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## Book Review

*Theory of the Transmission and Processing of Information*, by A. G. VITUSHKIN. Translated from the Russian by RUTH FEINSTEIN, 1961, 206 pages. (London: Pergamon Press Limited, 100s.)

This is a fragment of the mathematical theory of approximation, following work of A. N. Kolmogorov and others.

The title "Theory of Transmission and Processing of Information" is misleading, being copied from the first sentence of the Foreword. This states that the monograph is connected with that theory; but the connection seems rather weak. A better title appears at the head of alternate pages of the text, viz. "Complexity of Tabulation Problems." The author asserts that the formal definition of the concept of the complexity of a tabulation problem (i.e. the construction of tables for functions) is required in the automatization of programming, but does not explain why.

The monograph is concerned with estimating the "complexity" of tables of certain general classes of functions. A "table" of a function  $f(x)$ , defined over a range  $G$  of values of  $x$ , is understood to mean an ordered set of quantities  $y_1, y_2, \dots, y_p$ , each represented to a finite accuracy of (say)  $n$  binary digits, together with a "decoding rule" whereby any value of  $x$  in the range  $G$  is used to enter the table and yield a value  $\phi(x)$ . This is to differ from  $f(x)$  by at most  $\epsilon$ , the "accuracy" of the table, for each  $x$  in  $G$ . This decoding rule is in general to be a polynomial in the variables  $y_1, \dots, y_p$ , of degree at most  $k$  in each one, but its coefficients may depend in any way upon  $x$ . As a very simple example,  $G$  might be

the range  $0 \leq x \leq p$  and  $f(x)$  might differ from the constant  $y_i$  by less than  $\epsilon$  in the range  $i - 1 \leq x \leq i$ , for  $i = 1, 2, \dots, p$ .

Then the polynomial could be  $\sum_{i=1}^p u_i(x) \cdot y_i$ , where  $u_i(x) = 1$  for  $i - 1 \leq x \leq i$  and  $u_i(x) = 0$  elsewhere, so that  $k = 1$  in this case. The numbers  $p$  and  $k$  are taken as measuring the "complexity" of the table, and the total number of binary digits employed in it, i.e.  $np$ , is called the "volume" of the table. Any set of values  $y_1, \dots, y_p$ , each of  $n$  binary digits will define some function  $\phi(x)$  when used with a fixed decoding rule, and  $n^p$  different such functions are possible, forming a "function space"  $\Phi$ . If a set  $F$  of functions  $f(x)$  and a number  $\epsilon$  is given, there will be a certain smallest number  $N$  of functions  $\phi(x)$  which form an " $\epsilon$ -net" of  $F$ , i.e. which contain for any  $f$  in  $F$ , a representative  $\phi$  serving to approximate within  $\epsilon$  to  $f$  throughout  $G$ . Then  $\log_2 N$  is called the " $\epsilon$ -entropy" of the set  $F$  with respect to  $\Phi$ , and it is shown that this is (within 1 unit) the volume of the smallest possible table for functions  $f$  in  $F$ .

The  $\epsilon$ -entropy depends on the properties assumed for the functions  $f(x)$  which comprise  $F$ , as well as on the set  $\Phi$  of approximating functions. An "absolute"  $\epsilon$ -entropy is defined which is the lower bound of the above entropy with respect to all possible sets  $\Phi$ . Estimates are obtained for the absolute  $\epsilon$ -entropy for some general subspaces of analytic differentiable functions of one or several variables.

These results appear to be of limited interest to users of digital computers.

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