# The Embedding of Meshes and Trees into Degree Four Chordal Ring Networks

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**In this paper a particular computer interconnection network, the symmetric chordal ring network of degree four, is presented, and the mapping of meshes and binary trees onto chordal ring networks is analysed. Expressions for the network diameter (the maximum distance a message must travel between any pair of computers) and the mean inter-computer distance are derived for a sub-set of chordal ring networks. Such networks incorporate the maximum number of computers for a given diameter, and have a communications cost, measured either as network diameter or as the mean internode distance, of $O(\sqrt{N})$. While these networks provide attractive properties for mesh-based applications on small- and medium-sized multicomputer systems, binary trees are restricted to five levels (31 nodes).**

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## 1. INTRODUCTION

The application of networks of computers (a processor plus local memory) to applications such as machine vision depends on a number of factors:

1. The combined processing power of the computers.
2. The efficiency with which data can be transferred into and out of the network.
3. The efficiency with which data can be transferred between computers in the network.
4. The manner in which the performance of the system degrades when components fail.
5. The efficiency with which applications map onto the network.
6. The fault-tolerance characteristics of the network (this is dealt with in a separate paper; see Browne, 1995).

Multicomputer networks can be categorized by the following criteria (Feng, 1981):

1. The control of the network may be centralized or distributed.
2. The interconnection topology may be static or dynamic.
3. Inter-computer communications may be established by sender and receiver prior to transmission, or packet switching may be employed.

Transputer-based multicomputers are based on degree four network topologies and such a system was used as the basis of the work reported in this paper. Image processing in general, and machine vision in particular, is a task that is well suited to parallel processing. Accordingly, the performance of the transputer networks discussed in this paper will be related to the requirements of image processing. The principal operations involved in image-processing can be classified as point operations, local (neighbourhood) operations and global operations (Jain, 1989). The four-way connectivity of arrays of transputers provides a conceptually convenient basis for neighbourhood operations since images can be readily segmented in a way which maps onto a flat array. This does not mean, however, that a flat mesh is necessarily the best topology for image-processing, since if global operations predominate in a particular application the mean inter-computer distance of the network will be the most important factor.

In this paper graph theory terminology will be mixed freely with the description of the equivalent hardware. Thus a *node* in a graph is the same as a computer in a multi-computer and an *edge* in a graph corresponds to an inter-computer link. These links will be considered to be bidirectional, so that the network graphs are *undirected.* Expressions for the network *diameter* (the maximum distance a message must travel between any pair of computers) and the mean internode (inter-computer) distance are derived for a set of optimal networks, which incorporate the maximum number of computers for a given diameter. The results of simulating the performance of non-optimal chordal ring networks will be reported.

Local image-processing operators can be catered for by providing four-way links between adjacent computers, forming a mesh. By connecting the computer at the end of each row with the computer at the beginning of the next row, and connecting the first and last computer, a *Hamiltonian* graph is formed. Define the inter-computer distance $d(i,j)$ as the minimum number of links which must be traversed to interconnect computers $i$ and $j$. For global operators, the total execution time will be minimized if the mean inter-computer distance is minimized. If each computer can proceed asynchronously the mean distance will be a sufficient criterion, but if the algorithm must be synchronized at intermediate points (for instance, for the passing of computed values between computers for subsequent processing) the diameter (the maximum inter-computer distance) may be of significance.

If an image processing system is being used in a real-time application (for instance, in an automated inspection system) the total processing time available per picture is dictated by the application. If through external changes the rate at which pictures are presented to the system is increased, or if changes in the algorithms involve an increased amount of processing, the system architecture may need to be extended to incorporate added computers. If this can be done incrementally and without a major redesign the architecture will be more acceptable as the basis for practical systems. In comparison, the inability of networks such as the hypercube to be extended by small increments of processing power is a distinct disadvantage.

For a system which is to handle both local and global image processing operators the criteria for selecting an optimum interconnection strategy may be summarized as follows:

1. The network diameter should be minimized.
2. The mean inter-computer distance should be minimized.
3. The system should be incrementally extensible.
4. Any single faults should not lead to a complete failure of the system.
5. Algorithms should be able to be implemented as simply as possible.

