i'',i'), axiom (H.1) can be derived from (H.2).

Allen's attitude to time points changed somewhat over the years. Originally, he argues for abolishing time points altogether since, first, they do not appear to be necessary; second, instantaneous time points will present difficulties with the semantics of the temporal logic (2, 3). However, in the later paper (5), Allen and Hayes introduce the idea of very short intervals, termed moments, to be the times that some 'instant-like' events occupy.

One limitation to the definition of properties by means of axioms H.1, H.2 is that they do not treat time moments satisfactorily, for any property, *pro*, can be shown to hold over any time moment unconditionally from either axiom, since there is not any proper subinterval within any given time moment.

On the other hand, if all time elements are taken as infinitely decomposable intervals, Galton has shown in (10) that this will lead to the inadequacies for reasoning about continuous change.

Another limitation of Allen's predicate, **HOLDS**, axiomatized by (H.1)/(H.2), is that it characterizes only one way of ascribing properties to times, namely to assert that a property holds throughout an interval, which seems too restrictive, and represents only one category in the taxonomy of properties introduced by Shoham (17,18), which is based on a point-based interval logic. Additionally, it is interesting to note that, as shown by Galton in terms of his two different formulations, there are some problems with Allen's property-negation which is characterized by the following axiom

$$\text{(H.4)}\quad \mathbf{HOLDS}[\text{not}(pro),i] \Leftrightarrow \forall i_1\{\text{IN}(i_1,i) \Rightarrow \text{not}[\mathbf{HOLDS}(pro,i_1)]\}$$

4. GALTON'S REVISED THEORY RELATED TO PROPERTIES

In order to overcome the inadequacy of Allen's theory of action and time, Galton proposes a series of revisions which address time points in the theory as well as time intervals, and diversify the range of predicates assigning temporal locations to properties and occurrences.

Galton's theory of points and intervals is built up by means of adding two extra relationships between points and intervals, as the extension to Allen's temporal relations between intervals. Rejecting the question whether or not a given point is part of, or a member of a given interval, while retaining the idea of there being a point at the meeting place where two intervals meet, Galton introduces two additional temporal relations to Allen's time theory: First, the point where two intervals meet each other is said to fall 'WITHIN' the ordered union of these two intervals, and second, the same point is said to 'LIMIT' both of these two intervals, the former at its end, the latter at its beginning. Galton uses notions, WITHIN(p,i) and LIMITS(p,i), to represent that a point p falls 'within', and 'limits' an interval i, respectively.

Additionally, to develop his revised logic, Galton imposes some required rules (10) which may be represented as below:

(I.1) $\forall i \in I \exists p \in P$[WITHIN(p,i)],
(I.2) WITHIN(p,i) $\wedge$ IN(i,i_1) $\Rightarrow$ WITHIN(p,i_1),
(I.3) WITHIN(p,i_1) $\wedge$ WITHIN(p,i_2) $\Rightarrow \exists i_3 \in I$[IN($i_3,i_1$) $\wedge$ IN(i_3,i_2)],
(I.4) WITHIN(p,i_1) $\wedge$ LIMITS(p,i_2) $\Rightarrow \exists i_3 \in I$[IN($i_3,i_1$) $\wedge$ IN(i_3,i_2)],

where the predicate, IN, is as same as that defined in section 3, and I and P represent the set of time intervals, and time points, respectively.

Intuitively, it is easy to infer that the two relations, WITHIN and LIMITS, are exclusive to each other, from their definitions.

Whereas Allen recognises only one way of ascribing properties to times, namely to assert that a property holds throughout an interval, Galton introduces the following three forms:

HOLDS-ON(pro, i),
HOLDS-IN(pro, i),
HOLDS-AT(pro, p), where $i \in I$ and $p \in P$,

for three types of statement: a property *pro* holds throughout an interval i, holds during i (i.e. at some time during an interval, not necessary throughout all of it), holds at a point p, respectively.

Commenting that the problems with Allen' system can all be traced to the assumption that all properties should receive a uniform treatment with respect to the logic of their temporal incidence, Galton proposes one of his revisions by distinguishing sharply between two kinds of properties, namely *states of position* and *states of motion*, which have different temporal logics: States of position can hold at isolated points; and if a state of position holds throughout an interval, then it must hold at the limits of that interval, e.g. a body's being in particular position, or moving at a particular speed or in a particular direction. States of motion cannot hold at isolated points; if a state of motion holds at a point then it must hold throughout some interval within which that point falls, e.g. a body being at rest or in motion. In terms of the above classes of properties, Galton characterizes the formal constraints imposed on states of position (SP) and states of motion (SM) by the following axioms:

(SP) $\forall i \in I$[WITHIN(p,i) $\Rightarrow$ **HOLDS-IN**(*pro*,i)] $\Rightarrow$ **HOLDS-AT**(*pro*,p),

(SM) **HOLDS-AT**(*pro*,p) $\Rightarrow \exists i \in I$[WITHIN(p,i) $\wedge$ **HOLDS-ON**(*pro*,i)],

respectively (10).

