degrees of freedom. This fits the crude view that we have T_n observations to give us T_m results. Now in the usual way we can give confidence limits for the true smoothed power spectrum.

If we merely want to know if a peak in our results is significant, we can test against the hypothesis that the true $P(f)$ has no peak there.

We have also to consider the effect of the extra calculations involved in allowing for a possible non-zero mean. This is very like what happens when we estimate the variance of a population of unknown mean from a few samples: if we have k samples (degrees of freedom) the correct estimate has a factor $k/(k-1)$, which compensates for the loss of a degree of freedom in using the average of the samples to estimate the mean of the population.

In fact, the only change from the process of Section 4 is that we have subtracted off numbers roughly the same from each $C_3(r\delta t)$. This will alter V_0 substantially, but the other V_s only slightly. Speaking crudely, we may say that to compensate for the unknown mean will have led to a mis-estimation only of V_0. Now the total power in the record is measured by ΣV_s, and this has the nature of a variance. But if we have to estimate the variance of our set of $n+1$ readings we have to insert a factor $(n+1)/n$ to cope with the unknown mean, and we have to do this by a change in V_0 only. Hence before doing (4.4) we increase V_0 by $n^{-1}\Sigma V_s$.

Book Review

Adaptive Control Processes: A Guided Tour, by RICHARD BELLMAN, 1961; 255 pp. (Princeton University Press; London: Oxford University Press, 42s.)

Those who have not read Richard Bellman's previous volume, *Dynamic Programming*, will, if they go by the title, be surprised by the contents of the present work. Those who have will not. The theme of *Adaptive Control Processes* is the application of dynamic programming methods to the determination of optimal control policies for physical systems. The systems considered are divided into three classes, of increasing order of complexity. First is the fully determinate system, whose dynamic dependence on its own present state and an applied control vector is known. Within a given time interval, starting from a given state, it is desired to determine the control vector, as a function of time over this interval, which will maximize a stated criterion function. The author shows that solution by the calculus of variations, although applicable in principle, is subject to serious practical difficulties, concerned with the problems of relative maxima, of constraints on variables, and so on. Solution of dynamic programming is apparently the answer, given certain conditions which are often satisfied in practice.

The next class is the stochastic process, in which the dynamic dependence is known only in terms of probability distributions. Provided that we are satisfied with criteria stated in terms of expected values, such processes can be treated by entirely analogous methods.

The final class is the stochastic process in which the probability distributions are not known in advance, but must be estimated from results obtained as the process unfolds. This is what the author calls an "adaptive control process," and he shows that such processes may themselves be classified by hierarchy, in which, for instance, it may be known that the primary distribution belongs to a class (e.g. Gaussian) which can be characterized by parameters themselves belonging to a known distribution. And so *ad infinitum*. If this does not cover what everyone may mean by an adaptive control process, the reader must not complain. He will have read at least sixteen out of the eighteen chapters before discovering that the particular type of adaptive control in which he is interested is either not included in the definition or is in any case not amenable to practical solution by dynamic programming. It will have been well worth it.

The author hopes his readers will consist of "mathematicians and modern engineers, which is to say, mathematically trained and oriented engineers." Others are hereby warned off. Even many mathematicians and modern engineers will find it hard work, but it is a fascinatingly discursive exposition of a fascinating subject, written in a delightful style and rich with the author's inimitable humour. There are many intriguing sidelines, tantalisingly mentioned but not followed up. The treatment is indeed by the sweep of a broad brush, and many a reader will feel the lack of detailed, and preferably numerical, examples to assist his understanding. There is the usual sprinkling of misprints; the reference to a Chapter XIX which does not exist is perhaps the least important. A number of misprints in the mathematical sections add to the difficulties of a first reading, but may eventually cause the persistent reader to acquire a deeper understanding.

Does the author make his case for the use of dynamic programming methods in optimizing control? After disposing of the calculus of variations, he compares his method with a complete search for an optimum over all possible combinations of control. With a deterministic one-variable discrete process of N stages, with 100 possible values of control variable, the latter involves computation at 100^N points, the former at $100 \times N$. A big gain for dynamic programming. Unfortunately, $100 \times N$ is still a large number, especially as each computation involves finding, by trial, the maximum of a function over 100 points. As the author himself points out, an extension to a three-variable process takes us to the limit of practicability even with a large computer. As we go from deterministic to stochastic and adaptive systems, the order of complexity increases. Alas, the method is as yet applicable only to special cases, to which the low cunning of the mathematician, well expounded in many directions by the author, can be applied in mitigation of the formidable practical problems.

A first-class feature of the book is the extensive bibliography, with notes, at the end of each chapter. Serious students of the subject are strongly recommended to buy the book as a companion volume to *Dynamic Programming*. Other mathematicians and modern engineers should borrow it from a library, for general education and entertainment. Physicists and old-fashioned engineers should do the same, if only to read the reprint of M. G. Kendall's "Hiawatha Designs an Experiment" on pages 148 to 151.

R. H. TIZARD.