These requirements may lead to conflicting network descriptions, in which case the minimization of an objective function of these five factors would be required. This paper does not consider the fourth requirement, which is dealt with elsewhere (Browne, 1995).

The remainder of this paper is organized as follows. In the next section a class of optimal chordal ring networks is derived. In Section 3 the general mapping problem is presented. Meshes can be mapped onto a chordal ring in a number of ways, and these are presented and contrasted in Section 4. The mapping of binary trees onto chordal rings is analysed in Section 5.

## 2. OPTIMAL CHORDAL RING NETWORKS

Chordal ring networks of degree three, based on a fixed chord length, were proposed by Arden and Lee (1981). As part of a discussion of the $(d,k)$ graph problem (in which the number of nodes $n$ for a graph of degree $d$ and diameter $k$ is maximized) Doty (1984) has presented a generalization of the chordal ring network. However, for larger systems these networks lack the simplicity of those of Arden and Lee. Akers and Krishnamurthy (1986) have pointed out that solutions to the $(d,k)$ graph problem often ignore factors such as symmetry, ease of routing, and the structure of the graph.

McKeown (1985) has investigated chordal ring networks of fixed chord length (independent of the number of computers). The resultant networks are efficient in regard to local communication and have useful fault-tolerance, but a large system based on such a topology would be very inefficient if processing involved global references since the network diameter is of $O(n)$. Some of the properties of degree four chordal ring networks have been examined by Browne and Hodgson (1990) where the results presented in Section 2.2 of this paper were derived by a different technique.

Arden and Lee (1982) have investigated the properties of an alternative network, the multitree structured (MTS) graph. When drawn in circular form these graphs are seen to be related to chordal rings. Arden and Lee concentrated on MTS graphs of degree three, establishing bounds on the diameter. If applied to image processing, the mapping from an image onto the MTS graph nodes is unlikely to be simple.

This paper is concerned with chordal ring networks of degree four in which each node has two circumferential and two chordal links.

### 2.1. General description of chordal ring networks

Consider a ring of $N$ nodes (computers). These nodes will be referred to as nodes 0, 1, ..., $(N-1)$, and for convenience will be assumed to be in ascending order when the ring is traversed in a clockwise direction. An equivalent numbering system can be derived by traversing the ring in the reverse direction, i.e. $-1, -2, -3, ...$, so that $-1$ is the same as $(N-1)$. Each node is connected to its two nearest neighbours; thus node $i$ is connected to nodes $(i-1)$ and $(i+1)$. In particular node 0 is connected to nodes 1 and $(N-1)$. All arithmetic performed on node labels is modulo $N$.

In addition to the ring connections each node will have two auxiliary (or chordal) connections to other nodes. If a bidirectional link connects nodes $i$ and $j$, then the existence of the link is completely described if $i$ is said to be connected to $j$ or if $j$ is said to be connected to $i$. For convenience, each link is associated with just one node. Thus in the case in which each node has two auxiliary links (i.e. a total of four links, two being devoted to the ring), only one destination needs to be specified. Because of symmetry all nodes are equivalent and the network can be analysed in terms of the communication between node 0 and the other nodes in the network.

Let the chordal links of the network have a constant chord length (or displacement) $= d$. Then

$$i = (p_i d + q_i) \bmod N, \ p_i \text{ and } q_i \text{ integers,}$$

generates all the nodes $i$ in $[0, N-1]$. The number of chordal transfers is $p_i$ and the number of circumferential transfers is $q_i$. The number of full rotations involved in going from 0 to

$$i = (p_i d + q_i) \bmod N$$

is $(p_i d + q_i)$ div $N$ where mod' and div' represent the integer modulo and division operators.

The allowed transitions are such that either $p_i$ or $q_i$

(but not both) can change by one, corresponding to the inter-computer links. For example, $3d + 4$ can go to any one of $3d + 3$, $3d+5$, $2d + 4$ or $4d + 4$.

**Definition** The length $L_j$ of an internode link or connection from node 0 to node $j$ is the minimum number of edges of the graph that must be traversed.

The length is then given by

$$L_j = p_j + q_j.$$

The maximum of the set of minimal paths for the full network, that is the diameter, is given by the maximum of the set of values given by $L_j$.