Additionally, Galton lists a series of theorems which can be derived from the above axiomatization, some of them, i.e. (T1)–(T.10) hold for general properties, regardless of whether they are states of position or states of motion, others, i.e. (T.11P)–(T.15P) and (T.11M)–(T.15.M) hold so long as *pro* is a state of position, and a state of motion, respectively (10).

5. PROBLEMS WITH GALTON'S REVISED THEORY

The objective of Galton's revised theory is to accommodate the representation of facts concerning continuous change by means of addressing time points as well as time intervals in the system. However, as in Allen and Hayes's approach (4, 5), Galton defines time points as the meeting places of time intervals. Hence, from the view of the abstract axiomatization about time itself, time points are still relegated to a subsidiary status, not really treated on the same footing as time intervals. Additionally, to develop his revised logic, Galton imposes a very strict rule, that is, (I.1) (see section 4), which states that for any time interval, there exists a point which falls within this interval. It is easy to see from Galton's definition of time points, rule (I.1) implies that, for any time interval i, it can be decomposed to two proper subintervals i_1 and i_2, such that: MEETS(i_1,i_2). Further, it is straightforward to infer that any time interval is required to be infinitely decomposable. Hence, Galton's revised axiomatization definitely excludes the special time intervals that are non-decomposable, namely moments, in Allen and Hayes' theory (4, 5). This limitation is perhaps not too serious, since Allen and Hayes' conception of time moments are in fact introduced to characterized the times that some 'instant-like' events occupy (although time moments still have positive duration). We may simply utilize time points to play the role of moments.

However, it is the following problems with Galton's revisions which we shall show require revisions to the fundamental axioms about time itself so as to extend the abstract concept of time elements to include both intervals and points, and the temporal relations between intervals to address points as well.

Let $state_1$ and $state_2$ be two opposite states [i.e. $state_1 \Leftrightarrow$ not($state_2$)], that hold throughout intervals i_1 and i_2, respectively, where MEETS(i_1,i_2). If we use p to denote the point at which i_1 meets i_2, from the definition of 'LIMITS' we get:

LIMITS(p,i_1) $\wedge$ LIMITS(p,i_2).

According to Galton's classification of states, there are four possible cases:

(a) both $state_1$ and $state_2$ are states of motion,
(b) both $state_1$ and $state_2$ are states of position,

(c) $state_1$ is a state of position and $state_2$ is a state of motion,
(d) $state_1$ is a state of motion and $state_2$ is a state of position.

In case (a), if we assume that $state_1$ holds at point p, then it must hold throughout some interval i′ such that WITHIN(p,i′). Hence, together with MEETS(i_1,i_2), LIMITS(p,i_1) and LIMITS(p,i_2), we can infer that OVERLAPS(i′,i_2). Hence, both stats, $state_1$ and $state_2$, which are opposite to each other, will hold throughout an interval which is a common subinterval of both i′ and i_2. This is obviously an unsatisfactory result.

Similarly, if $state_2$ holds at point p, then it must hold throughout some interval i″ such that WITHIN(p,i″). Hence, together with MEETS(i_1,i_2), LIMITS(p,i_1) and LIMITS(p,i_2), we can infer that OVERLAPS(i_1,i″). Again, both $state_1$ and $state_2$ will hold throughout the common subinterval of i_1 and i″.

Hence, the above proof shows that neither $state_1$ nor $state_2$ will hold at the point p. This is obviously an unacceptable result to the logic.

Similarly, for case (b), if we again use p to denote the point at which i_1 meets i_2, then by Galton's definition of a state of position, $state_1$ must hold at point p, which is one of i_1's limits, since $state_1$ holds throughout interval i_1; similarly, since $state_2$ holds throughout interval i_2, it must holds at p as well, which is also one of i_2's limits. Hence both $state_1$ and $state_2$ hold at point p. Obviously, this is absurd.

For the remaining two cases, (c) and (d), the choice between which of them applies seems to be arbitrary and unresolved. For example, consider the following two states

$state_{yes}$: the car belongs to John,
$state_{no}$: the car does not belong to John,

which apply before and after John sells a car. Which should be considered as a state of position, and which should be addressed as a state of motion?

In fact, it is interesting to note that, in Galton's paper, it is not explicitly expressed whether states of position and states of motion are all the possible kinds of properties or not. However, it is not obvious what other kinds of property will be needed to avoid the problem outlined above.

