only determined non-zero element of b_{r+1} is computed, then we may replace elements $2r$, $2r + 1$ of b_1 and repeat this last part of the computation. Now if the cancellation is of an accidental nature, then changing the $2r + 1$ element of b_1 substantially will avoid this cancellation unless both $a_{2r,\,2r}$ and $a_{2r,\,2r+1}$ are very small.

24. As an example we take the matrix

$$\begin{bmatrix} 0\cdot1234 & 0\cdot2415 & & & \\ 0\cdot3123 & 0\cdot5163 & 0\cdot2167 & & \\ 0\cdot2157 & 0\cdot1143 & 0\cdot9121 & -0\cdot2806 & \\ 0\cdot3164 & 0\cdot2876 & 0\cdot5143 & 0\cdot3176 & 0\cdot4123 \\ 0\cdot4157 & 0\cdot3164 & 0\cdot4135 & 0\cdot6125 & 0\cdot3123 \end{bmatrix} \quad (36)$$

Taking the first three components of b_1 to be 1, 0, 0 we can proceed as far as the second element of b_2. No cancellation takes place and hence we accept 1, 0, 0 as satisfactory and add two more zeros. The vectors b_1, b_2 and Ab_2, as far as they may be calculated, are

This technique obviously has its limitations. Perhaps the greatest of these is that it cannot deal with any instabilities which arise in the vectors after $b_{\frac{1}{2}n}$, since by this stage all components of b_1 already have assigned values. A second weakness is that when the ith element of b_i is small it does not necessarily imply that there is going to be an instability, since all non-zero elements of b_i may be small. Nevertheless, on a desk computer it is well worth taking advantage of this facility. The method has in fact proved successful on a matrix of order 9 which was reduced on a desk machine by the author. There was an instability at the fourth stage and this was removed by changing the seventh element of b_1. Extreme instabilities are most common in the first few stages because it is more likely that the numbers arising in these stages are "special." Perhaps the main value of the technique is that it gives insight into the significance of the initial vector.

b_1	b_2	Ab_2	$(Ab_2 - h_{12}b_1)$	$(Ab_2 - h_{12}b_1 - h_{22}b_2)$	
1	0	0·2415	0	0	
0	1·0000	0·6660	0·6660	0	
0	0·6907	0·4600	0·4600	$(0\cdot4600 - 0\cdot6907 \times 0\cdot6660) < 10^{-4}$	(37)
0	1·0131				
0					

When the appropriate multiples of b_1 and b_2 are subtracted from Ab_2 the third element, which should be non-zero, is less than 10^{-4} as a result of cancellation. If we now alter the fifth element of b_1 to 1 then, of the elements which have been computed, only the fourth of b_2 and the third of Ab_2 have to be changed. There is now no cancellation and, in fact, the whole process may be completed without instabilities.

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Book Reviews

Statistical Analysis and Optimization of Systems, by E. L. PETERSON, 1961; 190 pages. (London: John Wiley and Sons Ltd., 74s.)

This book gives a compact treatment of control systems which are subject to random inputs or noise. The subject is a highly mathematical one, but the treatment here is simplified as far as possible. This is wise in an engineering text, though it does conceal some difficulties which become important in applications. There is, for example, no discussion of the problem of estimating statistics from records of finite length.

The approach used is fundamentally the one associated with Norbert Wiener. The state-vectors, dynamic-programming approach, which is conceptually simpler, is introduced only in the last chapter. Examples are taken chiefly from the military field and the book will probably be of most interest to young engineers already working on such problems. Production and illustrations are both admirable.

H. H. ROSENBROCK.