### 2.2. A special class of chordal ring network

In this section a special class of network is derived from the condition that the network diameter should be a minimum. It will be shown that this is equivalent to minimizing the mean internode distance. A consideration of the way in which the list of chordal transfers is constructed provides a technique for structuring a network so that the diameter is minimized. Some of the results presented in this section have already been reported by Browne and Hodgson (1990) but the method used here is more general, with potential application to other network topologies.

Consider chordal moves from node 0. In the forward direction these will be at nodes $d$, $2d$, $3d$ and so on. Similarly, reverse chordal moves will be at nodes $N\text{-}kd$, $N - (k - 1)d$, $N - (k - 2)d$ and so on (see Figure 1). These locations can be tabulated as shown in Table 1.

One strategy for moving to any given node from node 0 is to go to the nearest node reachable by chordal moves only and then to proceed by circumferential moves. Of interest is the minimum number of moves that must be made to reach any given node from node 0.

Referring to the $j$th entry in Table 1 as $(c_j, L_j)$, and the following entry as $(c_{j+1}, L_{j+1})$, the lengths of $L_j + 1$, $L_j + 2$, ... are generated by moving circumferentially up from $c_j$. Similarly, the lengths $L_{j+1} + 1$, $L_{j+1} + 2$, ... are generated by moving circumferentially down from $c_{j+1}$. Let these two sequences of moves in opposing directions around the ring meet at some point $(c_{jm}, L_{jm})$, so that this point is midway between $(c_j, L_j)$ and $(c_{j+1}, L_{j+1})$. This point is at a distance $c_{jm} - c_j$ beyond $(c_j, L_j)$, so that to get to this point from below the distance is $L_{jm} = L_j + (c_{jm} - c_j)$. Similarly, to get to this point from above requires moving a distance $c_{j+1} - c_{jm}$ back from $L_{j+1}$, giving $L_{jm} = L_{j+1} + (c_{j+1} - c_{jm})$. Thus

$$2c_{jm} = c_j + c_{j+1} + L_{j+1} - L_j$$

and

$$2L_{jm} = c_{j+1} - c_j + L_j + L_{j+1}.$$

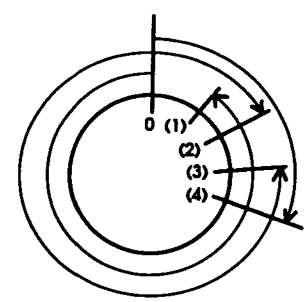


**FIGURE 1.** Node references: (1) $N - kd$, (2) $d$, (3) $N - 2kd$ and (4) $2d$.

**TABLE 1.** Locations of chordal moves

| *Chordal location (c)* | *Length (L) of move* |
|---|---|
| $N - kd$ | $k$ |
| $d$ | 1 |
| $N - (k - 1)d$ | $k - 1$ |
| $2d$ | 2 |
| $N - (k - 2)d$ | $k - 2$ |
| $3d$ | 3 |
| ... | ... |

The values of $2L_{jm}$ alternate between $(d + 1)(k + 1) - N$ and $K - kd + k$. Let these values be related by the expression

$$2K_a = 2K_b + K \tag{1}$$

where

$$2K_a = N - kd + k,\ 2K_b = (d + 1)(k + 1) - N \tag{2}$$

and $K$ is a constant for any given chordal ring. $K_a$ and $K_b$ correspond to the distance from node 0 to alternate mid-points between adjacent pairs of chordal moves. Clearly the maximum value of $K_a$ and $K_b$ is the diameter of the network, and the objective in this section will be to select values of $k$ and $K$ that minimize this value.

Combining (1) and (2), the following relationships are obtained:

$$k = \frac{2N - d - 1 - K}{2d}$$

$$K_a = \frac{1}{4}\left(d + K + \frac{2N - 1 - K}{d}\right)$$

$$K_b = \frac{1}{4}\left(d - K + \frac{2N - 1 - K}{d}\right).$$

Differentiating with respect to $d$, the minimum values for $K_a$ and $K_b$ both occur at

$$d^2 = 2N - 1 - K$$

Thus the minimum values of $K_a$ are $K_b$ are $(2d + K)/4$ and $(2d - K)/4$, respectively. Clearly the maximum of $K_a$ and $K_b$ is minimized when $K = 0$.