A further problem arises in connection with assignment of properties to time intervals. Noticing that it is necessary to extend Allen's single way of ascribing properties to times, namely to assert that a property holds throughout an interval, Galton introduces three different ways. For the initial, general treatment, he takes the locution **HOLDS-AT** (relating to time points) as primitive, and defines the other two, **HOLDS-IN** and **HOLDS-ON**, in terms of **HOLDS-AT** as below:

(D.1) **HOLDS-IN**(*pro*, i) $\Leftrightarrow \exists p \in P$[WITHIN(p,i) $\wedge$ **HOLDS-AT**(*pro*, p)],

(D.2) **HOLDS-ON**(*pro*, i) $\Leftrightarrow \forall p \in P$[WITHIN(p,i) $\Rightarrow$ **HOLDS-AT**(*pro*, p)].

Later in his paper (10), Galton shows that, for states of position, **HOLDS-IN** may be taken as primitive instead of **HOLDS-AT** by means of the following theorems for states of position:

(T.14P) **HOLDS-AT**(*pro*, p) $\Leftrightarrow \forall i \in I$[WITHIN(p,i) $\Rightarrow$ **HOLDS-IN**(*pro*, i)]
(T.15P) **HOLDS-ON**(*pro*, i) $\Leftrightarrow \forall i' \in I$[IN(i′,i) $\Rightarrow$ **HOLDS-IN**(*pro*, i′)] .

and, for states of motion, **HOLDS-ON** may be taken as primitive instead of **HOLDS-AT**, or **HOLD-IN**, by means of:

(T.14M) **HOLDS-AT**(*pro*, p) $\Leftrightarrow \exists i \in I$[WITHIN(p,i) $\wedge$ **HOLDS-ON**(*pro*, i)]
(T.15M) **HOLDS-IN**(*pro*, i) $\Leftrightarrow \exists i' \in I$[IN(i′,i) $\Rightarrow$ **HOLDS-ON**(*pro*, i′)]

However, consider the example of a moving arrow: let pro_1 represent the property that the arrow does *not change position*, and pro_2 represent the property that the arrow *changes position*, then

$\forall p \in P$[**HOLDS-AT**(pro_1, p)]

hence, by (D.2), we get that for any time interval,

HOLDS-ON(pro_1, i)

That is, the property, *not change position*, will hold **throughout** any time interval. This seems contrary to both human intuition and the corresponding axiomatization. On the other hand, for interval i, as well as any subinterval of i, we intuitively have that the property that the arrow *changes position* holds during interval i, that is:

HOLD-IN(pro_2, i)

However, by (D.1) we will get that there exists a point p within the interval i, such that

HOLDS-AT(pro_2, p)

that is, at this point the arrow *changes position*, which is again contrary to our assumptions.

The source of the above problems is indeed in the determination to define time points in terms of the meeting places of time intervals, and define the corresponding types of predicates ascribing properties to times, either according points conceptual priority over intervals, or regarding intervals as conceptually prior to points.

6. A REVISED THEORY BASED ON INTERVALS AND POINTS

In the light of those discussions given in the above, we propose in this section a further revised theory which utilizes a new axiomatization of time, given previously by the authors (14), which treats intervals and points on an equal conceptual basis. However, whereas Allen's interval-based logic and Galton's revised theory are set up as frameworks on which to hang assertions about the instantiation in time of *properties*, and *occurrences*, which are subdivided by Allen into *processes* and *events*, in this paper we shall concentrate on issues relating to properties. Occurrences, processes and events will be briefly discussed in the next section.

Based on the general time axiomatization given by the authors in the previous paper (14), the time model employed here addresses points and intervals on an equal footing by taking both intervals and points as time-elements, rather than defining points in terms of intervals, or constructing intervals out of points. Excepting the assumption that the duration of an interval is positive while the duration of a point is zero, the differentiating property between intervals and points which is proposed here is that although intervals may meet/(be met-by) points or other intervals, points are not allowed to meet/(be met-by) other points, although they must meet/(be met-by) other intervals. This characteristic is based on the intuition that between any two time-points, there is a time interval. In what follows, we give a brief outline of the main features of the general time axiomatization.

We use T to denote a nonempty set of time-elements, and *dur* to denote a duration assignment function from T to R_0^+, the set of non-negative real numbers. A time-element, t, is called a (time) interval if $dur(\mathrm{t}) > 0$, otherwise, t is called a (time) point. According to this classification, the set of time-elements, T, may be expressed as $T = I \cup P$, where I is the set of intervals, and P is the set of points (see above section).

