

level" with the aim of giving them "a reasonable grounding in numerical analysis and also to give them some understanding of how and when a computer can assist them in their work."

This, I believe, is only part of the battle. If engineers,

in particular, are not to be disappointed in their hopes, we must develop teams of engineering experts who have ready access to a computer, and who will make it their job to develop the necessary engineering programs and give advice and assistance in their use.

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## Correspondence

To the Editor,  
*The Computer Journal*.

Dear Sir,

Professor F. L. Bauer of the University of Mainz (private communication) has pointed out that a technique analogous to that we have described for the Givens process (*The Computer Journal*, Vol. 4, p. 177) may be applied to the Householder process.

The typical stage in the Householder process is the reduction of a matrix  $A_r$ , which is tri-diagonal in its first  $(r-1)$  rows and columns, to  $A_{r-1}$ , which is tri-diagonal in its first  $r$  rows and columns.  $A_{r-1}$  is defined by the relation

$$A_{r-1} = (I - 2w_r w_r^T) A_r (I - 2w_r w_r^T).$$

It has been shown (*The Computer Journal*, Vol. 3, p. 23) that if we define  $p_r$ ,  $K_r$  and  $q_r$  by the relations

$$\begin{pmatrix} A_r w_r & p_r \\ w_r^T p_r & K_r \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \end{pmatrix}$$

then

$$A_{r-1} = A_r - 2q_r w_r^T - 2w_r q_r^T.$$

The method suggested is that during the computation of  $A_{r-1}$ , which involves the reading of  $A_r$  from the backing store and its replacement by  $A_{r-1}$ ,  $w_{r+1}$  and  $p_{r+1}$  should also be computed.  $A_{r-1}$  differs from  $A_r$  only in rows and columns  $r$  to  $n$  and as soon as the  $(r-1)$  row of  $A_{r-1}$  is known,  $w_{r+1}$  can be computed. As each element of  $A_{r-1}$  is determined it can be used to compute the corresponding contribution to  $A_{r-1} w_{r+1}$ . When the calculation of  $A_{r-1}$

and its entry on the backing store is complete,  $w_{r+1}$  and  $p_{r+1}$  will be determined and hence  $K_{r+1}$  and  $q_{r+1}$  may be computed without further reference to the backing store. This technique halves the number of references to the backing store. (The gain is not so great as in the Givens process precisely because Householder's method is already very efficient).

It is usual to store only the upper triangle of each  $A_i$  and when this is done the computed  $(i, j)$  element of  $A_{r-1}$  must be used to determine the relevant contributions both to  $(A_{r-1} w_{r+1})_i$  and  $(A_{r-1} w_{r+1})_j$ . The working store must be able to accommodate the five vectors  $w_r$ ,  $q_r$ ,  $w_{r+1}$ ,  $p_{r+1}$  and the current row of  $A_r$  which is being transformed into a row of  $A_{r-1}$ . Further, if we are to realize the full accuracy obtainable by accumulating the inner products involved in  $p_{r+1}$  then each of its elements must be held in double-precision, since these inner products are built up piecemeal as  $A_{r-1}$  is computed. The requirement is then effectively that for six single-precision vectors.

Johansen (*J. Assoc. Comp. Mach.*, 1961, p. 331) has shown that the working storage requirements for the Givens process which we described can be reduced slightly. If transfers to and from the backing store are of  $k$  words at a time, then the requirement can be reduced from 4 vectors to 3 vectors plus  $k$  words. In a similar way the working storage for the Householder method can be reduced from 5 vectors to 4 vectors plus  $k$  words.

8 December 1961  
Oxford University Computing Lab.  
National Physical Laboratory.

Yours faithfully,  
J. S. Rollett.  
J. H. Wilkinson.