Therefore

$$N = 2k^2 + 2k + 1 \tag{3}$$

and

$$d = 2k + 1. \tag{4}$$

The network diameter is the maximum value of $L_{jm}$, which equals $k$. Thus

$$K_a = \frac{1}{2}\sqrt{2N - 1}$$

Since $K_a$ has been maximized for given $k$, the number of nodes $N$ is also a maximum for given $k$.

The sum of the minimum paths from node 0 to the nodes from $c_j$ to $c_{j+1} - 1$ is

$$S_j = \sum_{i=c_j}^{c_{jm}} (L_j + i - c_j) + \sum_{i=c_{jm}+1}^{c_{j+1}-1} (L_{j+1} - i + c_{j+1})$$

Substituting the expressions derived above:

$$S_j = 2j(k - j + 1) + k(k + 1)$$

The value of the index $j$ ranges from 1 to $k$. Summing over these values,

$$S = \frac{2k(k+1)(2k+1)}{3} \tag{5}$$

is the sum of the distances to all nodes from node 0. By symmetry, it is also the distance sum from any node. The mean internode distance is given by

$$\bar{D} = S/N \tag{6}$$

Thus minimizing $k$ is equivalent to minimizing $\bar{D}$.

An example of a chordal ring with $N = 13$, $d = 5$, $k = 2$ and $\bar{D} = 1.5385$ is shown in Figure 2.

Mean internode distance as a function of the number of nodes for general chordal ring networks (those that do not in general belong to the category defined by equations 1 and 2) is shown in Figure 3. For each network the chordal displacement was chosen to minimize the mean internode distance.

## 3. MAPPING OTHER NETWORKS ONTO CHORDAL RINGS

Sequences of operations to be performed on multicomputer systems commonly possess some fundamental pattern. Such patterns are usefully described by graphs, and the selection of multicomputer networks isomorphic to these graphs is intuitively the preferred approach. For instance, image processing, consisting inherently of matrix operations, maps easily onto meshes, while database searches map easily onto tree networks. The use of a chordal ring is not intuitive in either of these cases. In this and the next two sections the way in which this mapping can be achieved, and the resultant performance compared to the source network, will be studied with respect to two networks—the mesh in Section 4 and the tree network in Section 5. Graphs describing meshes and trees encompass the great majority of computation-intensive operations.

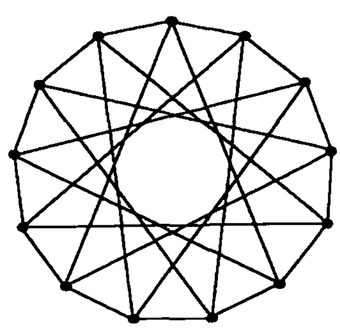

FIGURE 2.

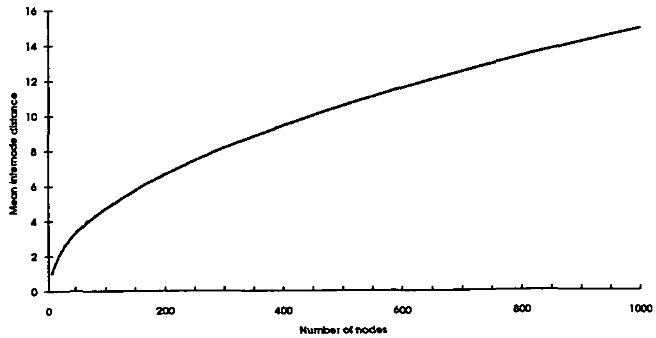


FIGURE 3.

Two types of mappings are defined as follows:

1. Direct: Given graphs $G = \{V,E\}$, $G' = \{V',E'\}$, then for every vertex $i$ in $G$ there exists a corresponding unique vertex in $G'$. The mapping of $G$ onto $G'$ is denoted by $V_i \rightarrow V_i'$. Furthermore,

$$V_i \rightarrow V_i' \text{ and } V_j \rightarrow V_j \Rightarrow E_{ij} \rightarrow E_{ij}'$$

2. Indirect: Given a graph $G = \{V,E\}$, $G' = \{V',E'\}$, then for every vertex $i$ in $G$ there exists a corresponding unique vertex in $G'$.

Note that $G$ and $G'$ are not, in general, isomorphic—$G$ is a subgraph of $G'$.