The primitive order over general time elements is a temporal predicate termed 'meets', which is axiomatized by the following axioms:

$$\langle \mathrm{A1}\rangle\ \forall t_1,t_2,t_3,t_4 \in T[\mathrm{MEETS}(t_1,t_2) \wedge \mathrm{MEETS}(t_1,t_3) \wedge \mathrm{MEETS}(t_4,t_2) \Rightarrow \mathrm{MEETS}(t_4,t_3)]$$

$$\langle \mathrm{A2}\rangle\ \forall t \in T \exists t',t'' \in T[\mathrm{MEETS}(t',t) \wedge \mathrm{MEETS}(t,t'')]$$

$$\langle \mathrm{A3}\rangle\ \forall t_1,t_2 \in T\{\exists t',t'' \in T[\mathrm{MEETS}(t',t_1) \wedge \mathrm{MEETS}(t_1,t'') \wedge \mathrm{MEETS}(t',t_2) \wedge \mathrm{MEETS}(t_2,t'')] \Rightarrow t_1 = t_2\}$$

$$\langle \mathrm{A4}\rangle\ \forall t_1,t_2 \in T\{\mathrm{MEETS}(t_1,t_2) \Rightarrow \exists t \in T \forall t',t'' \in T[\mathrm{MEETS}(t',t_1) \wedge \mathrm{MEETS}(t_2,t'') \Rightarrow \mathrm{MEETS}(t',t) \wedge \mathrm{MEETS}(t,t'')]\}$$

$$\langle \mathrm{A5}\rangle\ \forall t_1,t_2 \in T[\mathrm{MEETS}(t_1,t_2) \Rightarrow t_1 \in I \vee t_2 \in I]$$

$$\langle \mathrm{A6}\rangle\ \forall t_1,t_2 \in T[\mathrm{MEETS}(t_1,t_2) \Rightarrow dur(t_1 \oplus t_2) = dur(t_1) + dur(t_2)]$$

where $t_1 \oplus t_2$ denotes the ordered union interval from t_1 and t_2, such that $\mathrm{MEETS}(t_1,t_2)$.

It has been proved that, in terms of the primitive relation 'meets', there are totally 30 possible temporal relations over time elements, which can be classified into the following four groups (14):

Point–Point:
{EQUAL, BEFORE, AFTER}
which relate points to other points;

Interval–Interval:
{EQUAL, BEFORE, MEETS, OVERLAPS, STARTS, DURING, FINISHES, FINISHED-BY, CONTAINS, STARTED-BY, OVERLAPPED-BY, MET-BY, AFTER}
which relate intervals to intervals;

Point–Interval:
{BEFORE, MEETS, STARTS, DURING, FINISHES, MET-BY, AFTER}
which relate points to intervals;

Interval–Point:
{BEFORE, MEETS, FINISHED-BY, CONTAINS, STARTED-BY, MET-BY, AFTER}
which relate intervals to points.

In what follows, our revised theory of action and time shall be based on this new time specification.

Whereas Allen recognizes only one way, and Galton introduces three different ways of ascribing properties to times (see section 3 and section 4), in our revised theory, we shall take the following single form

hold-for(*pro*, t), $\mathrm{t} \in T$

as the primitive type of statement, presenting that for time element t, the property *pro* holds.

N.B. Here, in the form: **hold-for**(*pro*, t), when t is a variable, it refers to a general time element which may be either an interval or a point. To determine whether the time element refers to an interval or to a point, according to the definition given above, duration knowledge is needed. For instance, if we want state exactly that a property, pro_1, holds for a point, p, we may express it as:

$dur(\mathrm{p}) = 0$,
hold-for(pro_1, p).

Similarly, the knowledge that a property, pro_2, holds for an interval, i, may be expressed as:

$dur(\mathrm{i}) > 0$,
hold-for(pro_2, i).

It is important to note that the predicate **hold-for** does not assume homogeneity or any other connection between a property holding for a time element and its holding for any substructure of the time element.

For Allen's system, if we limit the set of time elements, T, to the set of time intervals I (i.e. for any i in I, $dur(\mathrm{i}) > \mathrm{o}$), we may simply replace Allen's (H.1) and (H.2) by means of the following axioms:

$$\mathbf{HOLDS}(pro, i) \Leftrightarrow \forall i' \in I[\mathrm{IN}(i',i) \Rightarrow \mathbf{hold\text{-}for}(pro, i')],$$

$$\mathbf{HOLDS}(pro,i) \Leftrightarrow \forall i_1 \in I\{\mathrm{IN}(i_1,i) \Rightarrow \exists i_2 \in I[\mathrm{IN}(i_2,i_1) \wedge \mathbf{hold\text{-}for}(pro,i_2)]\}$$

respectively.

Similarly, let p be a point [i.e. $dur(\mathrm{p}) = 0$], and i an interval [i.e. $dur(i) > 0$], we may simply define Galton's form **HOLDS-AT**(*pro*, p) as:

$$\text{HOLDS-AT}(pro, p) \Leftrightarrow \textbf{hold-for}(pro, p),$$

and replay his (D.1) and (D.2) by means of:

$$\text{HOLDS-IN}(pro, i) \Leftrightarrow \exists p \in P[\text{DURING}(p,i) \wedge \textbf{hold-for}(pro, p)],$$
$$\text{HOLDS-ON}(pro, i) \Leftrightarrow \forall p \in P[\text{DURING}(p,i) \Rightarrow \textbf{hold-for}(pro, p)]$$

respectively.

In fact, we may also give definitions characterizing that a property holds in a time element, and holds throughout a time element. However, since time elements may now be intervals or points, we must make sure these definitions are well-defined for general treatment. First, for convenience of representation, we need to extend Allen's predicates 'IN' to include the 'EQUAL' relation. We define:

$$\text{SUB-ELEMENT}(t_1, t_2) \Leftrightarrow \text{EQUAL}(t_1,t_2) \vee \text{DURING}(t_1,t_2) \vee \text{STARTS}(t_1,t_2) \vee \text{FINISHES}(t_1,t_2)$$

where $t_1,t_2 \in T$, and EQUAL, DURING, STARTS and FINISHES belong to the groups of temporal relations classified above.

The difference between Allen's predicate 'IN' and our 'SUB-ELEMENT' is that, while in Allen's system, $\text{IN}(i_1, i_2)$ summarizes the relationship that interval i_1 is a proper subinterval of interval i_2, our notation $\text{SUB-ELEMENT}(t_1, t_2)$ allows time element t_1 to be t_2 itself.

The following axiom defines what it is for a property to hold in a time element:

⟨d.1⟩ $\textbf{hold-in}(pro, t) \Leftrightarrow \exists t' \in T[\text{SUB-ELEMENT}(t', t) \wedge \textbf{hold-for}(pro, t')]$,

that is, a property is said to hold in a time element t, iff there is at least one sub-element of t for which the property holds.

Similarly, for a property to hold throughout a time element is that for any sub-element of the time element, including the whole element itself, the property holds.

⟨d.1⟩ $\textbf{hold-on}(pro, t) \Leftrightarrow \forall t' \in T[\text{SUB-ELEMENT}(t', t) \Rightarrow \textbf{hold-for}(pro, t')]$.

N.B. The above definitions overcome the problem with Allen's axiom (H.1)/(H.2) for the case in which the addressed time element is non-decomposable, i.e. a *moment* (see section 3), and therefore is well-defined for both intervals/moments and points. Additionally, time intervals and points are addressed here on the same footing, there is no necessary connection between the a property holding for intervals and its holding for points, while this connection is definitely axiomatized in Galton's theory.

Given the above definitions of **hold-in** and **hold-on**, by making use of our classification of temporal relations over intervals and points, we can straightforwardly prove the following theorems, which are in fact very similar to those (T.1–T.5) given by Galton in (10):

⟨t.1⟩ $\textbf{hold-in}(pro, t) \wedge \text{SUB-ELEMENT}(t, t') \Rightarrow \textbf{hold-in}(pro, t')$

which says that if a property holds in some time element t, then it holds in any time element of which t is a sub-element.

⟨t.2⟩ $\textbf{hold-on}(pro, t) \wedge \text{SUB-ELEMENT}(t', t) \Rightarrow \textbf{hold-on}(pro, t')$

which says that if a property holds throughout t, then it holds throughout every sub-element of t.

⟨t.3⟩ $\textbf{hold-on}(pro, t) \Rightarrow \textbf{hold-in}(pro, t)$

which says that if a property holds throughout t, then it holds in t.

⟨t.4⟩ $\textbf{hold-on}(pro, t) \Rightarrow \forall t' \in T[\text{SUB-ELEMENT}(t', t) \Rightarrow \textbf{hold-in}(pro, t'))$

which says that if a property holds throughout t, then it holds in every sub-element of t.

⟨t.5⟩ $\exists t \in T[\text{SUB-ELEMENT}(t, t') \wedge \textbf{hold-on}(pro, t)] \Rightarrow \textbf{hold-in}(pro, t')$

which says that if a property holds throughout some sub-element of t, then it holds in t.

N.B. Whereas Galton gives more theorems relating to his **HOLDS-AT** for points, that is (T.6) and (T.7), in our revised system, they are the same as ⟨t.5⟩ and ⟨t.2⟩, respectively, since points are treated now on the same footing as intervals. Examples relating to these theorems may be found in Galton's paper (10).

Similarly, we can define the negation of a property as: If properties pro_1 and pro_2 satisfy:

⟨d.3⟩ $\forall t(\textbf{hold-for}(pro_1, t) \Leftrightarrow \neg\textbf{hold-for}(pro_2, t))$

then we say property pro_2 is the negation of property pro_1, and write it as: $pro_2 = \text{not}(pro_1)$.