## 4. MAPPING MESHES ONTO CHORDAL RINGS

Define a mesh as a graph consisting of a set of nodes

$$P = \{(i,j),\ i \in [0, N_r - 1],\ j \in [0, N_c - 1]\}$$

and a set of edges

$$E = \{r_{i,j},\ i \in [0, N_r - 1], j \in [0, N_c - 2];$$
$$c_{i,j},\ i \in [0, N_r - 2],\ j \in [0, N_c - 1]\}$$

where the row' $r_{i,j}$ is the unordered pair $(p_{i,j}; p_{i,j+1})$ and the 'column' is the unordered pair $(p_{i,j};\ p_{i+1,j})$.

Define the communications cost as the mean number of nodes to be traversed in performing a given operation. In particular, the *global cost* is equal to the mean internode distance for the network, while the *local cost* is the mean distance to the four nearest neighbours in the mesh.

The direct mapping of meshes onto chordal rings is based on the selection of a chordal displacement equal to the number of columns $N_c$ in the array. The alternative of indirect mapping is less efficient for local (nearest-neighbour) references, but in some circumstances will result in an improved overall performance because of a better global efficiency. This is discussed in Section 4.2 where a comparison is made between direct and indirect mappings.

### 4.1. Direct mappings

Consider a mesh of $N_c$ by $N_r$ nodes. Consider a chordal

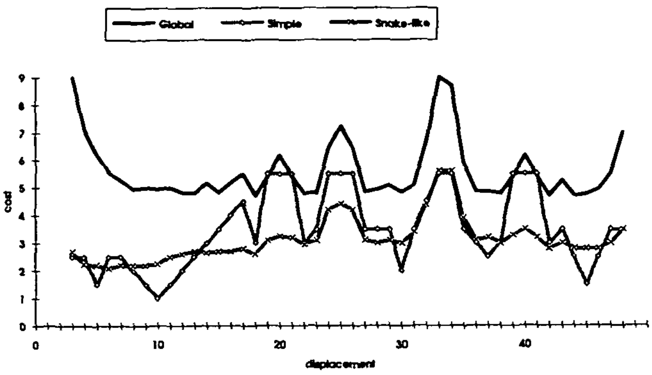


FIGURE 4.

ring of $N = N_c\ N_r$ nodes and a displacement of $N_c$. Define a mapping from node $(i,j)$ in the mesh to node

$$k = N_c i + j$$

in the chordal ring. The four edges incident on $k$ connect it to

$$\begin{aligned} &(k \pm 1) && \text{and } (k \pm d) \\ =&(k \pm 1) && \text{and } (k \pm N_c) \\ =&N_c i + (j \pm 1) && \text{and } N_c(i \pm 1) + j \\ =&(i, j \pm 1) && \text{and } (i \pm 1, j) \end{aligned}$$

which is a mesh (disregarding the additional edge connections). The local cost for communications is 1.0.

### 4.2. Indirect mappings

Two mappings from meshes onto chordal rings will be considered. Firstly, analogous to the simple direct mapping above, consider

$$k = N_c i + j$$

where in general the chordal ring displacement is not equal to $N_c$. This will be referred to as *simple* mapping. The alternative mapping to be reported here is a *snake-like* mapping, defined by

$$\begin{aligned} k &= N_c i + j && \text{for even-numbered rows,} \\ k &= N_c(i+1) - j && \text{for odd-numbered rows.} \end{aligned}$$

Such a mapping has been used by Miller and Stout (1985) for image processing on a mesh-connected computer. A comparison of simple and snake-like mappings with global mapping is shown in Figure 4, which deals with the case of a 10 by 10 mesh. The local cost based on simple mapping has a pronounced minimum at a displacement of 10, which corresponds to the case of direct mapping. For any particular application, if $\alpha$ is the proportion of the operations that are global then the total communications cost is

$$C = \alpha C_g + (1 - \alpha) C_l$$

in terms of global ($C_g$) and local ($C_l$) costs. In the case of the example in Figure 4, the requirements for $C$ to be minimized are shown in Table 2. Since the minimum value of $C_l = 1.0$ occurs when direct mapping is used, applications having a small value of $\alpha$ will have the smallest communications cost under direct mapping. The values of $\alpha$ above which alternative mappings are most efficient are shown in Figure 5 for the case of square meshes. For small arrays only applications with a large proportion of global operations will benefit from indirect mapping, but for meshes having more than about 25 rows and columns indirect mapping provides the best performance if global operations do not exceed 50% of the total.