From this definition, it is easy to prove the following theorems:

⟨t.6⟩ $\textbf{hold-on}[\text{not}(pro), t] \Leftrightarrow \neg\textbf{hold-in}(pro, t)$,
⟨t.7⟩ $\textbf{hold-in}[\text{not}(pro), t] \Leftrightarrow \neg\textbf{hold-on}(pro, t)$,
⟨t.8⟩ $\textbf{hold-for}\{\text{not}[\text{not}(pro)], t\} \Leftrightarrow \textbf{hold-for}(pro, t)$.

N.B. We omit the proofs of the above theorems in this paper since they are very straightforward and indeed will be very similar to the corresponding ones given in the appendix to Galton's paper (10).

It is interesting to note that, although intervals are taken in our theory as primitive, that is there are no definitions about the starting- and finishing-points for intervals, the time axiomatization allows the expression of the 'open' and 'closed' nature of intervals, by means of the following formal definitions:

interval i is **left-open** at point p iff
$dur(i) > 0 \wedge dur(p) = 0 \wedge \text{MEETS}(p, i)$;

interval i is **right-open** at point p iff
$dur(i) > 0 \wedge dur(p) = 0 \wedge \text{MEETS}(i,p)$;
interval i is **left-closed** at point p iff
$dur(i) > 0 \wedge dur(p) = 0 \wedge \exists i'[dur(i') > 0 \wedge \text{MEETS}(i',i) \wedge \text{MEETS}(i',p)]$;
interval i is **right-closed** at point p iff
$dur(i) > 0 \wedge dur(p) = 0 \wedge \exists i'[dur(i') > 0 \wedge \text{MEETS}(i,i') \wedge \text{MEETS}(p,i')]$.

In terms of the open and closed nature of intervals, we may given a formal and intuitive characterization for Galton's distinction between *states of position* and *states of motion* (see section 4), as below:

States of position can hold at isolated points; and if a state of position holds on an interval, then it must hold on the closure of that interval.
States of motion cannot hold at isolated points: the maximal interval on which a state of motion holds must be open.

7. ABOUT PROCESSES AND EVENTS

In addition to properties, in Allen's interval based system, *processes* and *events* are addressed as well, which are generally termed as *occurrences*. However, as Galton argues in his corresponding examination (10), by locating the distinction between *broad sense* and *narrow sense* (9) in processes rather than the time, it is possible to simplify the ontology by subsuming processes under properties. That is, it is unnecessary to introduce a category of processes separate from properties and events. Hence,. in this section we shall only consider some issues about events.

Allen introduces his additional predicate **OCCUR** with the intended meaning that **OCCUR**(e,i) is true only if the event e happens over the interval i, and there is no proper subinterval of i over which e happens. (In fact, we may also easily infer that there is no proper superinterval of i over which the event e happens.) Of course, in Allen's system, instantaneous events are definitely excluded, although they do occur in reality [examples are given in (10).

N.B. In Shoham's *reified interval logic* (17,18), a series of definitions have been given to categorise some entities, termed *temporal propositions*, with the intention of replacing Allen's trichotomy properties/events/processes by a more flexible scheme. However, in this reified logic, time intervals are defined in terms of pairs of time points. It is important to note that, to give a formal specification for the temporal relations between time intervals in terms of their ending-points, some careful consideration must be concerned (in particular, for the case that an interval is '*immediately after*', that is '*met-by*', another interval). Otherwise, some problems may appear when addressing whether the ending-points are included in intervals or not [see the problematic question illuminated by means of the 'LIGHT-ON/LIGHT-OFF' for example (3)].

In addition to his three different predicates **HOLDS-AT**, **HOLDS-IN** and **HOLDS-ON** for properties, Galton goes on to replace Allen's **OCCUR** by means of three predicates **OCCURS-AT**, **OCCURS-IN** and **OCCURS-ON**. The first of these is for locating an *instantaneous* (or termed *punctual*) event at the point at which it occurs, the second for locating an event (punctual or *durative*) in an interval within which it occurs, and the third is for locating a durative event which takes time on an interval over which it occurs (i.e., **OCCURS-ON** corresponds to Allen's **OCCUR**).

In our revised theory, we shall define the following type of statement:

occur(e, t), $t \in T$

as primitive, which locates the event e over the time element t, on which e happens.

N.B. Again, knowledge about whether the time element t refers to an interval or to a point may be added to the above statement.

The axiom for the predicate **occur** is

$\langle$o.1$\rangle$ $\mathbf{occur}(e,t) \Rightarrow \neg\exists t_1 \in T[\text{IN}(t_1,t) \wedge \mathbf{occur}(e,t_1)]$,

which, despite that it addresses time points as well, is very similar to Allen's (O.1) (3) or Galton's (O.2) (10).

In fact, Allen's (O.1) may be replaced with the following special form of $\langle$o.1$\rangle$ by means of simply limiting the set of time elements, T, to the set of time intervals I (i.e. time element i such that $dur(i) > 0$):

$\langle$o.1$\rangle'$ $\mathbf{occur}(e,i) \Rightarrow \neg\exists i_1 \in I[\text{IN}(i_1,i) \wedge \mathbf{occur}(e,i_1)]$.

Similarly, we can also define another predicates **occur-in**, for locating an event over a time element in which it occurs.

Formal definition for **occur-in** is:

$\langle$d.4$\rangle$ $\mathbf{occur\text{-}in}(e,t) \Leftrightarrow \exists t_1 \in T[\text{SUB-ELEMENT}(t_1,t) \wedge \mathbf{occur}(e,t_1)]$

N.B. In the extreme case where t is a point, since the 'SUB-ELEMENT' relation includes 'EQUAL' relation, we get that: $\mathbf{occur\text{-}in}(e,t) \Leftrightarrow \mathbf{occur}(e,t)$, where $dur(t) = 0$.

Note that our predicates **occur** and **occur-in** address both instantaneous and durative events: for an instantaneous event e, if we let $t \in P$ [i.e. $dur(p) = 0$], Galton's **OCCURS-AT**(e, p) can be simply taken as our **occur**(e, p); for a durative event e, if we let $i \in I$ [i.e. $dur(i) > 0$], again, Galton's **OCCURS-ON**(e, i) can be simply taken as our **occur**(e, i); and, for a general event e (instantaneous or durative), if we let $i \in I$ [i.e. $dur(i) > 0$], Galton's **OCCURS-IN**(e, i) can be simply taken as our **occur-in**(e, i).

From the definitions of **occur** and **occur-in**, we can straightforwardly prove the following theorems:

$\langle$t.9$\rangle$ $\mathbf{occur}(e,i) \wedge \text{SUB-ELEMENT}(i,i_1) \Rightarrow \mathbf{occur\text{-}in}(e,i_1)$
$\langle$t.10$\rangle$ $\mathbf{occur\text{-}in}(e,i) \wedge \text{SUB-ELEMENT}(i,i_1) \Rightarrow \mathbf{occur\text{-}in}(e,i_1)$

It is interesting to note that, from ⟨t.9⟩, ⟨t.10⟩ along with our ⟨o.1⟩ and ⟨d.4⟩, we can subsume Galton's (O.1)–(O.4), (O.5D), (O.5P), (T.16D) and (t.16P) (10).

8. THE EXPRESSIVE POWER OF THE NEW SYSTEM

In this section, by means of some critical examples, we will show that, on the one hand, approaches which address only time intervals, e.g. Allen's interval temporal logic, may be inadequate in reasoning about continuous change. On the other hand, approaches that relegate time points to a subsidiary status by means of addressing time points as the 'meeting places' or 'ending-elements' of time interval may lead back to the problem of modelling the open and closed nature of time intervals. The new theory proposed here retains many of the appealing characteristics of the systems of Allen, and of Galton, but without bearing their corresponding deficiencies as discussed in this paper. In fact, in a previous paper (14), we have shown that the general time axiomatization utilized here allows time structures such as linear time, non-linear time, dense time, discrete time, and so on. It is also proved to be powerful enough to subsume many existing temporal systems, such as the interval-based theories of Allen and Hayes, the point-based theories of Bruce, McDermott, and the interval- and point-based theories of Vilain, etc.

One important intuition which leads to Allen's interval-based logic is that most of human temporal knowledge, especially in the field of AI, is expressed without explicit reference to time points. As Allen argues again and again (1, 2, 3), if one insists on addressing the ending-points of time intervals, one must consider what knowledge one has at them about properties which are naturally associated only with the intervals. Allen's idea is therefore to take intervals as primitive, excluding the concept of points explicitly from the fundamental theory, and to maintain that only knowledge about properties associated with intervals is necessary. Hence, for example, the knowledge of the situation where a light is switched on can be represented by using an interval i to denote the time over which the light is off, and another interval j to denote the time over which the light is on [where interval j is immediately after interval i, i.e. MEETS(i, j)]. In Allen's notation, this may be expressed as:

HOLDS(*Light_Off*, i),
HOLDS(*Light_On*, j),
MEETS(i, j).

Knowledge about whether the light is on or off at the 'switching point' is not to be represented in this system, since we do not have any firm knowledge about the state of the system at this point.

However, as Galton has shown, excluding time points may lead to inadequacy in reasoning about continuous change. Also, in (14), we have illuminated the problem involved with reference to time points by means of the example of a ball thrown vertically into the air, and show that this situation cannot be satisfactorily expressed in terms of Allen's interval-based logic.