**TABLE 2.** A 10 by 10 mesh mapped onto a chordal ring

| *Condition* | *Optimum displacement* | *Optimum mapping* |
|---|---|---|
| $\alpha < 0.73$ | 10 | direct |
| $0.73 < \alpha < 0.94$ | 45 | simple |
| $\alpha > 0.94$ | 44 | simple |

$\alpha$ is the proportion of operations that are global.

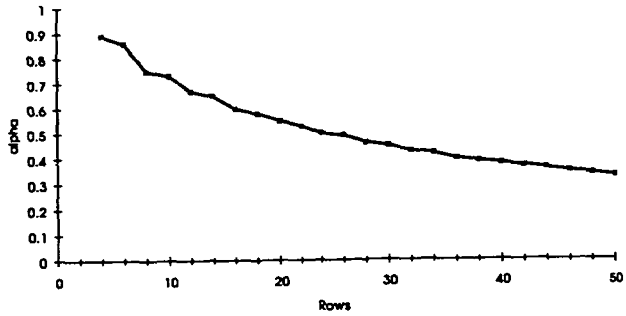


FIGURE 5.

The mean global distance (global cost) for four representative square meshes is illustrated in Figure 6. For large meshes there are clearly a number of local minima from which a suitable network displacement could be selected. The mean global distance for a square mesh of side $R$ is given by

$$\bar{D} = \frac{1}{N(N-1)} \sum_{x=0}^{R-1} \sum_{y=0}^{R-1} \sum_{i=0}^{R-1} \sum_{j=0}^{R-1} (|x - i| + |y - j|) = \frac{2}{3} R$$

where $N = R^2$. This is to be compared with the optimal mean global distances for the mappings shown in Figure 6. If equations (3), (5), and (6), are applied to the chordal ring of size $N = R^2$ (that is, regarding this as belonging to the special class of chordal rings), then

$$\bar{D} = \frac{(R^2 - 1)\sqrt{2R^2 - 1}}{3R^2} \approx \frac{\sqrt{2}}{3} R \quad \text{for large } R$$

The left-hand part of this formula is exact only for those in the class of special chordal rings, but it is accurate to within 1% for $R \geqslant 5$. The right-hand side of the formula proves to be a good approximation for all cases of $R \geqslant 10$. Thus

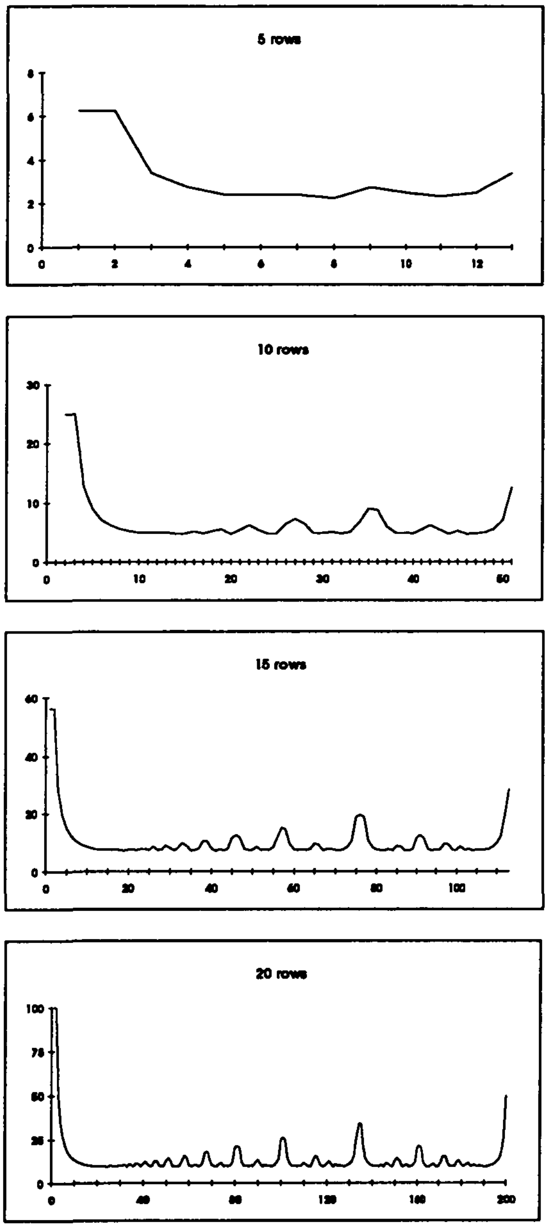


FIGURE 6.