In order to overcome this inadequacy, Galton proposes his series of revisions to Allen' system to accommodate the representation of facts concerning continuous change. In terms of Galton's terminology, one can now express the situation of a ball thrown vertically into the air as:

HOLDS-ON(*Ball_Going_Up*, i),
HOLDS-AT(*Ball_Stationary*, p),
HOLDS-ON(*Ball_Going_Down*, j),
MEETS(i, j),
LIMITS(p, i),
LIMITS(p, j),

where both *Ball_Going_Up* and *Ball_Going_Down* are states of motion, while *Ball_Stationary* is a state of position.

However, as we have shown in section 6, Galton's revisions are achieved by insisting on a point between any two intervals that meet. Since in this case there must be a point between any two intervals, the question will exist as to the value of any property at this point. This leads back to the very problem that Allen tries to overcome: *viz* do properties ascribed to the intervals apply to the point or not? This question cannot be avoided. For example, in modelling the situation that a property *pro* holds on interval i, and the opposite property, not(*pro*), holds on interval j, where i meets j: if we use p to denote the point which LIMITS both i and j, then we must address the question whether *pro* or not(*pro*) holds at p. However, as Allen, and Galton have claimed in their papers, the answer to this question must be artificial and hence unsatisfactory (2,10).

Allen's system avoids this awkward question entirely by abolishing points. In the system proposed here we do not insist that there is a point between two intervals, so that the question does not apply in general. For instance, the above situation may be simply expressed as:

dur(i) > 0,
dur(j) > 0,
holds-on(*pro*, i),
holds-on[not(*pro*), j],
MEETS(i, j).

Here we have expressed exactly the knowledge about the two opposite properties, without being forced into expressing disputable knowledge about the property at the meeting point.

In fact, in order to ask a question about a property at a point, one must specify the point, in particular its relation to the two intervals in question. In the above case, the answer to the question becomes straightforward. In fact, there are only two cases that can be specified:

1. MEETS(i, j),
   MEETS(i, p).
2. MEETS(i, j),
   MEETS(p, j).

It is easy to see that case 1 specifies the knowledge that not(*pro*) holds at point p, and case 2 specifies the knowledge that *pro* holds at point p: in fact, in case 1, the situation may be expressed as:

dur(i) > 0,
dur(p) = 0,
dur(j) > 0,
holds-on(*pro*, i),
holds-on[not(*pro*), j],
MEETS(i, j),
MEETS(i, p).

which will straightforwardly imply that:

holds-on[not(*pro*), p],

and in case 2, the situation may be expressed as:

dur(i) > 0,
dur(p) = 0,
dur(j) > 0,
holds-on(*pro*, i),
holds-on[(**not**(*pro*), j],
MEETS(i, j),
MEETS(p, j).

which implies that:

holds-on(*pro*, p).

Additionally, in the new system, we may also model cases where there is knowledge of properties which hold on isolated points, which stand between intervals. For example, the situation of the ball thrown vertically into the air may be conveniently expressed by means of:

dur(i_1) > 0,
dur(p_1) = 0,
dur(j_1) > 0,
holds-on(*Ball_Going_Up*, i_1),
holds-on(*Ball_Stationary*, p_1),
holds-on(*Ball_Going_Down*, j_1),
MEETS(i_1, p_1),
MEETS(p_1, j_1).

Hence, on the one hand, the new system still retains Allen's solution which allows the expression of knowledge of properties over time intervals only. On the other hand, since points are now addressed as primitive time elements on the same footing as intervals, the system proposed here also allows for correct reasoning about isolated points, which are necessary for modelling continuous change.

9. CONCLUSIONS

A revised theory of action and time based on both intervals and points has been presented as an extension of J. F. Allen's interval-based theory, and A. P. Galton's corresponding revisions. The revised system utilizes a new axiomatization of time, given previously by the authors, as the underlying temporal basis. The problems which have arisen in Allen's theory of time and action stem from one of the main ideas of the theory; the avoidance of time points, as being problematic. However, it has been shown that excluding points may lead to inadequacy in correct reasoning of continuous motion. Galton's revised theory overcomes this deficiency, but insists on explicit expression of a point between any two intervals that meet each other. The new theory proposed here addresses both intervals and points as primitive time elements of equal standing. This approach retains Allen's solution to the ending-point question by allowing the expression of properties which are only known to hold over intervals. Hence the question of whether they hold over ending-points does not arise. However, the system proposed here also allows for reasoning about continuous change, as Galton has suggested, by including points on an equal footing with intervals.

The revisions that have been proposed, while radical, are in fact very much in the spirit of Allen's and Galton's systems. In addition to these fundamental revisions, a diversification of the range of properties/occurrences over intervals and points is also proposed in the paper, which may replace both Allen's and Galton's results.

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