$$\frac{\bar{D}_{\text{ChordalRing}}}{\bar{D}_{\text{Mesh}}} \approx \frac{\sqrt{2}/3}{2/3} = \frac{1}{\sqrt{2}} \quad \text{for } R \geqslant 10$$

so that the cost of global communications in algorithms running on meshes implemented on chordal rings will be just 70% of that incurred on a plain mesh architecture.

## 5. MAPPING BINARY TREES ONTO CHORDAL RINGS

Consider a binary tree of $L$ levels (labelled 0 to $L-1$). Any chordal ring containing the tree must have at least $2^L - 1$ nodes. Consider a direct mapping from the tree onto the chordal ring $(N,d)$, with $N \geqslant 2^L - 1$. Consider the number of nodes that can be accessed from the root node by traversing a specified number of edges. In a binary tree this is $2^L - 2$ nodes at the $L$-th level (that is, all nodes excluding the root node). Let the root node

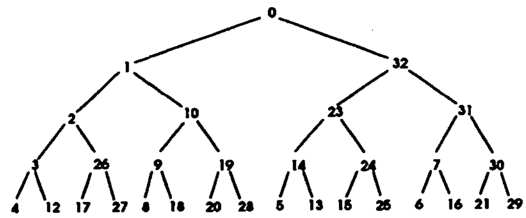

FIGURE 7.

correspond to node 0 in the chordal ring, and let the nodes at level $i$ map onto $pd + q$ ($p$ and $q$ integers).

If $i = 1$, the $(p,q)$ permutations are $(\pm 1,0)$ and $(0,\pm 1)$, a total of four selections. Similarly, for $i = 2$, the additional permutations are $(\pm 2,0)$, $(\pm 1,\pm 1)$, and $(0,\pm 2)$, which, when added to the original four selections, gives a total of twelve. In general, at the $i$-the level the additional permutations $(\pm(i-j),\pm j)$, $j = 0$ to $i$, are available, giving an overall total of $2i(i+1)$ selections. Thus the total number of nodes accessible within $i = L-1$ moves in the chordal ring is $2(L-1)L$. If the binary tree is to be able to map onto the chordal ring then the number of accessible nodes in the chordal ring must be at least as great as the number of accessible nodes in the tree. Thus

$$2(L-1)L \geqslant 2^L - 2 \Rightarrow (L-1)L \geqslant 2^{L-1} - 1 \Rightarrow L \leqslant 5$$

Therefore only trees of up to five levels can be directly mapped onto a chordal ring. A possible mapping for $L = 5$ is shown in Figure 7, based on a chordal ring with 33 nodes and a displacement of 9. Note that in the diagram the difference in the node numbers at each end of an edge is either 1 (i.e. adjacent along the circumference) or 9 (adjacent along a chord). The tree requires 31 nodes, but mappings of a 5-level tree onto chordal rings of either 31 or 32 nodes do not exist.

## 6. CONCLUSIONS

Symmetric chordal ring networks of degree four possess properties that make them attractive for small- and medium-sized multicomputer systems (networks of up to, say, 1000 nodes) for mesh-based applications. In particular:

1. Their symmetry means that the computers that form the network nodes are completely equivalent, thus simplifying the task of hardware implementation. A similar comment applies to the internode communications facilities and to the support software.
2. Their degree, i.e. four, makes them suited to implementation using transputers and to applications in areas such as image processing.
3. They are incrementally extensible, making the provision of additional computing power simply a matter of adding one or more extra processors.
4. Meshes map naturally onto chordal rings. Alternative mappings are available, but the efficiency will depend on the application.

However, applications involving binary trees are restricted to five levels (containing 31 nodes).

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